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Small fatigue crack propagation in notched components under combined torsional and axial loading

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Abstract

The present paper deals with two specific subjects related to the fatigue strength and life of notched bars under combined torsional and axial loading. The first subject is the fatigue thresholds of materials with small defects. The fatigue threshold of materials with small defects or sharp notches is not controlled by the initiation of fatigue cracks, but by their propagation. The R-curve method is very useful to predict the fatigue thresholds of notched components. Small cracks nucleated at the notch root becomes nonpropagating when the applied stress intensity factor drops below the resistance of the material. It is important that the R curve is independent of loading conditions and only the applied stress intensity factor depends on loading conditions. In the present paper, the R-curve method was successfully applied to predict the fatigue thresholds of holed tubes made of carbon steels under in-phase and out-of-phase combinations of cyclic torsion and axial loading.

The second subject is an anomalous phenomenon of notch strengthening found in torsional fatigue of circumferentially notched round bars of austenitic stainless steel. In torsional fatigue of circumferentially notched bars of austenitic stainless steel, the fatigue life of notched bars was found to be longer than that of smooth bars and to increase with increasing stress concentration under the same amplitude of the nominal torsional stress. On the basis of the electrical potential monitoring of the initiation and propagation of small cracks at the notch root, the crack initiation life decreased with increasing stress concentration, while the crack propagation life increased. The anomalous behavior of the notch-strengthening effect was ascribed to the larger retardation of fatigue crack propagation by crack surface contact for the cases of sharper notches. The superposition of static tension reduced the retardation due to the smaller amount of crack surface contact, which gave rise well-known notch-weakening of the fatigue strength. The crack initiation life is controlled not by the peak strain amplitude at the notch root, but by the strain amplitude about 0.1mm distant from the notch root where the strain distribution was calculated by the elastic-plastic finite element method using cyclic stress-strain relation. The propagation rate of cracks can be predicted using the J-integral range when the static tension is superposed on cyclic torsion. The shielding by crack face sliding contact greatly reduced the crack propagation rate under cyclic torsion without static tension.

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1. Introduction

Fatigue fracture of several engineering components such as transmission shafts, pipes and springs occurs under combined torsional and axial loading. Notches or stress concentrations are the common site of crack initiation. The assessment of the notch effect on the fatigue strength and life is essential in fatigue designs. In comparison with axial fatigue, torsional fatigue studies are relatively limited, and the combination of torsional and axial loading further complicates the assessment of the fatigue strength and life. Since fatigue fracture results from the initiation and propagation of fatigue cracks, the understanding of small crack behavior greatly helps to improve the accuracy of the fatigue life prediction of notched components. The present paper deals with two specific subjects related to the fatigue strength and fatigue life of notched bars under combined torsional and axial loading.

The first subject is the fatigue thresholds of materials with small defects. The fatigue threshold of materials with small defects or sharp notches is not controlled by the initiation of fatigue cracks, but by their propagation[1,2]. After crack initiation, the fatigue crack first decelerates and then stops when the applied stress amplitude is below the fatigue threshold. Tanaka and Nakai [2] was first to show that the development of crack closure with crack extension was primarily responsible for crack deceleration and stoppage at notches. Tanaka and Akiniwa [3-5] have proposed the $R$-curve method for predicting the fatigue thresholds of notched components, and have shown a good agreement with the experimental results for uniaxial loading cases. In the present paper, the $R$-curve method is successfully applied to the fatigue thresholds of notched components of carbon steels under in-phase and out-of-phase combinations of cyclic torsion and axial loading [6,7].

The second subject is an anomalous phenomenon of the notch-strengthening effect found in torsional fatigue of circumferentially notched round bars of austenitic stainless steel [8,9]. The fatigue life under cyclic torsion becomes longer as the stress concentration increases. On the other hand, when the static tension was superposed on cyclic torsion, the fatigue life decreased with increasing stress concentration. This anomalous phenomenon was studied by dividing the total fatigue life into crack initiation and propagation lives.

2. Fatigue thresholds of notched components

2.1. Resistance-curve method

Our previous studies have shown that the effective stress intensity factor range, $\Delta K_{\text{effth}}$, at the crack stoppage takes a constant value irrespective of the notch geometry or stress amplitudes [3-5]. The threshold value of the maximum stress intensity factor, $K_{\text{maxth}}$, is obtained by adding the stress intensity factor at the crack-tip opening at the threshold, $K_{\text{opth}}$, to the $\Delta K_{\text{effth}}$ value as

$$K_{\text{maxth}} = \Delta K_{\text{effth}} + K_{\text{opth}} \quad (1)$$

The value of $\Delta K_{\text{effth}}$ can be assumed as a material constant independent of the defect geometry or loading condition. Once the change of $K_{\text{maxth}}$ with crack extension is known, the fatigue thresholds and the nonpropagating crack length can be determined on the basis of the $R$-curve method. The $R$-curve method is illustrated in Fig. 1, where the $K_{\text{max}}$ value is taken as the ordinate. The $R$-curve is drawn with the solid line and the applied $K_{\text{max}}$ value at a constant stress amplitude with the dashed lines. The fatigue threshold for crack initiation, $\sigma_{\text{a1}}$, is the stress amplitude corresponding to the applied $K_{\text{max}}$ value equal to the $\Delta K_{\text{effth}}$ at the Stage I crack length $c_1$. The fatigue threshold for fracture, $\sigma_{\text{a2}}$, is the stress amplitude where the applied $K_{\text{max}}$ curve is the tangent of the $R$-curve. At stress amplitudes between $\sigma_{\text{a1}}$ and $\sigma_{\text{a2}}$, the length of nonpropagating cracks is determined as the intersection of two curves. The effect of the stress multiaxiality on the fatigue threshold can be evaluated simply by changing the applied $K_{\text{max}}$ curve, because nonpropagating cracks are mode I cracks even under combined loading and the same resistance curve is applicable [6,7].

For the case of an annealed medium-carbon steel with 0.43 % C (JIS S45C), the fatigue threshold properties obtained in our previous studies [3,4] are as follows. The change of $K_{\text{opth}}$ (MPam$^{1/2}$) with crack length $c_{\text{np}}$ (m) is expressed by
where \( c_2 \) is the crack length at \( K_{\text{opth}} = K_{\text{opth},o} \), \( c_1 \) is given by

\[
c_1 = \left( \Delta K_{\text{elfh}} / 1.122 \sigma_{\gamma,0} \right)^{1/2} \pi
\]

and \( K_{\text{maxth}} = 5.26 \text{MPa.m}^{1/2} \), \( \Delta K_{\text{elfh}} = 2.94 \text{ MPa.m}^{1/2} \), and the fatigue limit of the smooth specimen is \( \sigma_{\gamma,0} = 223 \text{ MPa} \). The values of the characteristic crack lengths are \( c_1 = 0.044 \text{ mm} \), \( c_2 = 0.595 \text{ mm} \). To apply the method to combined loading, we have to know the direction of crack propagation and the stress intensity factor of cracks.

For the case of in-phase combined torsional and axial loading, the propagation direction of fatigue cracks emanating from a hole can be determined as follows. The applied nominal stress and shear stresses of tension-compression and torsion are given by

\[
\sigma = \sigma_0 \sin \theta, \quad \tau = \tau_0 \sin (\theta + \delta)
\]

where \( \tau_0 \) and \( \sigma_0 \) are shear and normal stress amplitudes, respectively, and \( \delta \) is the phase shift. For in-phase loading, \( \delta = 0 \), the circumferential stress around a hole shown in Fig. 2(a) is given by

\[
\sigma_\theta = \sigma + 2\sigma_0 \cos \theta - 4\tau \sin 2\theta
\]

where \( \theta \) is the angle measured counter clockwise from the plane perpendicular to the loading axis. For the nominal stress, the stress components, \( \sigma_{\theta,0} \), \( \sigma_\tau \), and \( \tau_{\theta,0} \), are expressed by (see Fig. 2(b))

\[
\sigma_\theta = \frac{\sigma_0}{2} \pm \frac{\sigma}{2} \cos 2\theta - \frac{\tau}{2} \sin 2\theta
\]

\[
\sigma_\tau = \frac{\sigma_0}{2} - \frac{\sigma}{2} \cos 2\theta + \frac{\tau}{2} \sin 2\theta
\]

\[
\tau_{\theta,0} = \frac{\sigma_0}{2} \sin 2\theta + \frac{\tau}{2} \cos 2\theta
\]

From Eqs. (5) and (6), we have

\[
\overline{\sigma}_\theta = 4\sigma_\theta - \sigma
\]

For in-phase loading, \( \delta = 0 \), the direction of the maximum range of the tangential nominal stress \( \sigma_\theta \) is coincident with that for the tangential (circumferential) stress \( \sigma_\theta \) on the periphery of the hole as easily known from Eq. (7).

On the periphery of the hole, the stress state is uniaxial and fatigue cracks are expected to initiate at the angle where the range of the tangential stress takes the maximum. According to the crack propagation simulation based
on the criterion of the maximum range of near-tip tangential stress, the nucleated fatigue propagated straight along the radial direction of the hole [10]. As described later, the experimentally observed path of nucleated fatigue cracks agree with the prediction [6,7,11].

For the case of out-of-phase loading, the fatigue crack may initiate at the position of the maximum range of the tangential stress on the periphery of a hole stress, and is expected to propagate gradually turning to the direction perpendicular to the maximum range of the principal nominal stress. We denote the former position by $\theta_1$ and the latter direction of the principal nominal stress by $\theta_T$. Two directions are identical for the case of in-phase loading, while they are different for out-of-phase loading. Table 1 presents the angles, $\theta_1$ and $\theta_T$, for the cases of $\delta = 0^\circ, 45^\circ, 90^\circ$ where the experiments were conducted as described later. The experimental results show that fatigue cracks initiate at the angle of $\theta_1$ and propagate fairly straight along the radial direction of the hole for any value of the phase shift between axial and torsional stresses.

For mode I crack, the stress intensity factor was obtained from the finite element analysis of cracked tubular specimen combined with the correction factor for cracks emanating from a hole calculated by using the body force method [11]. The maximum value of the stress intensity factor during cycling was calculated for cracks emanating from the hole, and the fatigue thresholds for crack initiation and propagation were determined using the $R$-curve method described in the preceding section.
Figure 3 shows the $R$-curve method for the case of out-of-phase loading with and $\omega_a/V_a=1$. The solid line is the $R$ curve expressed by Eqs. (1) and (2), and the dashed and dot-dashed curves indicate the change of the maximum stress intensity factor at the fatigue thresholds, $\sigma_{w1}$ and $\sigma_{w2}$, for crack initiation and fracture, respectively.

The thresholds for crack initiation and fracture for combined loading with phase differences of $\delta = 0^\circ$, $45^\circ$, and $90^\circ$ are shown in Fig. 4, where the dotted lines indicate the threshold for crack initiation and the solid lines for fracture. The in-phase loading gives the lowest threshold values; the combined loading with $\delta = 90^\circ$ gives the highest fatigue thresholds both for crack initiation and fracture.

Fig. 3. $R$-curve predicting fatigue thresholds for $\delta = 90^\circ$, $\tau_0/\sigma_y=1$

Fig. 4. Prediction of fatigue threshold under combined loading.
2.2. Experimental procedure

The material in the present experiment was a medium-carbon steel (JIS S45C) used for mechanical components. The chemical compositions of the material were as follows (mass%): C0.43, Si0.19, Mn0.81, P0.022, S0.02, Cu0.01, Ni0.02, Cr0.14. A tubular specimen had an outer diameter of 16 mm and an inner diameter of 14 mm. A through thickness hole with a diameter of 0.5 mm was drilled in the middle of the specimens. All the specimens were annealed for stress relief at 923 K for 1 hr before fatigue testing. The microstructure of the material was composed with ferrite and pearlite structure. The ferrite grain size is 18 μm. The lower yield strength was 298 MPa, the tensile strength was 570 MPa, Young’s modulus was 216 GPa, and Poisson’s ratio was 0.279.

Fatigue tests were conducted in a computer-controlled electro-servo hydraulic tension-torsion fatigue testing machine (Shimadzu EHF-ED 10TQ-40L). The loading conditions were cyclic torsion, cyclic axial tension-compression, and the combination of cyclic torsion and tension-compression either in-phase or out-of-phase. The cyclic stress was completely reversed without mean stress. The stress wave form was sinusoidal and the frequency was 20 Hz. The tests were conducted in air at room temperature. The propagation of fatigue cracks from a hole was observed by using an optical microscope. Plastic replicas were also taken from the specimen and the crack length was measured with an optical microscope at magnifications of 50 to 100. The crack was assumed to be nonpropagating when the propagation rate was below 10^{-11}m/cycle.

2.3. Crack propagation behavior

For the cases of δ = 0° and 45°, the directions of $\bar{\theta}$ and $\theta_s$ of the maximum range of the principal stresses of the periphery stress and the nominal stress are identical as described above. Figure 5 shows fatigue cracks for the $\delta = 45°$ under the shear stress amplitudes $\tau_a=75$MPa and under $\tau_a=85$MPa. The former crack is nonpropagating and the latter propagating. For both cases, the crack path is nearly straight following the radial direction of the hole. The angle of crack path is nearly equal to the predicted angle presented in Table 1. Figure 6(a) and (b) show the cracks under two stress levels for the case of $\delta = 90°$. The crack path is nearly straight for $\tau_a/\sigma_y=2$, and is slightly curved for $\tau_a/\sigma_y=0.7$. In both figures, the cracks are numbered from 1 to 4 as indicated in an inserted illustration.

![Fig. 5. Optical micrographs of cracks emanating from hole for $\delta = 45°$.](image-url)
The experimentally observed crack path for the case of \( \delta = 90^\circ \) is compared with the predicted angles \( \theta_1 \) and \( \theta_1 \) in Fig. 7. For \( \tau_n / \sigma_n = 2 \), \( \theta_1 \) and \( \theta_1 \) are about the same (see in Table 1), and nearly equal to the experimentally observed path as shown in Fig. 7(a). For \( \tau_n / \sigma_n = 0.7 \), two angles are different as shown in Fig. 7(b). Cracks are initiated at the angle \( \theta_1 \) and begin to propagate following the radial direction. As they propagate, the direction tends to turn to the horizontal direction. When the cracks are small, the direction is nearly straight. In the prediction of the fatigue threshold for propagation and nonpropagation of fatigue cracks, the direction is assumed equal to \( \theta_1 \).

At low stress amplitude, cracks were arrested after initiation and became nonpropagating. At higher stresses, the crack propagation rate dropped after some extension and then increased, or cracks continued to propagate. Both propagation and nonpropagating of small cracks were observed for in-phase as well as out-of-phase loadings.
2.4. Fatigue thresholds

The experimental data of the threshold stresses for three cases of \( \delta = 0^\circ, 45^\circ \) and \( 90^\circ \) are shown in Fig. 8(a). The open circles indicate the stress levels under which no cracks were observed, the half-open circles correspond to nonpropagating cracks and the solid circles to fracture. A very good agreement was obtained for in-phase loading. It was also found that the measured length of nonpropagating crack had some scatter and the maximum length agreed fairly well with the predicted line [6].

Figures 8(b) and (c) show the data for the cases of out-of-phase loadings with the phase differences of \( \delta = 45^\circ \) and \( 90^\circ \), respectively. For the case of \( \delta = 45^\circ \), an agreement between prediction and experiment is very good. On the other hand, for the case of \( \delta = 90^\circ \), the threshold for fracture is below the prediction, and lies close to the crack initiation limit. No nonpropagating cracks were observed. This may come from the reduction of the crack closure due to the rubbing of the crack surfaces by mode II shear component during cycling as discussed next.

Because of the change of the principal stress direction during cycling, the mode II shear stress is applied to fatigue cracks and reduces the crack closure. The shear stress range, \( \Delta \tau_{\theta}/\sigma_a \), which is the difference of the maximum and minimum of \( \Delta \tau_{\theta}/\sigma_a \) during one cycle can be obtained from the variation of the stress values in one cycle. By multiplying \( \Delta \tau_{\theta}/\sigma_a \) by the threshold stresses \( \sigma_{w1} \) and \( \sigma_{w2} \), the change of \( \Delta \tau_{\theta Max} \) is determined. Figure 9

![Fatigue thresholds under combined loading of torsion and tension-compression.](image)

Fig. 8. Fatigue thresholds under combined loading of torsion and tension-compression.
Fig. 9. Relation between shear stress range and stress-ratio parameter.

shows the variation of mode II with the stress ratio parameter $\psi$ for $\delta = 0^\circ$, $45^\circ$ and $90^\circ$. The shear stress range is the maximum for $\delta = 90^\circ$ and zero for in-phase loading, $\delta = 0^\circ$. The shear stress range is zero at $\psi = 0^\circ$ and $90^\circ$, and has a maximum at around $\psi = 30^\circ$. In the experiment, the debris made by rubbing was observed near fatigue cracks only for the case of $\delta = 90^\circ$. It is interesting to note that the shear stress is nearly zero at the maximum of the tangential stress, while non-zero during cycling. The sliding of crack faces takes place during stress cycling, but not at the maximum stress, reduces the crack closure.

The reduction of fatigue crack closure by mode II loading may influence the relation between the crack propagation rate and the maximum stress intensity, $K_{\text{max}}$, for cracks emanating from holes. The relation is shown in Fig. 10, for the case of combined loading with $\tau_a/\sigma_a=1$. In the figure, the solid and diamond marks mean the relation of the crack propagation rate against the maximum stress intensity factor and the effective stress intensity range, $\Delta K_{\text{eff}}$, obtained for long cracks [12]. The data for cracks from holes fall between the relations of the crack

Fig. 10. Relation between crack propagation rate and maximum stress intensity factor.
propagation rate against \( K_{\text{max}} \) and \( \Delta K_{\text{eff}} \) obtained for long cracks. When compared with the data for \( \delta = 0\,^\circ, 45\,^\circ \), the data for \( \delta = 90\,^\circ \) lie closer to the relation between the rate and \( \Delta K_{\text{eff}} \). This is because of the reduction of crack closer by mode II shear stress. The development of crack closure for \( \delta = 90\,^\circ \) is less than that assumed in the \( R \) curve, resulting in the experimental threshold stress is smaller than the prediction.

At higher stresses, cracks emanating from holes propagate with large plastic deformation in the vicinity of the crack tip, and the propagating rate is controlled by the \( J \) integral range [11].

3. Fatigue life of circumferentially notched steel bars under cyclic torsion

3.1. Experimental procedure

The material used for the experiment is hot rolling austenite stainless steel (JIS SUS316L) for nuclear power plant. The chemical compositions of the material are as follows (mass%): 0.009C, 1.26Mn, 12.13Ni, 17.59Cr, 2.09Mo. The specimen was machined from solution-treated hot-rolled bars in the direction whose longer direction coincides with the rolling direction. The diameter of the gage section of smooth specimens is 16mm (named SM-M specimen) or 10.5mm (SM specimen), and the length is 40 mm. Figure 11 shows the dimensions of notched specimens which have circumferential notches with three different root radii. The specimens with the radii 4.5, 1.07, and 0.22mm are named NA, NB, and NC specimens. All the specimens were finished by buffing using diamond past with the grain size 9\( \mu \)m followed by alumina powder of 1.0, 0.3, 0.1\( \mu \)m. The elastic stress concentration factor for the shear stress under torsion for NA, NB, and NC specimens was calculated by the finite element method to be 1.17, 1.55, and 2.54, respectively. Moreover, specimens which have 1.5 times larger dimensions of NA specimens, named NA-M specimens, are used for microscopic observation near the notch root.

Fatigue tests were performed with a tension-torsion biaxial electro-servo-hydraulic testing machine. Fatigue testing was done under load-controlled conditions with the load ratio \( R \) of -1. The wave form of the cyclic load was triangular and the frequency was 0.2-2.0Hz. Fatigue testing under cyclic torsion without static tension is called, case A and that with static tension case B. The shear stress amplitude expressed in terms of the nominal stress for the minimum cross section was set to be 160, 180 and 200MPa for case A, and the tensile stress applied in case B is the same as the applied amplitude of the shear stress. The direct-current electrical potential method was used for monitoring crack initiation and propagation. The current of 15A from a stabilized power supply was sent to the specimen through copper wires screwed to the section distant from the notch. Electric potential was measured through the nickel wire with 0.5mm in diameter which was spot welded at the upper and lower ends of the notch.

![Fig. 11. Shape and dimensions of notched specimens (dimensions are in mm).](image-url)
3.2. S-N relation

The relation between the shear stress amplitude and the number of cycles to failure for case A is shown in Fig. 12(a). It should be noted that the fatigue life becomes longer, as the notch gets sharper and stress concentration factor larger. This anomalous notch-strengthening is more evident for the cases of 180 and 160 MPa. The S-N relation for case B with a superimposed static tension is shown in Fig. 12(b). In this case, the life of notched specimens gets shorter for sharper notches. It is also interesting to note that the fatigue life of smooth specimens at the stress amplitudes of 180 and 200MPa becomes longer for case B than for case A. The strain range on the surface of smooth specimens becomes smaller when the static tension is superposed as in case B, because of strain hardening of austenitic stainless steels.

3.3. Change of electrical potential during fatigue

The change in the electrical potential under cyclic shear stress amplitude 180MPa without static tension, case A, is shown in Fig. 13(a), where the electrical potential was normalized by the initial value. The number of stress cycles at point where the electrical potential shows a rapid increase is considered to correspond to crack initiation. It is interesting to note that NC specimen shows the earliest rise in the electrical potential, followed by NB specimen and NA specimen. The crack initiation life is shorter as the notch gets sharper. However, the rising speed of the electrical potential is slowest for NC specimen. The potential of NB and NA specimens catches up the potential of NC specimen. The final fatigue life is longer in the order of NA, NB, and NC specimens. In conclusion, the crack initiation life is shorter while the crack propagation life longer as the notch becomes sharper. Similar results were obtained for the case of the stress amplitudes of 160 and 200MPa.

When a static tension is superposed on cyclic torsion, in case B, both crack initiation and propagation lives were shorter as sharper notches. Figure 13(b) shows the change of the electrical potential during fatigue under 180MPa for case B. The rising point of the electrical potential and the final fracture are shorter for the sharper notches.

Table 2 summarizes the crack initiation life, \( N_c \), and the total life, \( N_f \), for SM-M, NA, NB, NC specimens for cases A and B. For SM-M specimens, the compliance method was used to determine the crack initiation [9]. For case A, the ratio of \( N_c/N_f \) decreases as the notch becomes sharper and the stress amplitude lower. When compared at the same stress amplitude, the crack propagation life decreases while the crack propagation life increases as the notch becomes sharper. Both lives increases with decreasing stress amplitude. For case B, the crack propagation life as well as the crack initiation life decreases with increasing notch sharpness. The ratio of \( N_c/N_f \) tends to decrease with increasing notch sharpness and decreasing stress amplitude. It is interesting to note the effect of superposed

![Fig. 12. S-N relation for circumferentially notched bars of austenitic stainless steel bars.](image)
Fig. 13. Change of electrical potential during fatigue under cyclic torsion with and without static tension.

Table 2(a). Fatigue test results for case A, cyclic torsion.

<table>
<thead>
<tr>
<th>Specimen number</th>
<th>Stress amplitude (MPa)</th>
<th>Number of cycle to fracture, Nf</th>
<th>Number of cycle to crack initiation, No</th>
<th>Number of cycle for crack propagation, Np</th>
<th>Cycle ratio Nf/No</th>
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<td>SM-M1</td>
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<td>5.0x10^4</td>
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Table 2(b). Fatigue test results for case B, cyclic torsion with static tension.

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<th>Stress amplitude (MPa)</th>
<th>Number of cycle to fracture, Nf</th>
<th>Number of cycle to crack initiation, No</th>
<th>Number of cycle for crack propagation, Np</th>
<th>Cycle ratio Nf/No</th>
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<td>SM-M7</td>
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static tension on crack initiation and propagation in smooth specimens. The crack initiation life is longer for case B, while the crack propagation life is shorter. The superposition of static tension on cyclic torsion reduces the strain range on the surface, resulting in the elongation of the crack initiation life, and accelerating crack propagation by reducing contact of crack surfaces.

3.4. Fatigue crack initiation life

The relation between the stress amplitude and the number of cycles to crack initiation is shown in Fig. 14. For all stress amplitudes, the crack initiation life is shorter for sharper notched specimens. When compared at the same stress amplitude, the crack initiation life of notched specimens is slightly shorter for case B than for case A. For smooth specimens, the opposite is true for the stress amplitudes of 180 and 200 MPa.

The elastic-plastic stress-strain analysis was conducted for three notched specimens by the finite element method (FEM). The cyclic stress-strain relation was obtained by using hollow cylindrical specimens with an outer diameter of 16 mm and an inner diameter of 14 mm. The relation between the stress amplitude and strain amplitude is shown in Fig. 15, where the circles indicate the relation at the half fatigue life under strain-controlled tests, the triangles under stress-controlled test, and the squares are obtained in multiple-incremental step tests. The relation can be expressed the following relation between Mises equivalent stress \( \sigma_{eq} \) and strain \( \varepsilon_{eq} \) as

\[
\varepsilon_{eq} = \frac{\sigma_{eq}}{E} \quad (\sigma_{eq} \leq \sigma_Y)
\]

\[
\varepsilon_{eq} = \frac{\sigma_{eq}}{E} + \left( \frac{\sigma_{eq} - \sigma_Y}{k} \right)^m \quad (\sigma_{eq} \geq \sigma_Y)
\]

where Young’s modulus \( E = 195 \text{GPa} \), yield stress \( \sigma_Y = 129 \text{MPa} \), hardening modulus \( k = 1468 \text{MPa} \), and hardening exponent \( m = 2.89 \). Poisson’s ratio is \( \nu = 0.3 \). The above relation was obtained under completely reversed loading. When the static tension was superposed on cyclic torsion, the strain amplitude is reduced and the amount of reduction increases with increasing static tension [9]. This hardening by superposed static tension is responsible for the elongation of the crack initiation life of smooth specimens, resulting in the longer total fatigue life. The strain range at the notch root is also increased by superposed static tension, which may contradict to shorter crack initiation life for case B shown in Fig. 14. Some amount of crack propagation accelerated by static tension may takes place before the rising point of the electrical potential.

Using the constitutive relation of Eq. (8), the amplitude of the equivalent strain, \( \varepsilon_{eq}^* \), at the root of notched specimens was calculated by FEM, and is plotted against the number of cycles to crack initiation \( N_c \) in Fig. 16(a).
For case A. For the case of smooth specimens, the surface strain amplitude calculated by FEM was plotted as the ordinate. The figure also includes the relation between the strain amplitude and the total fatigue life obtained using hollow cylindrical specimens where the crack propagation life occupies only a small fraction of the total life. The crack initiation life is longer for sharper notches. The crack initiation process is not controlled by the maximum value of the strain amplitude, but by the strain amplitude at a certain structural distance from the notch root. The strain amplitude at 0.1 mm from the notch root obtained by EFM is plotted as the ordinate in Fig. 16(b). All data for notched specimens with different root radii merge to the same relation as for smooth specimens.

Fig. 15. Cyclic stress-strain relation for austenitic stainless steel SUS316L.

Fig. 16. Relation between strain amplitude at notch root and crack initiation life.
3.5. Crack propagation behavior

Figure 17 shows optical micrographs taken at the notch root of notched NA-M specimens whose fatigue tests were stopped at a normalized electrical potential of $V/V_0=1.1$ for cases A and B. The loading direction is vertical. The features of cracks seen at the notch root are distinctly different between cases A and B. For case A, cracks propagate in Stage II fashion branching from Stage I cracks which are vertical or horizontal, and the propagation direction of branched is 45 degree with respect to the loading axis. Stage I cracks extend out of the notch zone where the stress is low, and then decelerate as they extends. The connection of 45 degree cracks results in zigzag crack path, producing factory-roof appearance on the fracture surface as described in the following section. The sliding contact of the zigzag shaped fracture surface gives rise to the retardation of crack propagation [13,14]. When the static tension is superposed on cyclic torsion as in case B, Stage II cracks branching from Stage I cracks are connected horizontally, and the degree of tortuosity is lower than in case A, suggesting less amount of retardation of crack propagation. The superposition of static tension opens cracks and enhances the horizontal propagation [15].

The contact of crack surfaces can be detected from the torque vs twist-angle curve and the change of electrical potential during one loading cycle. Figure 18 shows the results obtained from NB specimen fatigued just before

![Figure 17. Optical micrographs of NA-M specimens fatigued under $\tau_a=180$MPa at $V/V_0=1.1$.](image1)

![Figure 18. Changes of twist angle and potential with applied torque for NB specimen under cyclic torsion $\tau_a=200$MPa at $N=8.40\times10^6$ (case A).](image2)
fracture under 200MPa for case A. Inflection points are observed on the hysteresis loop in the left figure. Points with the numbers from ① to ⑤ in Fig. 18 are explained as follows. Crack surfaces on the one side of factory roof are contacted at the minimum torque(③). When the torque increased to point ②, the crack begins to open and the electrical potential begins to increase. The crack entirely opens at ③ and begins to close again at ④ on the crack surfaces on the opposite side to those at ①. At the maximum torque (⑤), crack surfaces on the opposite side are strongly contacted. Figure 19 shows the results for case B. The width of hysteresis loop is larger and the crack closure point is not observed in the hysteresis loop. The variation in electric potential is very small, suggesting weak contact of crack surfaces during one cycle.

3.6. Fractography

After the fatigue tests, the specimens were broken by tension. Examples of SEM micrographs of notched specimens fatigued under cyclic torsion without static tension are shown in Fig. 20. The factory-roof shape becomes finer as the notch gets sharper as seen from the comparison among Fig. 20(b), (d), and (e). More number of crack nucleation sites may operate at the root of sharper notches because the strain amplitude is larger for sharper notches, and also narrow width of notches inhibits the propagation of Stage II cracks making 45 degree with respect to the loading axis. The shape of the factory roof also becomes finer with increasing stress level as seen from the comparison among Fig. 20(a), (b), and (c), or between (c) and (f). More crack nucleation sites operate as the stress amplitude increases, resulting in finer size of factory roof. At a high stress of 200MPa, the edge of the triangular shaped roof is rounded by rubbing between fracture surfaces as seen in Fig. 20(c) and (f).

When the static tensile stress is superposed on cyclic torsion, the appearance of the factory roof becomes less evident. Figure 21 shows examples of SEM micrographs of notched specimens fatigued under the stress level of 180MPa for case B. The fracture surface of NC specimen is smeared and the crack propagation takes place by mode III sliding at high stress as seen in Fig. 21(b), while at lower stress factory roof exists at some part of the fracture surface of NA specimen as seen in Fig. 21(a). When the stress amplitude is higher, factory roof becomes less evident and shows mode III sliding crack propagation. Whether the crack propagates in mode III fashion may be controlled by the amplitude of the shear displacement at the crack tip.
Fig. 20 Fracture surfaces of notched specimens fatigue under cyclic torsion (case A).

(a) NB, $\tau_s = 160\text{MPa}, \sigma_m = 0\text{MPa}$  
$N_f = 1.31 \times 10^9$

(b) NB, $\tau_s = 180\text{MPa}, \sigma_m = 0\text{MPa}$  
$N_f = 3.22 \times 10^9$

(c) NB, $\tau_s = 200\text{MPa}, \sigma_m = 0\text{MPa}$  
$N_f = 8.54 \times 10^8$

(d) NA, $\tau_s = 180\text{MPa}, \sigma_m = 0\text{MPa}$  
$N_f = 2.63 \times 10^8$

(e) NC, $\tau_s = 180\text{MPa}, \sigma_m = 0\text{MPa}$  
$N_f = 1.28 \times 10^8$

(f) NC, $\tau_s = 200\text{MPa}, \sigma_m = 0\text{MPa}$  
$N_f = 1.26 \times 10^8$

Fig. 21 Fracture surfaces of notched specimens fatigue under cyclic torsion + static tension (case B).

(a) NA, $\tau_s = 180\text{MPa}, \sigma_m = 180\text{MPa}$  
$N_f = 2.05 \times 10^8$

(b) NC, $\tau_s = 180\text{MPa}, \sigma_m = 180\text{MPa}$  
$N_f = 1.83 \times 10^8$
3.7. Fracture mechanics approach to crack propagation

The propagation behavior of cracks was estimated from the change of electrical potential. The crack is assumed to propagate from the root of circumferential notch toward the center of the bar concentrically. The relation between the crack length and the electrical potential was determined by the FEM analysis of the direct current electric field. Using this relation, the crack length was estimated from the potential, and the crack propagation rate was calculated by the secant method.

Figure 22 shows the change of the crack propagation rate with crack length for NA, NB, and NC specimens under the shear stress amplitude of 180MPa in cases A and B. For cyclic torsion, case A, the crack propagation rate is nearly constant at short crack lengths and increases as the crack length increases. For NC specimen, the crack propagation rate decreases at short crack length and then turn to increase after taking the minimum growth rate at the crack length of 1mm. When compared at the same crack length, the crack propagation rate is lower for sharper notches, and the amount of retardation is larger. In case B, the crack propagation rate increases with increasing crack length. When compared at the same crack length, the crack propagation rate is higher for sharper notches, corresponding to the higher stress intensity factor.

The stress intensity factor is not enough to characterize crack propagation because excessive plastic deformation is observed as suggested from the expanded hysteresis loop shown in Figs. 18 and 19. The $J$-integral range was estimated from the following equation [14,16]:

$$
\Delta J_{\text{III}} = \frac{1 + \nu}{E} (\Delta K_{\text{III}})^2 + \frac{3}{2\pi b^2} U_p
$$

(9)

where $E$ is Young’s modulus, $\nu$ is Poisson ratio, $\Delta K_{\text{III}}$ is the stress intensity range of mode III, $b$ is the radius of the ligament, and $U_p$ is the energy obtained from the curve between torque and twist angle. The stress intensity factor was calculated by EFM for concentric crack emanating from the notch root. Figure 23 shows the crack propagation rate plotted against the $J$ integral range, $\Delta J$. The solid line in the figure indicates the following relation between the rate $da/dN$ (m/cycle) and $\Delta J$ (N/m) obtained for stainless steel in our previous study [14]:

$$
da / dN = 7.70 \times 10^{-13} (\Delta J)^{1.41}
$$

(10)

For case B, the data for three types of specimens fall close to the above relation. For case A, all the data lie below the relation, indicating crack-surface contact shielding the crack tip and reducing the effective crack driving force. Similar results were obtained for the other stress amplitudes.

![Fig. 22. Change of crack propagation rate with number of stress cycles under cyclic torsion $\tau_0=180$MPa with and without static tension.](image)
4. Concluding remarks

The fracture of engineering components usually starts from notches or some stress concentration, and their fatigue life is the sum of crack initiation life and propagation life. The fatigue threshold or the fatigue limit is not controlled by crack initiation but by crack propagation, when stress concentrators are small defects or sharp notches. Understanding of the propagation behavior of small cracks in the vicinity of notches are especially significant in assessing the fatigue strength and fatigue life.

The R-curve method is very useful to predict the fatigue thresholds of notched components. Small cracks nucleated at the notch root becomes nonpropagating when the applied stress intensity factor drops below the resistance of the material. It is important that the R curve is independent of loading conditions and only the applied stress intensity factor depends on loading conditions. In the present paper, the R-curve method was successfully applied to predict the fatigue thresholds of holed tubes made of carbon steels under in-phase and out-of-phase combinations of cyclic torsion and axial loading.

In torsional fatigue of circumferentially notched bars of austenitic stainless steel, the fatigue life of notched bars was found to be longer than that of smooth bars and to increase with increasing stress concentration under the same amplitude of the nominal shear stress. This notch-strengthening effect is anomalous for the conventional fatigue design criterion. On the basis of the electrical potential monitoring of the initiation and propagation of small cracks at the notch root, the crack initiation life decreased with increasing stress concentration, while the crack propagation life increased. The anomalous behavior of the notch-strengthening effect was ascribed to the larger retardation of fatigue crack propagation by crack surface contact for sharper notches. The superposition of static tension reduced the retardation due to the smaller amount of crack surface contact, which gave rise well-known notch-weakening of the fatigue strength.

The crack initiation life is controlled not by the peak strain amplitude, but by the strain amplitude about 0.1mm distant from the notch root in the strain distribution calculated by the elastic-plastic finite element method using cyclic stress-strain relation. The propagation rate of cracks can be predicted using the J-integral range when the static tension is superposed on cyclic torsion. The shielding by crack face sliding contact greatly reduces the crack propagation rate under cyclic torsion without static tension.
References


