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Abelian p-groups with no invariants*

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Abstract

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We prove for abelian p-groups a non-structure theorem relative to approximations of Ehrenfeucht–Fraïssé games of length ω_1 in terms of linear orderings with no uncountable descending sequences. Our result shows that there is a group which is too complicated to be characterized up to isomorphism by the Ehrenfeucht–Fraïssé game approximated by a fixed ordering. This means that such a group cannot have any complete invariants which are bounded in the sense of these approximations of the Ehrenfeucht–Fraïssé game. On the other hand, all the approximations characterize together the notion of isomorphism among groups of cardinality at most ω_1. From the point of view of Stability Theory, our result concerns certain stable theories with NDOP and NOTOP.

1. Introduction

There is an elegant and well-understood way of approximating the relation of being isomorphic among countable structures. Let $\mathfrak{A}$ and $\mathfrak{B}$ be two countable structures. Recall first that $\mathfrak{A}$ and $\mathfrak{B}$ are isomorphic, if and only if they are partially isomorphic, i.e., there is a set I of isomorphisms between substructures of $\mathfrak{A}$ and $\mathfrak{B}$ which can be extended in I 'back and forth'. The latter relation is by

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Karp's theorem equivalent to the structures being elementarily equivalent in the infinitary logic $L_{\infty\omega}$,

$$\mathfrak{A} \equiv_{\infty\omega} \mathfrak{B}$$

(see [1] and [11]). The set of sentences of $L_{\infty\omega}$ can be filtered according to a notion of quantifier rank which assigns an ordinal to every formula. Thus to every ordinal α we have a relation

$$\mathfrak{A} \equiv^{\alpha}_{\infty\omega} \mathfrak{B} ,$$

which holds exactly when the structures $\mathfrak{A}$ and $\mathfrak{B}$ satisfy the same sentences of quantifier rank $\leq \alpha$. This means that the countable structures $\mathfrak{A}$ and $\mathfrak{B}$ are isomorphic, if and only if $\mathfrak{A} \equiv^{\alpha}_{\infty\omega} \mathfrak{B}$ holds for all α. By a refined version of Karp's theorem, $\mathfrak{A} \equiv^{\alpha}_{\infty\omega} \mathfrak{B}$, if and only if $\mathfrak{A}$ and $\mathfrak{B}$ are α-partially isomorphic (see [1]).

These ideas have also a game-theoretic expression. The notion of being partially isomorphic can be expressed in terms of the well-known *Ehrenfeucht–Fraïssé game*. In it, the two players $\forall$ and $\exists$ choose elements $x_0, x_1, \ldots$ and $y_0, y_1, \ldots$ from the two structures in alternating turns, $\forall$ beginning. When $\forall$ has chosen x_n from one of the structures, $\exists$ has to reply with y_n from the other structure. Thus a play of the game creates a sequence $a_0, a_1, \ldots$ of elements of $\mathfrak{A}$ and a sequence $b_0, b_1, \ldots$ of elements of $\mathfrak{B}$. Player $\exists$ *wins*, if $a_n \mapsto b_n$ generates an isomorphism between some substructures of $\mathfrak{A}$ and $\mathfrak{B}$; otherwise $\forall$ *wins*. It is easy to see that $\mathfrak{A}$ and $\mathfrak{B}$ are partially isomorphic exactly when player $\exists$ has a winning strategy in the Ehrenfeucht–Fraïssé game between $\mathfrak{A}$ and $\mathfrak{B}$.

This game becomes more interesting when we observe that it easily yields a game-theoretic interpretation for Karp's characterization of elementary equivalence up to given quantifier rank. Indeed, $\mathfrak{A} \equiv^{\alpha}_{\omega_1\omega} \mathfrak{B}$, if and only if player $\exists$ has a winning strategy in the following approximation of the Ehrenfeucht–Fraïssé game which we call the α-game. In the *α-game* the players make the same moves as in the Ehrenfeucht–Fraïssé game with the addition that simultaneously with every move x_n of $\forall$, $\forall$ also has to give an element t_n of α in such a way that the elements t_n form a descending sequence. This means especially that x_n is chosen only if t_n is chosen, and therefore the game ends when t_n no more can be chosen. So every play of the α-game is necessarily finite.

These observations give the fact that the countable structures $\mathfrak{A}$ and $\mathfrak{B}$ are isomorphic, if and only if for every countable ordinal α, player $\exists$ has a winning strategy in the α-game between $\mathfrak{A}$ and $\mathfrak{B}$. Also it is not difficult to give direct proof of this.

There is even more to say. For every countable structure $\mathfrak{A}$ (over a countable language) there is by Scott's theorem a countable ordinal α which satisfies for every structure $\mathfrak{B}$ that if player $\exists$ has a winning strategy in the α-game between $\mathfrak{A}$ and $\mathfrak{B}$, then $\mathfrak{A}$ and $\mathfrak{B}$ are isomorphic. The smallest such α is called the *Scott* *height* of $\mathfrak{A}$. Moreover, every countable structure $\mathfrak{A}$ has in $L_{\omega_1\omega}$ a *Scott sentence* $\sigma(\mathfrak{A})$ which is such that for every countable $\mathfrak{B}$, $\mathfrak{B} \models \sigma(\mathfrak{A})$ implies that $\mathfrak{A} \simeq \mathfrak{B}$. Barwise and Eklof have shown in [2] that the relation $\equiv^{\alpha}_{\infty\omega}$ and the Scott heights are closely connected to the Ulm invariants for p-groups.

After recalling these well-known properties of countable structures we shall see how this picture can be carried over for uncountable structures. For the sake of concreteness we consider throughout this paper only structures of cardinality ω_1.

The Ehrenfeucht–Fraïssé game of length ω_1 is exactly like that of length ω, but now the players choose the elements x_ν and y_ν for all $\nu < \omega_1$. Let η be a linear ordering containing no uncountable descending sequences. The η-game between two structures is like the Ehrenfeucht–Fraïssé game of length ω_1 but simultaneously with every move x_ν, player $\forall$ chooses also an element t_ν in such a way that the elements t_ν form a descending sequence. Since there is no uncountable descending sequence in η, every play of the η-game is countable. If η is an ordinal, then this definition coincides of course with the previous one. In the literature, the Ehrenfeucht–Fraïssé game of length ω_1 is most commonly approximated in terms of trees with no uncountable branches. These two approaches are closely connected. The η-game is in the other terminology the Ehrenfeucht–Fraïssé game approximated by the tree of all descending sequences of η. On the other hand, the ordering of a tree with no uncountable branches can be extended to a linear ordering with no uncountable ascending sequences (see [3]).

The notion of isomorphism can be approximated by means of these η-games as the following proposition shows. Here, as well as in all of our results, we shall assume the Continuum Hypothesis.

Lemma 1 (Hyttinen, see [9]). (CH) *If $\mathfrak{A}$ and $\mathfrak{B}$ are of cardinality $\leq\omega_1$, then $\mathfrak{A} \simeq \mathfrak{B}$, if and only if, player $\exists$ has a winning strategy in the η-game between $\mathfrak{A}$ and $\mathfrak{B}$ for every linear ordering η of cardinality $\leq\omega_1$ which contains no uncountable descending sequences.* □

The situation with Scott heights is essentially more complicated for uncountable structures than for the countable ones. In [9] a version of Scott heights is developed for describing the difference between two given structures. Under CH, no universal notion of a Scott height, which would distinguish $\mathfrak{A}$ from every other structure $\mathfrak{B}$ of the same cardinality, is possible for uncountable structures as the following result of Hyttinen and Tuuri shows.

Lemma 2 [8]. (CH) *There is a linear ordering η of cardinality ω_1, and for every linear ordering θ of cardinality $\leq\omega_1$ which contains no descending uncountable sequences, there is a linear ordering η_θ of cardinality ω_1, where*

(1) *$\exists$ has a winning strategy in the θ-game between η and η_θ,*
(2) *η contains an uncountable descending sequence,*
(3) *η_θ contains no uncountable descending sequences.* □

Notice that we can assert in Lemma 2 that also $\eta \simeq \eta + \eta$, since the result remains true, if we replace η and η_θ by $\eta \otimes Q$ and $\eta_\theta \otimes Q$.

Hyttinen and Tuuri use Lemma 2 to prove non-structure theorems for unstable theories in [8]. We use the lemma here to prove a non-structure theorem for abelian p-groups. For the sake of concreteness we shall consider only groups of cardinality $\leq \omega_1$ but it is clear that our constructions can easily be generalized for higher cardinalities, too.

2. Hahn powers

In this section we describe the kind of groups that we shall use in our constructions. We show that in the specific applications it suffices to prove that our construction leads to non-isomorphic groups. The existence of winning strategies for $\exists$ will follow from a general result proved in this section. The reader is asked to consult [5] and [10] for general information and notation concerning abelian groups.

An abelian torsion group H is a *p-group* for a prime p, if for all $x \in H$, there is some $n \in \omega$ with $p^n x = 0$. The *socle* of a p-group H is the subgroup $H[p] = \{x \in H: px = 0\}$. A p-group H is *reduced*, if it has no non-trivial divisible subgroups. Let H be a reduced p-group. Consider the descending sequence of subgroups $H_0 \supseteq H_1 \supseteq \cdots$ where $H_0 = H$, $H_{\nu+1} = pH_\nu$ and $H_\alpha = \bigcup_{\nu<\alpha} H_\nu$ for limit ordinals α. Since H is reduced, there is a smallest ordinal $\alpha = l(H)$ with $H_\alpha = \{0\}$ called the *length* of H. For every element $x \neq 0$, there is ν with $x \in H_\nu \setminus H_{\nu+1}$. This ordinal ν is called the *height* of x.

Let V be a vector space. A *valuation* of V is a function $v : V \to X$ where X is a linear ordering with suprema and where

(1) $v(a) = \sup X$, if and only if $a = 0$,
(2) $v(\lambda a) = v(a)$ for all scalars $\lambda \neq 0$,
(3) $v(a + b) \geq \min(v(a), v(b))$.

If H is a reduced p-group, then the socle $H[p]$ is a Z_p vector space and $h : H[p] \to l(H)$ is a valuation of it. The readers can find more about vector spaces with valuations in [6] and [7].

It has turned out that structures of the form of an abstract *Hahn power* $\mathfrak{A}^\eta$ are fruitful sources of examples. The Hahn powers and products come from the theory of ordered algebraic structures (see [4] and [7]), but there are also other applications of the same idea. By $\mathfrak{A}^\eta$ we mean the set of those $x : \eta \to \mathfrak{A}$ where the *carrier* $\{t \in \eta: x(t) \neq 0\}$ is of the order type α^* for some (countable) ordinal α. Here $\mathfrak{A}$ must contain a distinguished element 0, of course, and α^* denotes the opposite of α. Typical cases where this kind of Hahn powers give interesting structures are those where $\mathfrak{A}$ is either a linear order with a smallest element 0 or an algebraic structure like a group. In case $\mathfrak{A}$ and η are ordinals, the Hahn power $\mathfrak{A}^\eta$ coincides with the usual ordinal exponentiation.

In [11] Karp proves that in case $\alpha = \omega^\alpha$ and A is a linear order with a smallest element, then

$$\alpha \equiv^{\alpha}_{\infty\omega} \alpha \otimes A .$$

In [12] this is generalized for the η-game and uncountable linear orderings of the form of a Hahn power. The original proof of Lemma 2 gave η in the form of a Hahn power where a saturated linear order is an essential building block of the exponent. (The construction presented in [8] is quite different.)

We are here interested in the following kind of Hahn powers. Let η be any linear ordering and H any group (or more generally, any algebraic structure with a distinguished zero element). Then H^η is the set of those functions $x : \eta \to H$ where the order type of the carrier $\mathrm{carr}(x) = \{t \in \eta : x(t) \neq 0\}$ is the opposite α^* of some countable ordinal α. Clearly H^η is a subgroup of the product $\prod_\eta H$. The torsion subgroup of H^η is denoted by $G(\eta, H)$.

If V is a vector space and $v : V \to X$ is a valuation of it, then we consider the function $v^\eta : V^\eta \to X$, where

$$v^\eta(a) = \inf\{v(a(t)) : t \in \eta\} .$$

The following two lemmas are obvious.

Lemma 3. (1) *If H is a (reduced) p-group, then also $G(\eta, H)$ is a (reduced) p-group.*

(2) *If V is a vector space and $v : V \to X$ is a valuation of V, then V^η is a vector space and v^η is a valuation of V^η.* □

Lemma 4. (CH) *If H, V and η are of cardinality $\leq \omega_1$, then H^η, V^η and $G(\eta, H)$ are of cardinality $\leq \omega_1$.* □

It is relatively easy to arrange a winning strategy for $\exists$ in the η-game between the groups $G(\eta', H)$ and $G(\eta'', H)$. Below, $\omega^* \otimes \eta$ is understood to mean η copies of ω^* and ω^* denotes the opposite ordering of ω. Vector spaces with valuations are considered there as two sorted structures.

Theorem 5. *If $\exists$ has a winning strategy in the $\omega^* \otimes \theta$-game between η and η', then $\exists$ has a winning strategy in the θ-game between*

(1) *H^η and $H^{\eta'}$,*

(2) *$G(\eta, H)$ and $G(\eta', H)$, and*

(3) *(V^η, X, v^η) and $(V^{\eta'}, X, v^{\eta'})$.*

Proof. We consider here only case (2) which will be used later. We fix a winning strategy S for $\exists$ in the $\omega^* \otimes \theta$-game between η and η'. It is a winning strategy for

$\exists$ in the θ-game between the two groups to simulate the $\omega^* \otimes \theta$-game between η and η'. The same argument works for both of the two cases. Assume that the game has been played according to the strategy to be described so that on round ν player $\forall$ has just played an element x_ν from one of the groups and an element t_ν from η. During the earlier rounds $\exists$ has found out his moves by a simulation which has determined an initial segment of a play of the $\omega^* \otimes \theta$-game between η and η'. Let $\text{carr}(x_\nu) = \{u_n : n < \omega\}$, not necessarily in descending order. Then the simulation is continued so that we let for $n < \omega$ $\forall$ move u_n and (n, t_ν) to which the winning strategy S replies with some v_n. Then we can let $\exists$ play in the game between the groups the element y_ν where $\text{carr}(y_\nu) = \{v_n : n < \omega\}$ and $y_\nu(v_n) = x_\nu(u_n)$.

It is easy to see that it is a winning strategy for $\exists$ to play according to this strategy. Indeed, $G \simeq G'$ where G is the subgroup of $G(\eta, H)$ consisting of those elements x whose carrier contains only points played in the simulated game, and G' is defined similarly for $G(\eta', H)$. $\square$

When we combine this result with Lemma 2, we see that our real task lies in proving that for a suitable choice of H, $G(\eta, h)$ and $G(\eta', H)$ (H^η and $H^{\eta'}$) cannot be isomorphic if exactly one of the exponents η and η' contains descending uncountable sequences.

Corollary 6. (CH) *Let η and η_θ be as in Lemma 2. Then*:

(1) *If $\eta' = \eta_{\omega^* \otimes \theta}$, then $\exists$ has a winning strategy in the θ-game between $G(\eta, H)$ and $G(\eta', H)$, between H^η and $H^{\eta'}$, and between (V^η, X, v^η) and $(V^{\eta'}, X, v^{\eta'})$.*

(2) *η contains uncountable descending sequences.*

(3) *$\eta_{\omega^* \otimes \theta}$ contains no uncountable descending sequences.* $\square$

Let η be as in Lemma 2 and assume that $\eta \simeq \eta + \eta$. Recall that this property can be included in Lemma 2. Let G be H^η or $G(\eta, H)$ and let $T = Th(G, +, 0)$. Then one can easily see that $G \simeq G^2$. This implies that $T = T^{\aleph_0}$ (see [13, p. 41]). Hence by Theorem 5.41 in [13], T has NOTOP. On the other hand, every complete theory of modules is stable (see Theorem 3.1 in [13]), and is non-multidimensional and hence has NDOP (see p. 143 and Corollary 6.21 in [13].) Thus our results fall in the case left uncovered by those of [8].

3. *p*-groups

We prove in this section the following result.

Theorem 7. (CH) *There is a p-group G of cardinality ω_1 and for every linear ordering θ of cardinality $\leq \omega_1$ which contains no descending uncountable sequences, there is a p-group G_θ of cardinality $\leq \omega_1$ which satisfy*

(1) $\exists$ *has a winning strategy in the θ-game between G and G_θ,*
(2) $G \not\cong G_\theta$.

The rest of this section is devoted to a proof of Theorem 7. It follows from Corollary 6 that it suffices to show for a suitable p-group H of cardinality $\leq \omega_1$ that $G(\eta, H) \not\cong G(\eta', H)$ whenever exactly one of the orderings η and η' contains uncountable descending sequences. We prove actually as an application of a result on vector spaces with valuations that $G(\eta, H)[p] \not\cong G(\eta', H)[p]$.

Let V be a vector space with a valuation $v : V \to X$ and let $(V_\nu)_{\nu<\omega_1}$ be an ascending sequence of its countable subspaces. We assume that there is an ascending sequence $(\xi_\nu)_{\nu<\omega_1}$ of elements of X where for every $\alpha < \omega_1$ the subspace V_α contains an element x_α with $v(x_\alpha) = \xi_\alpha$, but $v(x) < \xi_\alpha$ for all $x \in \bigcup_{\nu<\alpha} V_\nu$. We shall consider the vector space

$$W = \bigoplus_{\nu<\omega_1} V_\nu$$

and its Hahn powers.

If $a \in W$, then we denote by spt(a) the set of those ν where a has nonzero projection in V_ν. Because W is a direct sum, spt(a) has to be finite for all a.

Lemma 8. *The Hahn powers W^η and $W^{\eta'}$ are not isomorphic, if exactly one of the two linear orderings η and η' contains an uncountable descending sequence.*

Proof. Let S be the set of countable ascending sequences of elements of ω_1 ordered according to the initial segment relation.

Let U be any vector space with a valuation $u : U \to X$. We say that U has *property* $(*)$, if in U there are elements z_s, $s \in S$, where $z_{\langle\rangle} = 0$ and with the notation $z_{s,r} = z_r - z_s$ and $y_{s,\delta} = z_{s\frown\langle\delta\rangle} - z_s$, it holds that

$$u(z_{s,r}) = u(y_{s,\delta}) = \xi_\delta$$

when $r = s\frown\langle\delta\rangle\frown s'$. This means intuitively that z_r is a sum of a sequence of countably many terms $y_{s,\delta}$ of ascending values.

We shall show that W^η has property $(*)$, if and only if η contains an uncountable descending sequence.

Claim 1: If η contains an uncountable descending sequence, then W^η has property $(*)$.

To prove this, we let $(t_\nu)_{\nu<\omega_1}$ be a descending sequence of elements of η. Consider a sequence $(a_\nu)_{\nu<\omega_1}$ of elements of W, where for all $\nu < \omega_1$, $u(a_\nu) = \xi_\nu$. The elements z_s, $s \in S$ are defined as follows. Let the length of s be μ and $s = \langle \delta_\nu \rangle_{\nu<\mu}$. Then carr$(z_s) = \{t_\nu : \nu < \mu\}$ and for $\nu < \mu$, $z_s(t_\nu) = a_{\delta_\nu}$. It is immediate to see that $(*)$ becomes true.

Claim 2: If W^η has property (*), then there is an uncountable descending sequence in η.

The rest of the proof of this lemma is devoted to a verification of this assertion. Assume that the system $\langle z_s \rangle_{s \in S}$ witnesses that W^η has property (*).

We observe first that for all $s \in S$ there is an ordinal δ where for all $r \in S$ with $s^\frown\langle \delta \rangle \leq r$, it holds that $\text{carr}(z_s) \subseteq \text{carr}(z_r)$ and for all $t \in \text{carr}(z_s)$, $\text{spt}(z_s(t)) \subseteq \text{spt}(z_r(t))$. Indeed, we choose

$$\delta = \sup(\{i+1: i \in \text{spt}(z_s(t)) \text{ for some } t \in \text{carr}(z_s)\}) .$$

Notice that this is a supremum of a countable set of countable ordinals. If r is as in the claim, then $v(z_r - z_s)$ is ξ_δ which is too high to cancel any $z_s(t) \neq 0$ or any $z_s(t)(i) \neq 0$.

Consider then an ascending sequence $\langle s_\nu \rangle_{\nu < \omega_1}$ of elements of S, where for all ν there is some δ with

$$\delta \geq \sup(\{i+1: i \in \text{spt}(z_{s_\nu}(t)) \text{ for some } t \in \text{carr}(z_{s_\nu})\}) ,$$

and $s_\nu{}^\frown\langle \delta \rangle \leq s_{\nu+1}$.

We observe next that for all $t \in \eta$, the projection $z_{s_\nu}(t)$ is constant when ν is large enough.

To prove this, fix t and assume to the contrary that $z_{s_\nu}(t)$ is not eventually constant. Then certainly $t \in \text{carr}(z_{s_\nu})$ for some ν, and hence for sufficiently large ν. (Otherwise $z_{s_\nu}(t)$ is constantly 0 for all ν.) By (*), $v(z_{s_\nu}(t) - z_{s_\mu}(t)) \geq \xi_\mu$ if $\nu > \mu$, and since $z_{s_\nu}(t) - z_{s_\mu}(t)$ is not 0, the sets $\text{spt}(z_{s_\nu}(t))$ cannot be constant. Since by the choice of the sequence $(s_\nu)_\nu$ they form an ascending sequence, there exists ν with $\text{spt}(z_{s_\nu}(t))$ infinite, contrary to the definition of W as a direct sum.

By the choice of $\langle s_\nu \rangle_{\nu < \omega_1}$, also the sets $\text{carr}(z_{s_\nu})$ form an ascending sequence. It cannot become constant by (*) and the previous observation. So there are elements t_ν, $\nu < \omega_1$, where for all $\alpha < \omega_1$ there is some $\mu < \omega_1$ with $\{t_\nu: \nu < \alpha\} \subseteq \text{carr}(z_{s_\mu})$. Hence $\{t_\nu: \nu < \omega_1\}$ must be an inversely well-ordered subset of η. Especially, there is an uncountable descending sequence in η. □

We shall finally apply this result to the Hahn powers of the socle of a suitable p-group in order to prove Theorem 7.

For a proof of the following fact, see p. 85 in Vol. II of [5].

Lemma 9. *There is for every $\nu < \omega_1$ a countable reduced p-group H_ν of Ulm-length $\nu + 1$ and containing therefore an element x with $h(x) = \nu$ but no elements $y \neq 0$ with $h(y) > \nu$. Here $h(x)$ denotes the Ulm-height of an element x.* □

Let H_ν be as in this lemma. Notice that if $x \in H_\nu$ and $h(x) = \nu$, then $px = 0$. Let us consider the p-groups $H = \bigoplus_{\nu < \omega_1} H_\nu$ and $G(\eta, H)$ for various η.

It is easy to see that

$$G(\eta, H)[p] = H^{\eta}[p] = \Big(\bigoplus_{\nu<\omega_1} H_\nu[p]\Big)^{\eta} .$$

So the valuated vector space $G(\eta, H)[p]$ is of the form W^{η} where W satisfies the assumptions of Lemma 8. Hence we obtain from Lemma 8 that

$$G(\eta, H)[p] \not\cong G(\eta', H)[p]$$

whenever exactly one of the two orderings η and η' contains an uncountable descending sequence. This completes the proof of Theorem 7. □

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