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Numerical Modeling of the Compression Process of Elastic Open-cell Foams

ZHANG Jia-lei, LU Zi-xing*

Institute of Solid Mechanics, Beijing University of Aeronautics and Astronautics, Beijing 100083, China

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Abstract

The random models of open-cell foams that can reflect the actual cell geometrical properties are constructed with the Voronoi technique. The compression process of elastic open-cell foams is simulated with the nonlinear calculation module of finite element analysis program. In order to get the general results applicable to this kind of materials, the dimensionless compressive stress is used and the stress-strain curves of foam models with different geometrical properties are obtained. Then, the influences of open-cell geometrical properties, including the shape of strut cross section, relative density and cell shape irregularity, on the compressive nonlinear mechanical performance are analyzed. In addition, the numerical results are compared with the predicted results of cubic staggering model. Numerical results indicate that the simulated results reflect the compressive process of foams quite well and the geometrical properties of cell have significant influences on the nonlinear mechanical behavior of foams.

Keywords: Voronoi technique; open-cell foam; nonlinear; compressive stress

1 Introduction

Low-density foams are widely used in aviation and spaceflight fields, such as lightweight sandwich panels, energy absorbing materials, impact damping and sound insulation materials, due to their unique mechanical performances. In order to study the mechanical behavior of foams, people usually use unit cell models with regular cell shapes to determine the relationships between the basic mechanical properties and the cell structure parameters, such as cubic staggering model[1] and Kelvin model[2]. But regular model can't accurately represent some important geometrical features of foams, including random variation of cell structure, the shape of strut cross-section and non-uniform solid distribution between strut and cell wall. Thus, constructing more complex models to represent the random variation becomes an important approach to study mechanical properties of foams. Many works have been conducted in this field[3], but most works are concentrated on studying the influences of cell geometrical properties on the linear elastic behavior of foams[4-8]. Only few works are concerned with the nonlinear mechanical properties of foams at present. Shulmeister et al.[9]started from regular BCC and FCC lattice nuclei distributions, then gave the nuclei positions an random disturbing offset and constructed the random foam model with the Voronoi technique, thereby simulated the nonlinear compressive and tensile mechanical behavior of elastic open-cell foams. But the stiffness and yield stresses of these models increased due to the struts at the boundary were normal to the boundary surface, also the influence of random variation of cell structure on the

*Corresponding author. Tel.: +86-10-82317507.
E-mail address: luzixing@263.net
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nonlinear mechanical behavior was not considered. Zhu et al.[10] built the periodic Voronoi open-cell foam model, and simulated the high strain compression of low-density open-cell polymer foams through finite element analysis. They considered the influence of strut geometrical nonlinearity on stress-strain curve and compared the numerical results with analytical results of tetrakaidecahedron model, but they didn't analyzed the effects of different strut cross-section shapes, and some of resulting curves were not complete. Thus, it is necessary to place more emphasis on investigating the nonlinear mechanical behavior of low-density foams.

In this paper, the Voronoi technique is used to generate the open-cell foams random model with irregular cell shapes and cell sizes. In order to consider the strut geometrical nonlinear effect, the compressive deformation process of elastic open-cell foams before being contacted with struts is simulated with the nonlinear calculation module of finite element program. In addition, the influences of shapes of the cell strut cross-section, relative density and cell shape irregularity on the nonlinear mechanical behavior of elastic open-cell foams, especially the compressive deformation modes' transitions when the cell structures change from regular to irregular shape are investigated. The main objective of this paper is to investigate the compressive deformation mechanism of open-cell foams at the microstructure level and predict the compressive stress-strain relationships. Thereby we can further set up more accurate macroscopic constitutive relationships. Finally, the numerical results are compared with Zhu's results[10] and Gibson's semi-empirical formula[1] which is based on the cubic staggering model to verify the conclusions of this paper.

2 Construction of the Geometrical Model

The method used to construct Voronoi random models of foams is introduced in detail in our previous papers[11-12]. The procedure[12] is: firstly generate the closed-cell random foam models, then delete all the walls of closed-cell models, finally trim off the struts on the boundary, so the open-cell random foam models are obtained. Different from Zhu's open-cell foam random models, these models have preserved the short struts because most real open-cell foams contain short struts. In this paper, irregularity degree α is defined to represent the grade of cell shape random variations, for example, when $\alpha = 0$, the random model is a tetrakaidecahedron model, when $\alpha = 1.0$, the model represents completely irregular one. As $\alpha = 0.7$, the model is very closed to completely irregular one, here four kinds of model ($\alpha = 0.1$, 0.3, 0.5 and 0.7) are used. These foam models with different irregularity degrees are shown in Fig.1. It is observed that the cell structure changes from quasi-tetrakaidecahedron structure to completely irregular one. The model size is defined as the ratio of cubic model length to the average cell size, and in Fig.1 the model size is 5 for each model. Here the average cell size is set as100 μm.

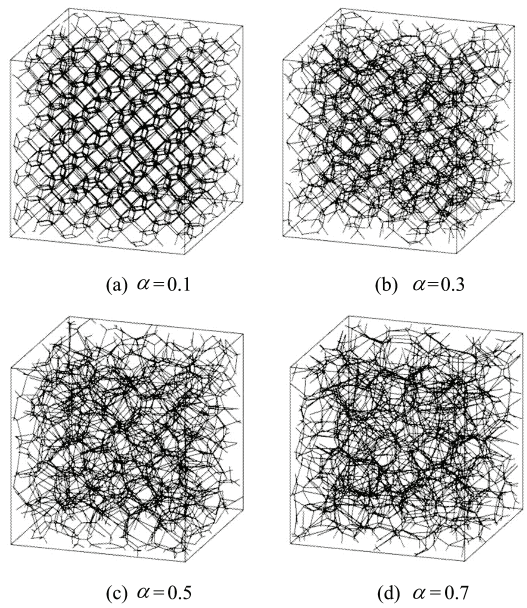


Fig.1 Random models of open-cell foams with different irregularity degrees (model size = 5).

3 The Finite Element Model

The geometrical model is drawn into commercial finite element analysis software ANSYS. Each strut is represented by a beam element BEAM188 based on Timoshenko beam theory, which accounts for shear deformation effects. Before the strut lines being meshed with beam element, the beam cross-sections (such as circle, square and equilateral triangle) should be defined, and then we assign beam cross sections as attributes of the element. In order to mesh beam element uniformly, we control the element size and make the element edge length be equal. Finally, an important issue is to determine the appropriate number of elements making relative errors of results be less than 0.5%, to obtain an accurate solution. In real foams, the strut cross-sections are very irregular, and changing along the struts. Usually cross-section size is smaller at the middle of strut while bigger at the vertices. Shulmeister[9] thinks that the solid material is primarily concentrated at the vertices for high-density foams, while for low-density foams, the solid material is distributed more uniformly along the struts and the struts are relatively slender. To simplify the model, it is assumed that all the struts have a constant cross-section and cross-section shapes are circle, square or equilateral triangle. The relative density of open-cell foams is determined by the cross-section area and defined by

$$R = \frac{\rho}{\rho_{\mathrm{s}}} = \frac{A\sum_{i=1}^{N} l_i}{V} \tag{1}$$

where R is the relative density, ρ is the foam density, ρ_{s} is the matrix density, l_i is the cell strut length of strut i, N is the total number of cell struts, V is the model volume and A is the strut cross-section area.

Under quasi-static axial compression, most elastic open-cell foams usually show three distinct regions of deformation: elastic region, plateau region and densification region[1]. The deformation mechanisms of microstructure are very complex in the three regions, because the deformation state of struts involves material nonlinearities, geometric nonlinearities and contact nonlinearities. So this paper mainly focuses on modeling the deformation state of low-density open-cell foams (relative density $R \leqslant 0.10$) in linear elastic and plateau stages (compressive strain less than 0.42). At this time, most struts are slender beams and in the state of bending and buckling, the solid material is still under linear stage, so the nonlinear mechanical behavior of foam is mainly determined by the geometrical nonlinear effect of struts. The Poisson ratio is set to be 0.5. As the random foam models are not periodic in space, the appropriate boundary conditions should be applied to ensure that the predicted properties of the models are representative for those foam materials, and the boundary has little effect on macroscopic mechanical properties of foams. The boundary conditions imposed in this paper are similar to those Silva and Gibson[13] have used, that is, to restrain rotational displacements of all boundary nodes, keep four boundary-planes along loading direction in-plane through the constraint equations, and impose opposite compressive displacement on two plane normal to the loading direction. Using these boundary conditions, the cubic model still keep its cube shape after compression and it is convenient to analyze compressive behavior of foams.

Similar to Zhu et al.[10], the same dimensionless stress $\bar{\sigma}$ is defined for open-cell foams with different elastic modulus. The stress can be regarded as a function of cell shape irregularity degree α, influence coefficient of strut cross-section β, relative density ρ/ρ_{s} and compressive strain ε, and defined by

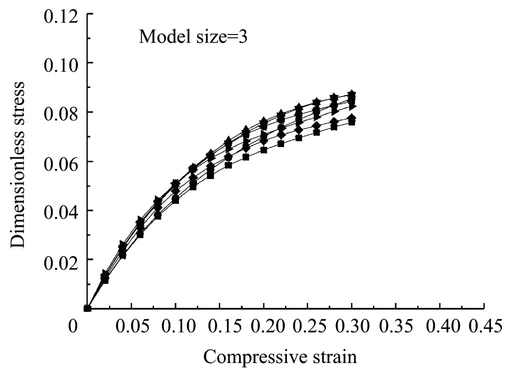
$$\bar{\sigma} = \frac{\sigma}{E_{\mathrm{s}}(\rho/\rho_{\mathrm{s}})^2} = F(\alpha, \beta, \rho/\rho_{\mathrm{s}}, \varepsilon) \tag{2}$$

where σ is the compressive stress of foam, E_{s} is the elastic modulus of matrix and ρ/ρ_{s} is the relative density. The effects of these parameters on compressive mechanical behavior of open-cell foam are analyzed as follows.

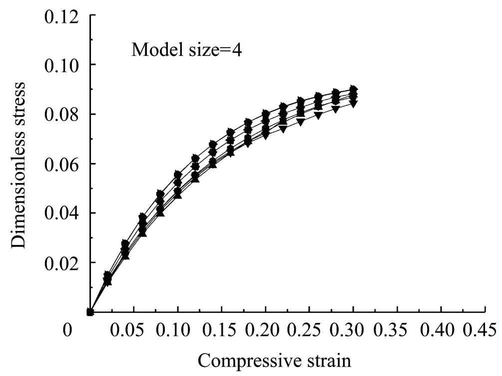
4 Numerical Results and Discussion

4.1 Effects of model size

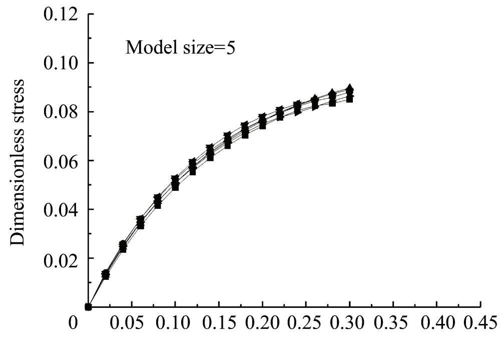
In order to investigate the effects of the model size on the stress-strain curves, a number of random models are analyzed for different model sizes (for 3, 4 and 5) at $R = 0.01$, $\alpha = 0.5$ with equilateral triangle as strut cross-section shape. Considering that the calculations of some samples aren’t convergent due to the instability of nonlinear calculations, several samples with same model size have been analyzed and the mean value of these results is taken as the final result. For three kinds of model size, the dimensionless stress-strain curves are given in Fig.2(a), Fig.2(b) and Fig.2(c). These figures reveal that there are some differences between the curves; especially as the strains become higher the differences will be more obvious. As the model size increases, the differences between the curves become smaller, and the differences can be ignored when model size is more than 5. As shown in Fig.2(d), the mean curves of these models with different model sizes are almost the same and the curve of model size being 3 is some lower than the others. While the model size increases, the computation time will increase accordingly. Here, the model size is fixed at 3 to reduce the computation expenses. The numerical results are the mean values of several samples, which can ensure the calculating precision, and can reflect the statistic characteristics of random models as well.



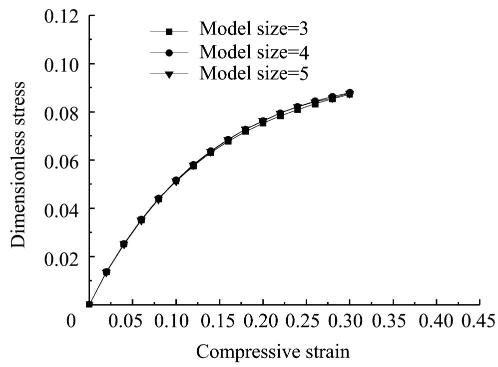
(a) Model size=3, model number=10



(b) Model size=4, model number=10



(c) Model size=5, model number=7



(d) The average value curves

Fig.2 The dimensionless stress-strain curves.

4.2 Effects of strut cross-section shape

Previous works show that strut cross-section shape has significant effects on the elastic properties of foams. In order to investigate the effects of cross-section shape on the nonlinear behavior of foams, we assume that the strut cross-section shapes have three types (circle, square and equilateral triangle) and set irregularity degree $\alpha = 0.5$, relative density $R = 0.01$. The dimensionless stress-strain curves of open-cell foams with different strut cross-section shapes are given in Fig.3. It can be observed that the compressive stress-strain curves haven't obvious plateau region, and increasing monotonously almost parallel with each other. Especially, the strength of foams with equilateral triangle is higher than the others; thereby the energy absorbing capability is enhanced. The Poisson ratio-strain curves of open-cell foams with different strut cross-section shapes are given in Fig.4. From this figure, it is found that Poisson ratio decreases as the compressive strain increase, and the curves which Zhu et al.[10] obtained also have the similar trend. For different cross-sections, the Poisson ratio-strain curves almost the same, which indicates that the strut cross-section shape has negligible influence on Poisson ratio in the linear elastic stage and the larger compression strain state.

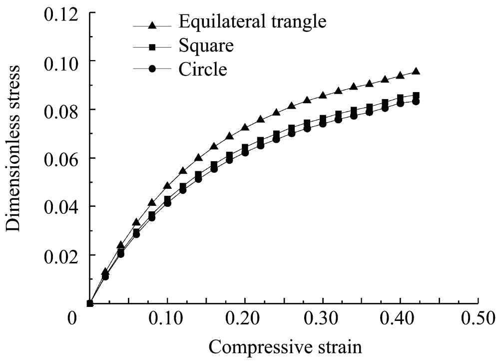


Fig.3 The dimensionless stress-strain curves of models with different strut cross-section shapes.

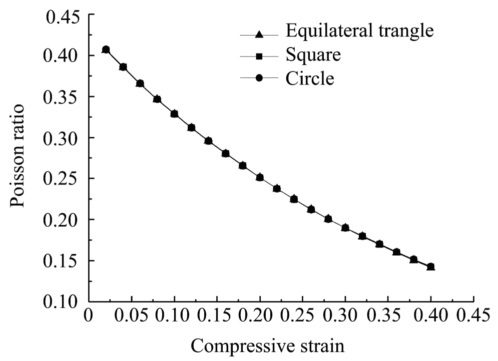


Fig.4 The Poisson ratio-strain curves of models with different strut cross-section shapes.

4.3 Effects of relative density

Relative density is the most important parameter determining the mechanical properties of foam. Both the elastic modulus and strength predicting formulae given by Gibson et al.[1] are proportional to the square of relative density. In this paper, the effect of relative density on mechanical behavior of open cell foam is studied through the dimensionless stress-strain curves, and the random models with α= 0.5 and equilateral triangle as strut cross-section shape are used for different relative density foams. As shown in Fig.5, the dimensionless stress decreases as relative density increases. When the strain is up to 0.3, the dimensionless stress for R = 0.01 is 18.2% higher than that for R = 0.08. As a result, the relative density has significant effect on mechanical behavior of foam, and the elastic modulus and strength aren't completely proportional to R^2. In Fig.6, the effect of relative density on the Poisson ratio-strain curve is shown. When the strain is small, the smaller the relative density the bigger Poisson ratio is. When the strain is higher, the bigger the relative density the bigger Poisson ratio is. When strain is around 0.10, these curves will be intersected with each other. As the Poisson ratio reflects the ratio of transverse deformation to the deformation along the loading direction, the deformation state changes as the relative density is changing, and the situation can be explained as follows: the strut slenderness ratio, which has great effect on the bending and bulking deformation of strut, will be varied due to the variation of relative density, then the compression mode of the whole model changes accordingly.

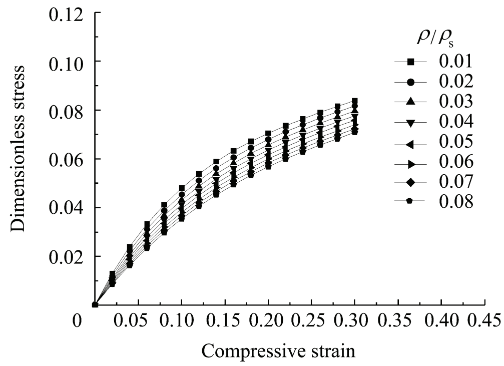


Fig.5 The dimensionless stress-strain curves of models with different relative density.

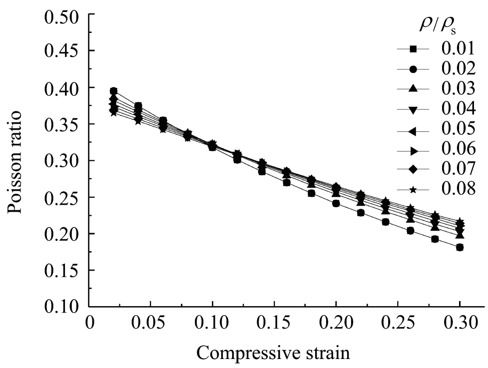


Fig.6 The Poisson ratio-strain curves of models with different relative density.

4.4 Effects of cell shape irregularity

In order to study the effects of cell shape irregularity on mechanical behavior of foam, the random models with α= 0.1, 0.3, 0.5 and 0.7 (here R =

0.01 and the strut cross-section is equilateral triangle) are analyzed. The dimensionless stress-strain curves of open-cell foam models with different irregularity degrees are shown in Fig.7, where the curve shapes change remarkably as the cell shapes change from being regular to irregular. It is observed that, when the cell shape is relatively regular, the stress-strain curves haven't obvious plateau region, but when the cell shape becomes completely irregular the curves have obvious long plateau region. Comparing with the relatively regular foam models, the irregular foam model has larger tangent modulus at low strain and lower dimensionless stress at higher strain. In Fig.8, the Poisson ratio-strain curves with different irregularity degrees are shown. When cell shape changes from regular to irregular, the curves change remarkable. At the same strain, the more regular foam model has larger Poisson ratio than that of the more irregular foam model. The results indicate that under different irregularity degrees the deformations of models in the lateral direction are different.

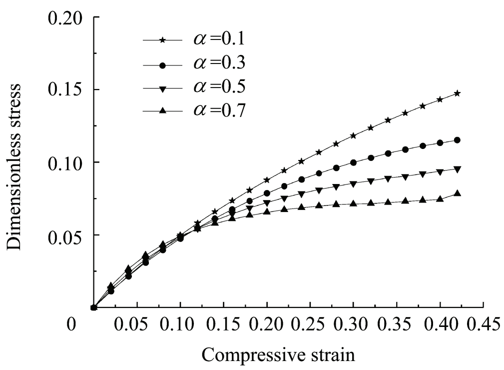


Fig.7 The dimensionless stress-strain curves of models with different irregular degrees.

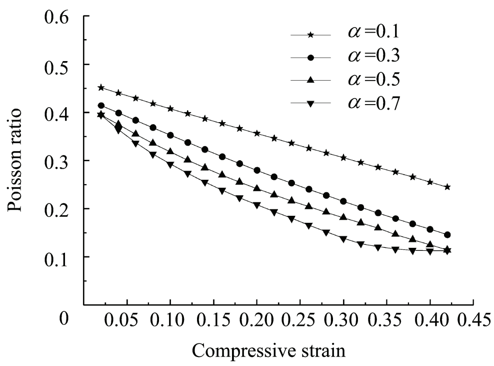


Fig.8 The Poisson ratio-strain curves of models with different irregular degrees.

4.5 Comparison of deformation modes

In order to describe deformation state of cell structure directly, the compressive deformation process pictures of models with irregular degree 0.1 and 0.7 (at the same relative density $R = 0.01$) are shown in Fig.9 and Fig.10, respectively. As shown

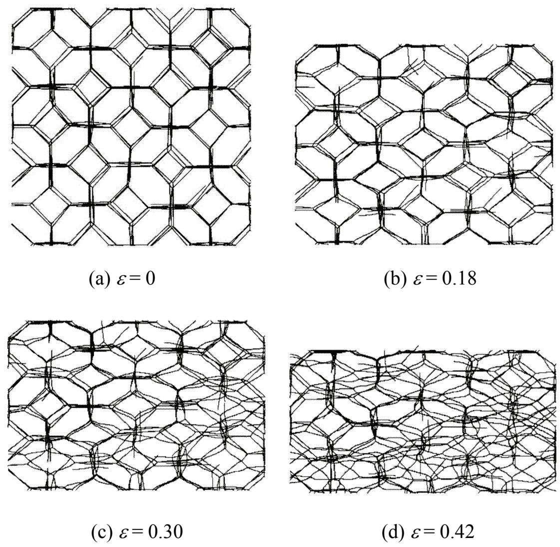


Fig.9 The compressive deformation process of open-cell foam ($\alpha = 0.1$).

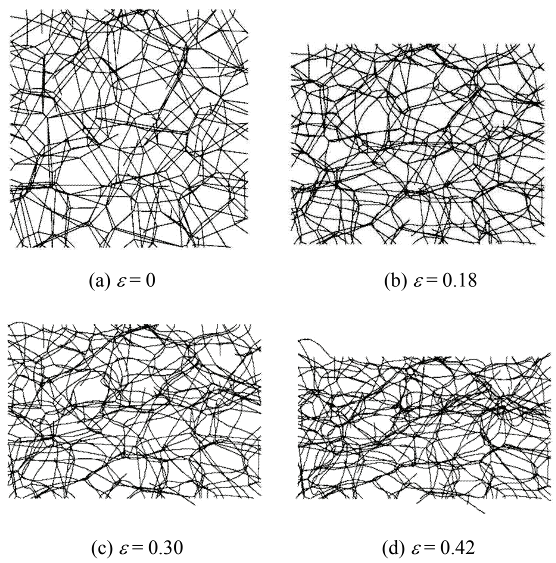


Fig.10 The compressive deformation process of open-cell foam ($\alpha = 0.7$).

in Fig.9, the cell shape keeps relatively regular, the relative displacements of adjacent struts are small, the main deformation mechanism is bending of cell struts and no obvious bulking or collapse occurs in struts. As shown in Fig.10, in compressive process not only bending deformation but also bulking deformation occurs in the struts, some cells collapses. Comparing Fig.9 with Fig.10, it is found that as the cell structure changes from the regular to irregular shape, the compressive deformation mode changes from homogeneous to non-homogeneous deformation, which is consistent with the characteristics of model stress-strain curves. These results indicate that the shape irregularity of cell has more significant effect on nonlinear behavior than elastic behavior of foam. The regular model is appropriate for predicting the elastic behavior before cell being collapsed, but isn't appropriate for predicting the nonlinear behavior of foam.

4.6 The elastic collapse stress

Zhu et al.[14] analyzed the high strain compression of open-cell foam with a regular tetrakaidecahedron model. Considering the symmetry, they simplified the model and analyzed strut deformation with beam theory. They predicted the compressive mechanical behavior of open-cell foam, but there is no a plateau region in their compressive stress-strain curves, which is similar to the present model with irregularity of 0.1. Gibson et al.[1] conducted analyses with staggered cube model and obtained the elastic-collapse stress σ_{el}^* as follows

$$\frac{\sigma_{\mathrm{el}}^*}{E_{\mathrm{s}}}=0.05(\frac{\rho^*}{\rho_{\mathrm{s}}})^2 \tag{3}$$

where E_{s} is the elastic modulus of matrix and ρ/ρ_{s} is the relative density. As the equilateral triangle cross-section is similar to real foam strut cross-section and the foam model with irregularity of $\alpha = 0.7$ has obvious plateau region along its stress-strain curve, we choose the initial stress in plateau region as elastic collapse stress and the corresponding initial strain as elastic collapse strain.

Then, write the critical stress as

$$\frac{\sigma_{\mathrm{el}}^*}{E_{\mathrm{s}}}=C(\frac{\rho^*}{\rho_{\mathrm{s}}})^2 \tag{4}$$

where $C = 0.049$, the elastic collapse strain equals 0.1. Although the elastic collapse stress is consistent with Gibson's semi-empirical result, the initial elastic collapse strain is higher than Gibson's prediction. Similarly, Zhu et al.[10] used the curved triangle (Plateau Border) as the strut cross-section shape, their results indicates that the value of C is closed to 0.1 (corresponding to higher elastic collapse stress), but their elastic collapse strain is higher than Gibson's prediction.

5 Conclusions

In this paper, the large compressive deformation process of low-density polymer foam is simulated with random foam model. The effects of strut cross-section, relative density and cell shape irregularity on compressive mechanical behavior of foam are investigated. The results indicate that the strut cross-section and relative density have great effects on the elastic modulus and compressive strength of open-cell foam and the cell shape irregularity directly affects the compressive deformation mode of open-cell foam. When cell structures change from regular to irregular shape, the deformation modes change from homogeneous to non-homogeneous deformation and the stress-strain relationships also change a lot. The stress-strain curves of the models with relatively regular cells do not have significant initial elastic collapse stress. Nevertheless, the stress-strain curves of these models with completely irregular cells have distinct plateau region due to strut buckling. In addition, the critical stress (initial elastic collapse stress) is consistent with Gibson's semi-empirical result but initial elastic collapse strain is higher than Gibson's prediction.

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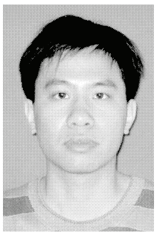

ZHANG Jia-lei Born in Sichuan Province in 1980. He received his B.S. from Beijing University of Aeronautics and Astronautics in 2004, and now he is as a graduate student in Institute of Solid Mechanics in BUAA. His research interests include foam mechanics and FEA.

E-mail: zhangjialei@ase.buaa.edu.cn

LU Zi-xing Born in Hebei Province in 1960. He received his B.S., M.S. and Ph.D. from Peking University in 1982, 1991 and 1995 respectively. He is a Professor and the head of Institute of Solid Mechanics in BUAA. His research interests include foam mechanics and composite mechanics.

E-mail: luzixing@263.net