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Weak phases from topological-amplitude parametrization

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Abstract

We propose a parametrization for two-body nonleptonic B meson decays, in which the various topologies of amplitudes are counted in terms of powers of the Wolfenstein parameter $\lambda \sim 0.22$. The weak phases and the amplitudes are determined by comparing this parametrization with available measurements. It is possible to obtain the phase ϕ_3 from the $B \to K\pi$ data up to theoretical uncertainty of $O(\lambda^2) \sim 5\%$. The recently measured $B_d^0 \to \pi^0\pi^0$ branching ratio implies a large color-suppressed or penguin amplitude, and that the extraction of the phase ϕ_2 from the $B \to \pi\pi$ data may suffer theoretical uncertainty more than the expected one, $O(\lambda^2) \sim 5\%$.

One of the major missions in B physics is to determine the weak phases in the Kobayashi–Maskawa ansatz for CP violation [1]. The phase ϕ_1 can be extracted from the CP asymmetry in the $B \to J/\psi K_S$ decays in an almost model-independent way, which arises from the B-$\bar{B}$ mixing. The application of the isospin symmetry to the $B \to \pi\pi$ decays [2] and to the $B \to \rho\pi$ decays [3] has been considered as giving a model-independent determination of the phase ϕ_2. However, this strategy in fact suffers the theoretical uncertainty from the electroweak penguin, which is expected to be about 5–10%. The phase ϕ_3 can be extracted in a theoretically clean way from the modes involving only tree amplitudes, such as $B \to \pi D$ [4] and $B \to KD$ [5,6]. The difficulty is that one of the modes, such as $B_d^0 \to \pi^- D^+$ or $B^+ \to K^+ D^0$, has a very small branching ratio and is not experimentally feasible [7]. The alternative modes $B \to K^* D$ [8] and $B_c \to D_s D$ [9] improve the feasibility only a bit. It has been pointed out that the $B^\pm \to K^\pm(D^0 \to f)$ and $B^\pm \to K^\pm(\bar{D}^0 \to f)$ amplitudes, with $\bar{D}^0 \to f$ being a doubly-Cabibbo suppressed decay, exhibits a strong interference [7,10,11]. For this strategy, the strong phase difference between $D^0 \to f$ and $\bar{D}^0 \to f$ is a necessary input. Another possibility is to measure the $B \to D^* V$ decays for the vector meson $V = \rho, K, \ldots$, since an angular analysis involves many observables, which are sufficient for extracting ϕ_3 model-independently [12].

Instead of resorting to theoretically clean modes, which are usually experimentally difficult, one considers the modes with higher feasibility and tries to con-

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strain the decay amplitudes and the weak phases. The problem is that available measurements are usually insufficient to make the constraint, and theoretical inputs are unavoidable. For example, one adopts the (imaginary) tree-over-penguin ratio obtained from the perturbative QCD (PQCD) formalism [13–17] or from the QCD-improved factorization (QCDF) [18], so that the phase ϕ_2 can be extracted from the CP asymmetries of the $B_d^0 \to \pi^+\pi^-$ decays. One may also employ symmetries to relate the amplitudes of the relevant modes, such as $SU(3)$ [19] and U-spin [20], in order to reduce the number of free parameters. However, the theoretical calculations are subject to subleading corrections, and the symmetry relations are broken with unknown symmetry breaking effects. For these strategies to work, the theoretical uncertainty must be under control.

In this Letter we shall propose counting rules for the various topologies of amplitudes [21] in two-body nonleptonic B meson decays in terms of powers of the Wolfenstein parameter $\lambda \sim 0.22$ [22]. The relative importance among the topological amplitudes has been known from some physical principles: helicity suppression (color transparency) implies that tree annihilation (nonfactorizable) contributions are smaller than leading factorizable emission contributions. Here we shall assign an explicit power of λ to each topology, such that the relative importance becomes quantitative. This assignment is supported by the known QCD theories [16,18,23,24], and differs from that assumed in [22]. We drop the topologies with higher powers of λ until the number of free parameters are equal to the number of available measurements. The weak phases and the decay amplitudes can then be solved by comparing the resultant parametrization with experimental data. Afterwards, it should be examined whether the solved amplitudes obey the power counting rules. If they do, the extracted weak phases suffer only the theoretical uncertainty from the neglected topologies. If not, the inconsistency could be regarded as a warning to QCD theories for two-body nonleptonic B meson decays. For example, the long-distance rescattering effect has been neglected in PQCD and in QCDF. If this effect is important, the hierarchy among the various topological amplitudes will be destroyed [25]. The comparison of our parametrization with data can tell whether the above assumption is reliable [26].

As shown below, dropping the electroweak penguin amplitude, the phase ϕ_2 can be extracted from the $B \to \pi\pi$ data. In principle, the theoretical uncertainly of the ignored amplitudes is around $O(\lambda^2) \sim 5\%$, the same as in the extraction based on the isospin symmetry [2]. Similarly, the phase ϕ_3 can be best determined from the $B \to K\pi$ data up to the uncertainty from the neglect of the $O(\lambda^3) \sim 1\%$ tree annihilation and color-suppressed electroweak amplitudes. Note that the determination of the phase ϕ_1 from the $B \to J/\psi K^{(*)}$ decays also bears about 1% theoretical uncertainty. Certainly, a CP asymmetry is an $O(\lambda)$ quantity itself. Precisely speaking, the above determination of ϕ_2 and ϕ_3, involving the data of CP asymmetries, in fact carries the uncertainly of $O(\lambda) \sim 20\%$ and $O(\lambda^2) \sim 5\%$, respectively. Because the $B \to \pi\pi$, $K\pi$ measurements are not yet complete, we shall drop more topologies in order to match the currently available data. In this simple demonstration, we observe that the amplitudes solved from the $B \to K\pi$ data more or less obey the hierarchy in λ. That is, an almost model-independent determination of ϕ_3 is promising. The solution from the $B \to \pi\pi$ analysis is, unfortunately, not consistent with the power counting rules, indicating that the extraction of ϕ_2 may suffer theoretical uncertainty larger than stated above. Hence, our work casts a doubt to the strategy based on the isospin symmetry [2] and gives a warning to the QCD calculations of the $B \to \pi\pi$ modes [17,18,27].

We start with the $B \to K\pi$ decays. The branching ratio of a two-body nonleptonic B meson decay is written as

$$B(B \to M_1 M_2) = \frac{\tau_B}{16\pi m_B}\left|A(B \to M_1 M_2)\right|^2, \qquad (1)$$

where the light-meson masses m_π and m_K have been neglected, and the B meson mass and the B meson lifetimes take the values $m_B = 5.28$ GeV, $\tau_{B^\pm} = 1.674 \times 10^{-12}$ s, $\tau_{B^0} = 1.542 \times 10^{-12}$ s. The effective Hamiltonian for the flavor-changing $b \to s$ transition is [28]

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_{qs}^* V_{qb}$$

$$\times\left[C_1(\mu)O_1^{(q)}(\mu)+C_2(\mu)O_2^{(q)}(\mu)\right.$$
$$\left.+\sum_{i=3}^{10}C_i(\mu)O_i(\mu)\right], \tag{2}$$

with the Cabibbo–Kobayashi–Maskawa (CKM) matrix elements V and the operators

$$\begin{aligned}
O_1^{(q)} &= (\bar{s}_i q_j)_{V-A}(\bar{q}_j b_i)_{V-A},\\
O_2^{(q)} &= (\bar{s}_i q_i)_{V-A}(\bar{q}_j b_j)_{V-A},\\
O_3 &= (\bar{s}_i b_i)_{V-A}\sum_q(\bar{q}_j q_j)_{V-A},\\
O_4 &= (\bar{s}_i b_j)_{V-A}\sum_q(\bar{q}_j q_i)_{V-A},\\
O_5 &= (\bar{s}_i b_i)_{V-A}\sum_q(\bar{q}_j q_j)_{V+A},\\
O_6 &= (\bar{s}_i b_j)_{V-A}\sum_q(\bar{q}_j q_i)_{V+A},\\
O_7 &= \frac{3}{2}(\bar{s}_i b_i)_{V-A}\sum_q e_q(\bar{q}_j q_j)_{V+A},\\
O_8 &= \frac{3}{2}(\bar{s}_i b_j)_{V-A}\sum_q e_q(\bar{q}_j q_i)_{V+A},\\
O_9 &= \frac{3}{2}(\bar{s}_i b_i)_{V-A}\sum_q e_q(\bar{q}_j q_j)_{V-A},\\
O_{10} &= \frac{3}{2}(\bar{s}_i b_j)_{V-A}\sum_q e_q(\bar{q}_j q_i)_{V-A},
\end{aligned} \tag{3}$$

i, j being the color indices. For the characteristic scale $\mu \sim \sqrt{m_b\bar{\Lambda}} \sim 1.5$ GeV involved in two-body B meson decays [16], $\bar{\Lambda} = m_B - m_b$ being the B meson and b quark mass difference, the values of the Wilson coefficients are

$$\begin{aligned}
C_1 &= -0.510, & C_2 &= 1.268,\\
C_3 &= 2.7\times10^{-2}, & C_4 &= -5.0\times10^{-2},\\
C_5 &= 1.3\times10^{-2}, & C_6 &= -7.4\times10^{-2},\\
C_7 &= 2.6\times10^{-4}, & C_8 &= 6.6\times10^{-4},\\
C_9 &= -1.0\times10^{-2}, & C_{10} &= 4.0\times10^{-3}.
\end{aligned} \tag{4}$$

The above characteristic scale has been confirmed by the dynamical penguin enhancement exhibited in the $B \to VP$ data [24]. The Wolfenstein parametrization for the CKM matrix is given by

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub}\\ V_{cd} & V_{cs} & V_{cb}\\ V_{td} & V_{ts} & V_{tb}\end{pmatrix}
= \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho-i\eta)\\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2\\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1\end{pmatrix}
= \begin{pmatrix} O(1) & O(\lambda) & O(\lambda^4)\\ O(\lambda) & O(1) & O(\lambda^2)\\ O(\lambda^3) & O(\lambda^2) & O(1)\end{pmatrix}, \tag{5}$$

with the parameters $\lambda = 0.2196 \pm 0.0023$, $A = 0.819 \pm 0.035$, and $R_b \equiv \sqrt{\rho^2+\eta^2} = 0.41 \pm 0.07$ [29]. Note that the product $AR_b \sim 0.3$ should be regarded as being of $O(\lambda)$, and that $|V_{ub}|$ is in fact $O(\lambda^4)$. The phases ϕ_1 and ϕ_3 are defined via $V_{td} = |V_{td}|\exp(-i\phi_1)$ and $V_{ub} = |V_{ub}|\exp(-i\phi_3)$, respectively.

Considering all possible topologies of amplitudes, the $B \to K\pi$ decay amplitudes are given by

$$A\big(B^+ \to K^0\pi^+\big) = P\left(1 - \frac{P_{\rm ew}^c}{P} + \frac{T^a}{P}e^{i\phi_3}\right), \tag{6}$$

$$A\big(B_d^0 \to K^+\pi^-\big) = -P\left(1 - \frac{P_{\rm ew}^a}{P} + \frac{T}{P}e^{i\phi_3}\right), \tag{7}$$

$$\sqrt{2}A\big(B^+ \to K^+\pi^0\big) = -P\left[1 + \frac{P_{\rm ew}}{P} + \left(\frac{T}{P} + \frac{C}{P} + \frac{T^a}{P}\right)e^{i\phi_3}\right], \tag{8}$$

$$\sqrt{2}A\big(B_d^0 \to K^0\pi^0\big) = P\left(1 - \frac{P_{\rm ew}}{P} - \frac{P_{\rm ew}^c}{P} - \frac{P_{\rm ew}^a}{P} - \frac{C}{P}e^{i\phi_3}\right), \tag{9}$$

which satisfy the quadrangle relation

$$A\big(B^+ \to K^0\pi^+\big) + \sqrt{2}A\big(B^+ \to K^+\pi^0\big) = A\big(B_d^0 \to K^+\pi^-\big) + \sqrt{2}A\big(B_d^0 \to K^0\pi^0\big). \tag{10}$$

The amplitude $P_{\rm ew}$ ($P_{\rm ew}^c$, $P_{\rm ew}^a$) comes from the color-allowed (color-suppressed, annihilation) topology through the electroweak penguin operators. The amplitude P includes the emission and annihilation topologies through both the QCD and electroweak penguins:

$$P = P_{\rm QCD} + e_u P_{\rm ew}^c + e_u P_{\rm ew}^a, \tag{11}$$

with the u quark charge $e_u = 2/3$. The amplitude T $(C,\ T^a)$ comes from the color-allowed (color-suppressed, annihilation) topology through the tree operators. The penguin contributions from the c quark loop can be included using the relation $V_{cs}^* V_{cb} = -V_{us}^* V_{ub} - V_{ts}^* V_{tb}$, and the expressions in Eqs. (17)–(20) remain unchanged.

It has been shown in PQCD that a nonfactorizable amplitude $M^{\rm nf}$, a factorizable annihilation amplitude $F^a_{(V-A)}$ from the $(V-A)(V-A)$ current, and a factorizable annihilation amplitude $F^a_{(V+A)}$ from the $(V-A)(V+A)$ current are suppressed, compared to the leading factorizable emission amplitude F^e, by the factors of [30]

$$\begin{aligned} &\frac{M^{\rm nf}}{F^e} \sim \left[\ln\frac{m_B}{\Lambda_{\rm QCD}}\right]^{-1} \sim \lambda, \\ &\frac{F^a_{(V-A)}}{F^e} \sim \frac{\Lambda_{\rm QCD}}{m_B} \sim \lambda^2, \\ &\frac{F^a_{(V+A)}}{F^e} \sim \frac{2m_0}{m_B} \sim \lambda^0, \end{aligned} \tag{12}$$

respectively, where m_0 is the chiral enhancement scale, and the CKM matrix elements and the Wilson coefficients are excluded. We list the power counting rules for the Wilson coefficients in Eq. (4)

$$\begin{aligned} &O(1):\quad a_1, \\ &O(\lambda):\quad a_2, 1/N_c, \\ &O(\lambda^2):\quad C_4, C_6, a_4, a_6, \\ &O(\lambda^3):\quad C_3, C_5, C_9, a_3, a_5, a_9, \\ &O(\lambda^4):\quad C_{10}, \\ &O(\lambda^5):\quad C_7, C_8, a_7, a_8, a_{10}, \end{aligned} \tag{13}$$

with $a_1 = C_2 + C_1/N_c$, $a_2 = C_1 + C_2/N_c$, $a_i = C_i + C_{i+1}/N_c$ for $i = 3, 5, 7, 9$, and $a_i = C_i + C_{i-1}/N_c$ for $i = 4, 6, 8, 10$.

According to Eqs. (12) and (13), we assign the powers of λ to the following ratios of the various topological amplitudes:

$$\begin{aligned} &\frac{T}{P} \sim \frac{V_{us}V_{ub}^*}{V_{ts}V_{tb}^*}\frac{a_1}{a_{4,6}} \sim \lambda, \qquad \frac{P_{\rm ew}}{P} \sim \frac{a_9}{a_{4,6}} \sim \lambda, \\ &\frac{C}{T} \sim \frac{a_2}{a_1} \sim \lambda, \\ &\frac{T^a}{T} \sim \frac{F^a_{(V-A)}}{F^e} \sim \frac{M^{\rm nf}}{F^e}\frac{C_1}{a_1 N_c} \sim \lambda^2, \\ &\frac{P^c_{\rm ew}}{P} \sim \frac{a_{8,10}}{a_{4,6}} \sim \frac{M^{\rm nf}}{F^e}\frac{C_9}{a_{4,6}N_c} \sim \lambda^3, \\ &\frac{P^a_{\rm ew}}{P} \sim \frac{F^a_{(V+A)}}{F_e}\frac{a_{8,10}}{a_{4,6}} \sim \frac{M^{\rm nf}}{F^e}\frac{C_9}{C_{4,6}N_c} \sim \lambda^3. \end{aligned} \tag{14}$$

For the latter three ratios, we present the power counting rules derived from both the factorizable and nonfactorizable contributions, which are of the same order of magnitude. Compared to the power counting rules in [22] based on the conventional scale $\mu \sim m_b$, $P^c_{\rm ew}/P$ is down by one more power of λ due to $a_{10} \sim O(\lambda^5)$ in PQCD.

Whether a factorizable amplitude or a nonfactorizable amplitude is important depends on the decay modes. In the $B \to K\pi$ case, C mainly comes from the factorizable color-suppressed diagrams, since there is a strong cancellation between a pair of nonfactorizable diagrams. The factorizable and nonfactorizable annihilation contributions to T^a, $P^c_{\rm ew}$, and $P^a_{\rm ew}$ are of the same order of magnitude as shown in Eq. (14). In the $B \to D\pi$ decays, C, being of the same order of magnitude as T, mainly comes from the nonfactorizable color-suppressed diagrams, since the above cancellation does not exist [15,31]. For T^a in the $B \to D\pi$ case, the nonfactorizable diagrams dominate, because of

$$\frac{M^{\rm nf}}{F^e}\frac{C_2}{a_1 N_c} \sim \lambda^2 \gg \frac{F^a_{(V-A)}}{F^e}\frac{a_2}{a_1} \sim \lambda^3. \tag{15}$$

Employing the reparametrizations

$$\begin{aligned} &P - P^a_{\rm ew} \to P, \qquad P_{\rm ew} + P^a_{\rm ew} \to P_{\rm ew}, \\ &P^c_{\rm ew} - P^a_{\rm ew} \to P^c_{\rm ew}, \end{aligned} \tag{16}$$

we arrive at the most general parametrization of the $B \to K\pi$ decay amplitudes

$$A(B^+ \to K^0\pi^+) = P\left(1 - \frac{P^c_{\rm ew}}{P} + \frac{T^a}{P}e^{i\phi_3}\right), \tag{17}$$

$$A(B^0_d \to K^+\pi^-) = -P\left(1 + \frac{T}{P}e^{i\phi_3}\right), \tag{18}$$

$$\begin{aligned} &\sqrt{2}A(B^+ \to K^+\pi^0) \\ &\quad = -P\left[1 + \frac{P_{\rm ew}}{P} + \left(\frac{T}{P} + \frac{C}{P} + \frac{T^a}{P}\right)e^{i\phi_3}\right], \end{aligned} \tag{19}$$

$$\sqrt{2}A\left(B_d^0 \to K^0\pi^0\right) = P\left(1 - \frac{P_{\rm ew}}{P} - \frac{P_{\rm ew}^c}{P} - \frac{C}{P}e^{i\phi_3}\right). \qquad (20)$$

There are totally 6 independent amplitudes, namely, 11 unknowns, because an overall phase can always be removed. Hence, we choose the amplitude P as a positive real value. Plus the weak phase ϕ_3, the 12 unknowns are definitely more than the 9 experimental inputs: the branching ratios and the direct CP asymmetries of the four modes, and the mixing-induced CP asymmetry of the $B_d^0 \to K^0\pi^0$ mode. Dropping the $O(\lambda^3)$ terms, T^a/P and $P_{\rm ew}^c/P$, we have 8 unknowns. Then the data of the direct CP asymmetry in the $B^+ \to K^0\pi^\pm$ decays should be excluded for consistency. Hence, we have 8 experimental inputs, and thus all unknowns can be solved exactly assuming the phase ϕ_1 is already known from the measurement of the mixing-induced CP asymmetry in the $B \to J/\psi K^{(*)}$ modes. The determination of ϕ_3 from this parametrization is then accurate up to the theoretical uncertainty of $O(\lambda^2) \sim 5\%$.

We emphasize the consequence from the different power counting rules in [22] and in this work: the smaller $P_{\rm ew}^c$ is crucial for claiming that the determination of ϕ_3 from the $B \to K\pi$ data is accurate up to 5% theoretical uncertainty. Following the counting rules in [22], both C and $P_{\rm ew}^c$ will be included at $O(\lambda^2)$, such that the 10 unknowns are more than the 9 available measurements. In this case we cannot solve for C and $P_{\rm ew}^c$ exactly, and have to rely on symmetry relations to reduce the number of unknowns. It is then difficult to estimate the involved theoretical uncertainty. Using the counting rules in Eq. (14), which are supported by the PQCD calculation, we include only C at $O(\lambda^2)$, and the number of unknowns can be equal to the number of measurements. Solving for C, and assuring that the solution obeys our counting rule as a self-consistency check, the uncertainty from the neglected topologies is under control.

The measurement of the time-dependent asymmetry in the $B_d^0 \to K_S\pi^0$ decay still suffers a large error. To demonstrate our method, we reduce the number of unknowns by further dropping the $O(\lambda^2)$ terms, C/P, arriving at

$$A\left(B^+ \to K^0\pi^+\right) = P, \qquad (21)$$

$$A\left(B_d^0 \to K^+\pi^-\right) = -P\left(1 + \frac{|T|}{P}e^{i\phi_3}e^{i\delta_T}\right), \qquad (22)$$

$$\sqrt{2}A\left(B^+ \to K^+\pi^0\right) = -P\left(1 + \frac{|P_{\rm ew}|}{P}e^{i\delta_{\rm ew}} + \frac{|T|}{P}e^{i\phi_3}e^{i\delta_T}\right), \qquad (23)$$

$$\sqrt{2}A(B_d^0 \to K^0\pi^0) = P\left(1 - \frac{|P_{\rm ew}|}{P}e^{i\delta_{\rm ew}}\right), \qquad (24)$$

where δ_T and $\delta_{\rm ew}$ denote the strong phases of T and $P_{\rm ew}$, respectively. The $B \to K\pi$ decay amplitudes in Eqs. (21)–(24) are the expansion up to the power of λ, at which the determination of ϕ_3 suffers the theoretical uncertainty of $O(\lambda) \sim 20\%$.

We shall solve for the 6 unknowns: P, $|P_{\rm ew}|$, $|T|$, ϕ_3, and the strong phases $\delta_{\rm ew}$ and δ_T, from the 6 experimental inputs [32,33],

$$\begin{aligned}
&{\rm Br}\left(B^\pm \to K^0\pi^\pm\right) = (20.6 \pm 1.4) \times 10^{-6},\\
&{\rm Br}\left(B_d^0 \to K^\pm\pi^\mp\right) = (18.2 \pm 0.8) \times 10^{-6},\\
&{\rm Br}\left(B^\pm \to K^\pm\pi^0\right) = (12.8 \pm 1.1) \times 10^{-6},\\
&{\rm Br}\left(B_d^0 \to K^0\pi^0\right) = (11.5 \pm 1.7) \times 10^{-6},\\
&\mathcal{A}\left(B_d^0 \to K^\pm\pi^\pm\right) = -(10.2 \pm 5.0)\%,\\
&\mathcal{A}\left(B^\pm \to K^\pm\pi^0\right) = -(9.0 \pm 9.0)\%.
\end{aligned} \qquad (25)$$

The $B^\pm \to K^0\pi^\pm$ and $B_d^0 \to K^0\pi^0$ modes indeed have very small direct CP asymmetries, consistent with the parametrization in Eqs. (21)–(24). The bounds on the various amplitudes and phases can be derived unambiguously from Eq. (25).

The allowed ranges of the ratios T/P and $P_{\rm ew}/P$ are exhibited in Figs. 1 and 2, respectively. The prescription for deriving the two figures is briefly explained below. The data for each branching ratio and for each CP asymmetry are expressed as a set, whose elements are the central value with $+1\times$ error bar, $0\times$ error bar, and $-1\times$ error bar. For a combination of the element from each set, we solve the coupled equations, and the solution is represented by a dot in the figure. Scanning all the combinations, we obtain the ranges in the figures. The central values of the solutions are

$$\begin{aligned}
&\frac{|T|}{P} = 0.23, \qquad \delta_T = -13^\circ,\\
&\frac{|P_{\rm ew}|}{P} = 0.50, \qquad \delta_{\rm ew} = -88^\circ.
\end{aligned} \qquad (26)$$

The above result of T/P is in agreement with the PQCD prediction, $T/P \sim 0.20\exp(-27^\circ i)$ [16,32,34,

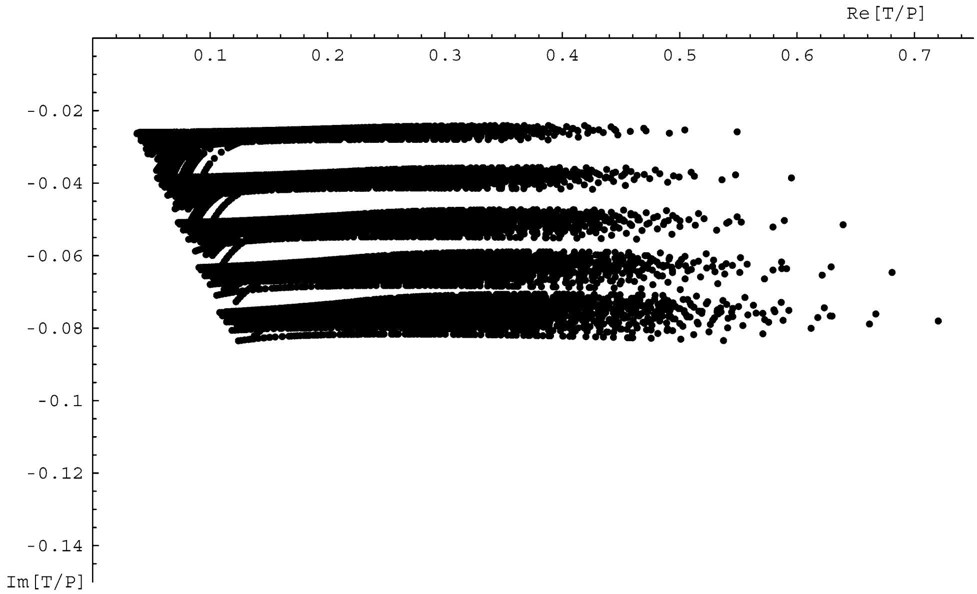

Fig. 1. The allowed range of T/P determined from the $B \to K\pi$ data.

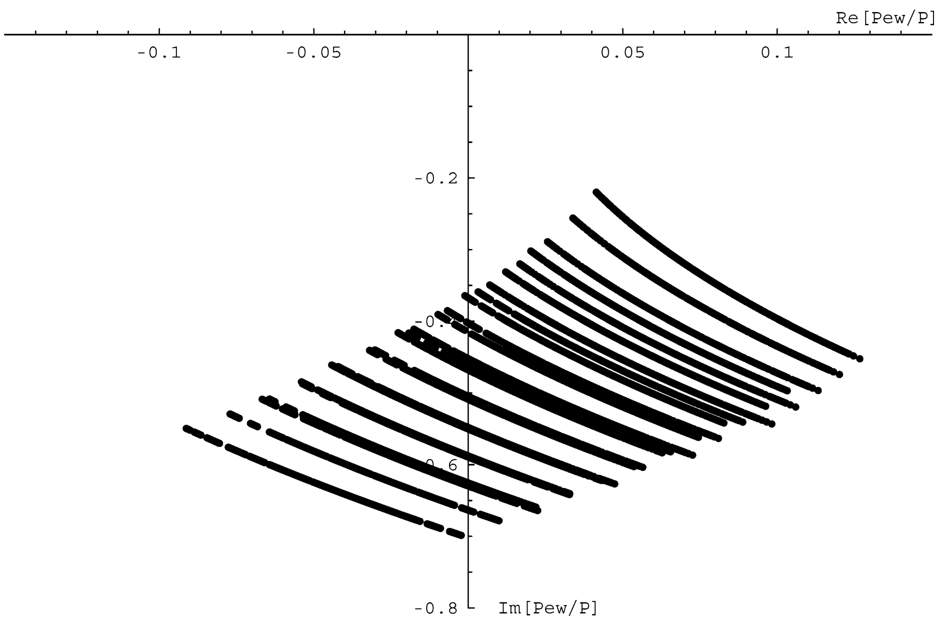

Fig. 2. The allowed range of P_{ew}/P determined from the $B \to K\pi$ data.

35], while the central values of $|P_{ew}|/P$ and of δ_{ew} differ from the PQCD prediction, $|P_{ew}|/P \sim 0.2$ and $\delta_{ew} \approx \delta_T$, respectively. The latter PQCD prediction is consistent with the almost model-independent relation between the electroweak penguin and tree amplitudes obtained in [25,36]. The ratio $|P_{ew}|/P = 0.5$ and the nearly 90° phase between P_{ew} and P in the above fit have been speculated in [27,37,39]. We also derive the allowed ranges $0.06 < |T|/P < 0.72$ and $0.22 < |P_{ew}|/P < 0.70$, implying that the extracted ratios

$|T|/P$ and $|P_{\rm ew}|/P$ deviate a bit from the power counting rules in Eq. (14). Hence, the $B\to K\pi$ data are indeed puzzling, especially from the viewpoint of the dramatically different strong phases $\delta_{\rm ew}$ and δ_T shown in Figs. 1 and 2. Because of the large central values of $|P_{\rm ew}|/P$ and of $\delta_{\rm ew}$, a strong hint of new physics has been claimed in [27,38,39]. A more convincing examination of the self-consistency can be made by solving for the amplitude C, when more complete data are available. At last, the central value and the allowed range of the phase ϕ_3 are given by

$$\phi_3=102^\circ,\quad 26^\circ<\phi_3<151^\circ, \tag{27}$$

respectively, with the theoretical uncertainty of about 20%.

We emphasize that our fitting differs from the global fitting based on the QCDF approach [18,41]. For example, the penguin contributions have been split into the factorizable type depending on a transition form factor, the nonfactorizable type depending on the imaginary infrared cutoff ρ_H for an end-point singularity, and the annihilation type depending on the imaginary infrared cutoff ρ_A in QCDF. Taking into account only the $B\to PP$ modes, such as $B\to K\pi$ and $\pi\pi$, the fitting result of the phase $\phi_3\sim 110^\circ$ [42] is close to that extracted in this work. Our method also differs from those based on the isospin relations [43], with which some combinations of the $B\to K\pi$ branching ratios can be described by the functions of the parameters $P_{\rm ew}/T$, T/P and the relative strong phases. The $SU(3)$ flavor symmetry is then employed to fix $P_{\rm ew}/T$ and T/P. Finally, only the strong phases and the weak phase ϕ_3 are treated as unknowns, and determined by the data. The conclusion is similar: the $B\to K\pi$ data favor $\phi_3\geqslant 90^\circ$. Our approach does not rely on the $SU(3)$ symmetry, and the ratios $P_{\rm ew}/T$ and T/P are treated as unknowns. Including the $B\to VP$ modes in the QCDF fitting, the value of ϕ_3 could be smaller than 90° [41]. Using the parametrization for the $B\to VP$ modes based on $SU(3)$ flavor symmetry, an phase $\phi_3<90^\circ$ was also obtained [44]. In a forthcoming paper we shall apply our parametrization to the $B\to VP$ modes, and make a comparison with the above works.

Next we apply our method to the $B\to\pi\pi$ decays. Considering all possible topologies of amplitudes, their decay amplitudes are given by

$$\begin{aligned}&\sqrt{2}A\left(B^+\to\pi^+\pi^0\right)\\&\quad=-T\left[1+\frac{C}{T}+\left(\frac{P_{\rm ew}}{T}+\frac{P^c_{\rm ew}}{T}+\frac{P^a_{\rm ew}}{T}\right)e^{i\phi_2}\right],\end{aligned} \tag{28}$$

$$A\left(B_d^0\to\pi^+\pi^-\right)=-T\left(1+\frac{T^a}{T}+\frac{P}{T}e^{i\phi_2}\right), \tag{29}$$

$$\begin{aligned}&\sqrt{2}A\left(B_d^0\to\pi^0\pi^0\right)\\&\quad=T\left[\left(\frac{P}{T}-\frac{P_{\rm ew}}{T}-\frac{P^c_{\rm ew}}{T}-\frac{P^a_{\rm ew}}{T}\right)e^{i\phi_2}\right.\\&\qquad\left.-\frac{C}{T}+\frac{T^a}{T}\right],\end{aligned} \tag{30}$$

which satisfy the triangle relation

$$\begin{aligned}&\sqrt{2}A\left(B^+\to\pi^+\pi^0\right)\\&\quad=A\left(B_d^0\to\pi^+\pi^-\right)+\sqrt{2}A\left(B_d^0\to\pi^0\pi^0\right).\end{aligned} \tag{31}$$

In the above expressions the amplitude P has been defined in Eq. (11), and the annihilation contribution T^a comes only from the nonfactorizable diagrams. Based on Eqs. (12)–(14), we assign the power counting rules to the following ratios of the topological amplitudes:

$$\begin{aligned}&\frac{P}{T}\sim\frac{V_{td}V^*_{tb}}{V_{ud}V^*_{ub}}\frac{a_{4,6}}{a_1}\sim\lambda,\qquad \frac{C}{T}\sim\lambda,\\&\frac{P_{\rm ew}}{T}\sim\lambda^2,\qquad \frac{T^a}{T}\sim\frac{M_{\rm nf}}{M_e}\frac{C_2}{a_1N_c}\sim\lambda^2,\\&\frac{P^c_{\rm ew}}{T}\sim\frac{P^a_{\rm ew}}{T}\sim\lambda^4.\end{aligned} \tag{32}$$

Employing the reparametrizations

$$\begin{aligned}&T+T^a\to T,\qquad C-T^a\to C,\\&P_{\rm ew}+P^c_{\rm ew}+P^a_{\rm ew}\to P_{\rm ew},\end{aligned} \tag{33}$$

the most general parametrizations of the $B\to\pi\pi$ decay amplitudes are written as

$$\sqrt{2}A\left(B^+\to\pi^+\pi^0\right)=-T\left[1+\frac{C}{T}+\frac{P_{\rm ew}}{T}e^{i\phi_2}\right], \tag{34}$$

$$A\left(B_d^0\to\pi^+\pi^-\right)=-T\left(1+\frac{P}{T}e^{i\phi_2}\right), \tag{35}$$

$$\sqrt{2}A\left(B_d^0\to\pi^0\pi^0\right)=T\left[\left(\frac{P}{T}-\frac{P_{\rm ew}}{T}\right)e^{i\phi_2}-\frac{C}{T}\right]. \tag{36}$$

There are 4 independent amplitudes, namely, 8 parameters including the phase ϕ_2, which are more than the available measurements. Neglecting the $O(\lambda^2)$ term, $P_{\rm ew}/T$, the resultant expressions are the same as in [22]:

$$\sqrt{2}A\left(B^+ \to \pi^+\pi^0\right) = -T\left(1 + \frac{|C|}{T}e^{i\delta_C}\right), \tag{37}$$

$$A\left(B_d^0 \to \pi^+\pi^-\right) = -T\left(1 + \frac{|P|}{T}e^{i\phi_2}e^{i\delta_P}\right), \tag{38}$$

$$\sqrt{2}A\left(B_d^0 \to \pi^0\pi^0\right) = T\left(\frac{|P|}{T}e^{i\phi_2}e^{i\delta_P} - \frac{|C|}{T}e^{i\delta_C}\right), \tag{39}$$

for which we have 6 unknowns T, $|C|$, $|P|$, δ_C, δ_P and ϕ_2. Similarly, we have removed the strong phase of T, and assumed it to be real and positive.

In this case we have to exclude the data of the direct CP asymmetry in the $B^+ \to \pi^+\pi^0$ decay, and 6 experimental inputs are relevant: the three CP-averaged branching ratios, the direct and mixing-induced CP asymmetries in $B_d^0 \to \pi^+\pi^-$, and the direct CP asymmetry in $B_d^0 \to \pi^0\pi^0$. At this level of accuracy, our parametrization is equivalent to that based on the isospin triangle [2,47], in which the electroweak penguin contribution to the $B^+ \to \pi^+\pi^0$ decay is also ignored. We mention that the electroweak penguin amplitude has been included in the isospin analysis of the $B \to \pi\pi$ decays, and that the CP asymmetry in the $B^\pm \to \pi^\pm\pi^0$ modes still vanishes [45]. After extracting ϕ_2 from the $B \to \pi\pi$ data and ϕ_3 from the $B \to K\pi$ data, we can check whether they, together with ϕ_1 from the $B \to J/\psi K^{(*)}$ data, satisfy the unitarity constraint, when the data precision improves.

The time-dependent CP asymmetry of the $B_d^0 \to \pi^+\pi^-$ mode is expressed as

$$\begin{aligned}
&\mathcal{A}\left(B_d^0(t) \to \pi^+\pi^-\right) \\
&\quad \equiv \frac{B(\bar{B}_d^0(t) \to \pi^+\pi^-) - B(B_d^0(t) \to \pi^+\pi^-)}{B(\bar{B}_d^0(t) \to \pi^+\pi^-) + B(B_d^0(t) \to \pi^+\pi^-)} \\
&\quad = -C_{\pi\pi}\cos(\Delta M_d t) + S_{\pi\pi}\sin(\Delta M_d t),
\end{aligned} \tag{40}$$

where the direct asymmetry $C_{\pi\pi}$ and the mixing-induced asymmetry $S_{\pi\pi}$ are defined by

$$C_{\pi\pi} = \frac{1 - |\lambda_{\pi\pi}|^2}{1 + |\lambda_{\pi\pi}|^2}, \qquad S_{\pi\pi} = \frac{2\,{\rm Im}(\lambda_{\pi\pi})}{1 + |\lambda_{\pi\pi}|^2}, \tag{41}$$

respectively, with the factor,

$$\lambda_{\pi\pi} = e^{2i\phi_2}\frac{1 + e^{-i\phi_2}P/T}{1 + e^{i\phi_2}P/T}. \tag{42}$$

The data are summarized as [46]

$$\begin{aligned}
&{\rm Br}\left(B^\pm \to \pi^\pm\pi^0\right) = (5.2 \pm 0.8) \times 10^{-6}, \\
&{\rm Br}\left(B_d^0 \to \pi^\pm\pi^\mp\right) = (4.6 \pm 0.4) \times 10^{-6}, \\
&{\rm Br}\left(B_d^0 \to \pi^0\pi^0\right) = (1.97 \pm 0.47) \times 10^{-6}, \\
&C_{\pi\pi} = -(38 \pm 16)\%, \\
&S_{\pi\pi} = -(58 \pm 20)\%.
\end{aligned} \tag{43}$$

Since the data of the direct CP asymmetry in the $B_d^0 \to \pi^0\pi^0$ mode is not yet available, we shall assign a plausible range to it,

$$\mathcal{A}\left(B_d^0 \to \pi^0\pi^0\right) = (-50 \sim +50)\%. \tag{44}$$

The central values of the measured $B^\pm \to \pi^\pm\pi^0$ and $B_d^0 \to \pi^\pm\pi^\mp$ branching ratios are close to each other, implying that either C is large and constructive in order to enhance the $B^\pm \to \pi^\pm\pi^0$ modes, or P is large and destructive (after including the weak phase ϕ_2) in order to suppress the $B_d^0 \to \pi^\pm\pi^\mp$ modes [27]. In either case the $B_d^0 \to \pi^0\pi^0$ branching ratio exceeds the expected order of magnitude, $O(10^{-7})$. There exist four solutions associated with each set of data input: two solutions correspond to the large C and P cases, and the other two are the reflections of the first two with respect to the $B^\pm \to \pi^\pm\pi^0$ side of the isospin triangle. Note that the relations of the phase ϕ_2 to the measured quantities have been given in [48] without numerical results. Here we shall not present the central values of the solutions, because the central values of the experimental data of the $B_d^0 \to \pi^0\pi^0$ direct CP asymmetry are not yet available.

The ranges of P/T and C/T, shown in Figs. 3 and 4, respectively, collect all allowed solutions. These ranges indicate that the hierarchy in Eq. (32) is not satisfied, since both $|P|/T$ and $|C|/T$ can be as large as 1, much greater than $O(\lambda) \sim 0.22$. There is then no reason for believing that the effect of the electroweak penguin would be as small as $O(\lambda^2) \sim 5\%$ according to the relation between $P_{\rm ew}$ and T [25,36]. Our analysis implies that the extraction of ϕ_2 from the $B \to \pi\pi$ data based on the isospin symmetry may suffer the theoretical uncertainty more than expected. It also casts a doubt to the PQCD (also QCDF) calculation of the $B \to \pi\pi$ decays. To complete our numerical study, we present the allowed range of ϕ_2 corresponding to the data in Eq. (43),

$$51^\circ < \phi_2 < 176^\circ. \tag{45}$$

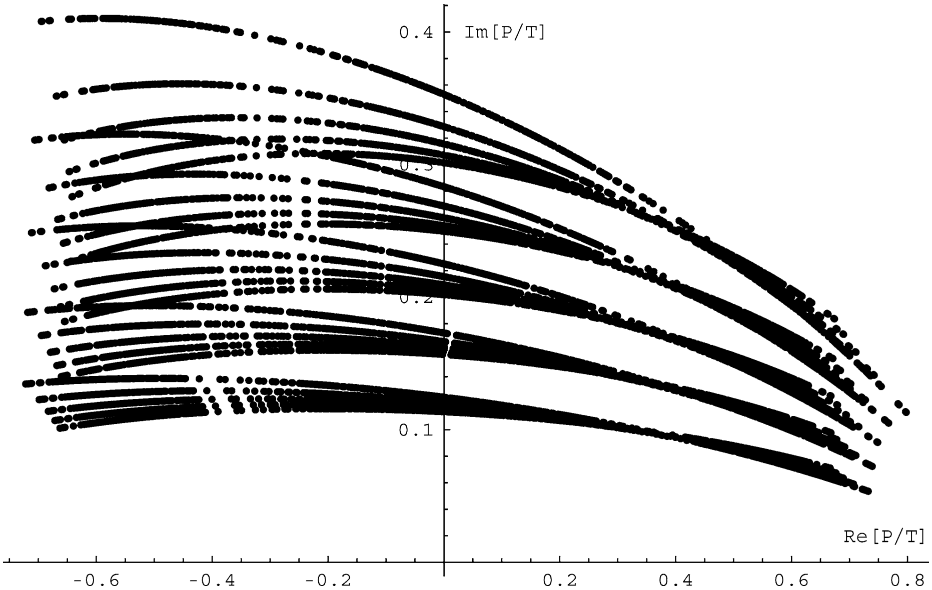

Fig. 3. The allowed range of P/T determined from the $B \to \pi\pi$ data.

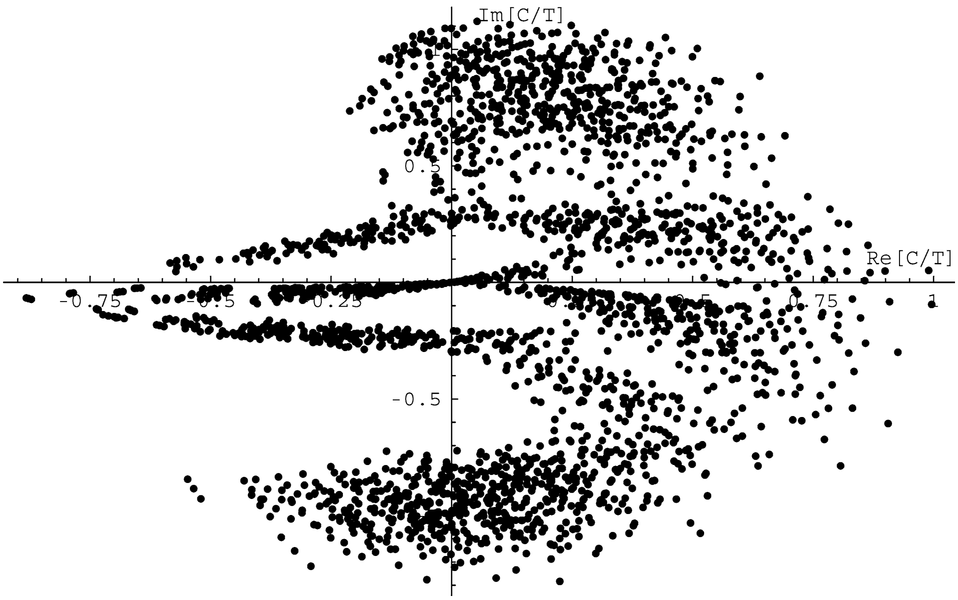

Fig. 4. The allowed range of C/T determined from the $B \to \pi\pi$ data.

As explained above, the theoretical uncertainty associated with the above range may not be under control.

When the data become more precise, and when the data of more CP asymmetries, such as the mixing-induced CP asymmetry in the $B_d^0 \to K_S\pi^0$ mode, are available, the allowed range will shrink, and the theoretical uncertainty can reduce. Our method then tells whether the $B \to K\pi$ data indicate a solid signal of new physics. Besides, our parametrization extends straightforwardly to the other relevant modes, such as $B \to K^*\pi$, from which the phase ϕ_3 can also be extracted [40]. Considering the overlap of the extractions from different modes, the allowed ranges of the decay amplitudes and of ϕ_3 will shrink too. An evaluation of the next-to-leading-order corrections to the $B \to \pi\pi$ decays in the PQCD framework is now in progress, whose result will clarify whether the large $|P|$ or $|C|$ is understandable. If not, new dynamics, such as the rescattering effect, might be important. The $B \to \pi\pi$ decays and the extraction of the phase ϕ_2 then demand more theoretical effort.

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