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## Holographic dark energy with varying gravitational constant

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## ABSTRACT

We investigate the holographic dark energy scenario with a varying gravitational constant, in flat and non-flat background geometry. We extract the exact differential equations determining the evolution of the dark energy density-parameter, which include  $G$ -variation correction terms. Performing a low-redshift expansion of the dark energy equation of state, we provide the involved parameters as functions of the current density parameters, of the holographic dark energy constant and of the  $G$ -variation.

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## 1. Introduction

Recent cosmological observations obtained by SNe Ia [1], WMAP [2], SDSS [3] and X-ray [4] indicate that the universe experiences an accelerated expansion. Although the simplest way to explain this behavior is the consideration of a cosmological constant [5], the two relevant problems (namely the “fine-tuning” and the “coincidence” one) led to the dark energy paradigm. The dynamical nature of dark energy, at least in an effective level, can originate from various fields, such as a canonical scalar field (quintessence) [6], a phantom field, that is a scalar field with a negative sign of the kinetic term [7], or the combination of quintessence and phantom in a unified model named quintom [8].

Although going beyond the above effective description requires a deeper understanding of the underlying theory of quantum gravity [9] unknown at present, physicists can still make some attempts to probe the nature of dark energy according to some basic quantum gravitational principles. An example of such a paradigm is the holographic dark energy scenario, constructed in the light of the holographic principle [10–13] (although the recent developments in Horava gravity could offer a dark energy candidate with perhaps better quantum gravitational foundations [14]). Its framework is the black hole thermodynamics [15] and the connection (known from AdS/CFT correspondence) of the UV cut-off of a quantum field theory, which gives rise to the vacuum energy, with the largest distance of the theory [10]. Thus, determining an appropri-

ate quantity  $L$  to serve as an IR cut-off, imposing the constraint that the total vacuum energy in the corresponding maximum volume must not be greater than the mass of a black hole of the same size, and saturating the inequality, one identifies the acquired vacuum energy as holographic dark energy:

$$\rho_{\Lambda} = \frac{3c^2}{8\pi GL^2}, \quad (1)$$

with  $G$  the Newton's gravitational constant and  $c$  a constant. The holographic dark energy scenario has been tested and constrained by various astronomical observations [16–20] and it has been extended to various frameworks [21–23].

Until now, in all the investigated holographic dark energy models a constant Newton's “constant”  $G$  has been considered. However, there are significant indications that  $G$  can be varying, being a function of time or equivalently of the scale factor [24]. In particular, observations of Hulse–Taylor binary pulsar [25,26], helio-seismological data [27], Type Ia supernova observations [1] and astero-seismological data from the pulsating white dwarf star G117-B15A [29] lead to  $|\dot{G}/G| \lesssim 4.10 \times 10^{-11} \text{ yr}^{-1}$ , for  $z \lesssim 3.5$  [30]. Additionally, a varying  $G$  has some theoretical advantages too, alleviating the dark matter problem [31], the cosmic coincidence problem [32] and the discrepancies in Hubble parameter value [33].

There have been many proposals in the literature attempting to theoretically justified a varying gravitational constant, despite the lack of a full, underlying quantum gravity theory. Starting with the simple but pioneering work of Dirac [34], the varying behavior in Kaluza–Klein theory was associated with a scalar field appearing in the metric component corresponding to the 5-th dimension

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[35] and its size variation [36]. An alternative approach arises from Brans–Dicke framework [37], where the gravitational constant is replaced by a scalar field coupling to gravity through a new parameter, and it has been generalized to various forms of scalar-tensor theories [38], leading to a considerably broader range of variable- $G$  theories. In addition, justification of a varying Newton's constant has been established with the use of conformal invariance and its induced local transformations [39]. Finally, a varying  $G$  can arise perturbatively through a semiclassical treatment of Hilbert–Einstein action [40], non-perturbatively through quantum-gravitational approaches within the “Hilbert–Einstein truncation” [41], or through gravitational holography [42,43].

In this work we are interested in investigating the holographic dark energy paradigm allowing for a varying gravitational constant, and extracting the corresponding corrections to the dark energy equation-of-state parameter. In order to remain general and explore the pure varying- $G$  effects in a model-independent way, we do not use explicitly any additional, geometrical or quintessence-like, scalar field, considering just the Hilbert–Einstein action in an affective level, as it arises from gravitational holography [42, 43]. In other words, we effectively focus on the dark energy and dark matter sectors without examining explicitly the mechanism of  $G$ -variation, which value is considered as an input fixed by observations. Additionally, generality requires to perform our study in flat and non-flat FRW universe. The plan of the work is as follows: In Section 2 we construct the holographic dark energy scenario with a varying Newton's constant and we extract the differential equations that determine the evolution of dark energy density-parameter. In Section 3 we use these expressions in order to calculate the corrections to the dark energy equation-of-state parameter at low redshifts. Finally, in Section 4 we summarize our results.

## 2. Holographic dark energy with varying gravitational constant

### 2.1. Flat FRW geometry

Let us construct holographic dark energy scenario allowing for a varying Newton's constant  $G$ . The space-time geometry will be a flat Robertson–Walker:

$$ds^2 = -dt^2 + a(t)^2(dr^2 + r^2 d\Omega^2), \tag{2}$$

with  $a(t)$  the scale factor and  $t$  the comoving time. As usual, the first Friedmann equation reads:

$$H^2 = \frac{8\pi G}{3}(\rho_m + \rho_\Lambda), \tag{3}$$

with  $H$  the Hubble parameter,  $\rho_m = \frac{\rho_{m0}}{a^3}$ , where  $\rho_m$  and  $\rho_\Lambda$  stand respectively for matter and dark energy densities and the index 0 marks the present value of a quantity. Furthermore, we will use the density parameter  $\Omega_\Lambda \equiv \frac{8\pi G}{3H^2} \rho_\Lambda$ , which, imposing explicitly the holographic nature of dark energy according to relation (1), becomes

$$\Omega_\Lambda = \frac{c^2}{H^2 L^2}. \tag{4}$$

Finally, in the case of a flat universe, the best choice for the definition of  $L$  is to identify it with the future event horizon [12,13,43, 44], that is  $L \equiv R_h(a)$  with

$$R_h(a) = a \int_t^\infty \frac{dt'}{a(t')} = a \int_a^\infty \frac{da'}{Ha'^2}. \tag{5}$$

In the following we will use  $\ln a$  as an independent variable. Thus, denoting by dot the time-derivative and by prime the derivative with respect to  $\ln a$ , for every quantity  $F$  we acquire  $\dot{F} = F'H$ . Differentiating (4) using (5), and observing that  $\dot{R}_h = HR_h - 1$ , we obtain:

$$\frac{\Omega'_\Lambda}{\Omega_\Lambda^2} = \frac{2}{\Omega_\Lambda} \left[ -1 - \frac{\dot{H}}{H^2} + \frac{\sqrt{\Omega_\Lambda}}{c} \right]. \tag{6}$$

Until now, the varying behavior of  $G$  has not become manifested. However, the next step is to eliminate  $\dot{H}$ . This can be obtained by differentiating Friedman equation (3), leading to

$$2 \frac{\dot{H}}{H^2} = -3 + \Omega_\Lambda \left( 1 + \frac{2\sqrt{\Omega_\Lambda}}{c} \right) + \frac{G'}{G} (1 - \Omega_\Lambda), \tag{7}$$

where  $G$  is considered to be a function of  $\ln a$ . In the extraction of this relation we have additionally used the auxiliary expression

$$\rho'_\Lambda = \rho_\Lambda \left( -\frac{G'}{G} - 2 + \frac{2\sqrt{\Omega_\Lambda}}{c} \right), \tag{8}$$

which arises from differentiation of (1). Therefore, substituting (7) into (6) we finally obtain:

$$\Omega'_\Lambda = \Omega_\Lambda (1 - \Omega_\Lambda) \left[ 1 + \frac{2\sqrt{\Omega_\Lambda}}{c} \right] - \Omega_\Lambda (1 - \Omega_\Lambda) \frac{G'}{G}. \tag{9}$$

The first term is the usual holographic dark energy differential equation [13]. The second term is the correction arising from the varying nature of  $G$ . Note that  $G'/G$  is a pure number as expected.

Finally, for completeness, we present the general solution for arbitrary  $c$  and  $G'/G \equiv \Delta_G$ , which in an implicit form reads

$$\begin{aligned} & \frac{\ln a}{c} + x_0 \\ &= \frac{\ln \Omega_\Lambda}{c(1 - \Delta_G)} - \frac{\ln(1 - \sqrt{\Omega_\Lambda})}{2 + c(1 - \Delta_G)} \\ &+ \frac{\ln(1 + \sqrt{\Omega_\Lambda})}{2 + c(\Delta_G - 1)} - \frac{8 \ln[c(1 - \Delta_G) + 2\sqrt{\Omega_\Lambda}]}{c(\Delta_G - 1)[c^2(\Delta_G - 1)^2 - 4]}. \end{aligned} \tag{10}$$

The constant  $x_0$  can be straightforwardly calculated if we determine  $a_0$  and  $\Omega_\Lambda^0$  today (for example choosing  $a_0 = 1$   $x_0$  is equal to the left-hand side with  $\Omega_\Lambda$  replaced by  $\Omega_\Lambda^0$ ). Clearly, for  $\Delta_G = 0$  and  $c = 1$ , expression (10) coincides with that of [13].

### 2.2. Non-flat FRW geometry

In this subsection we generalize the aforementioned analysis in the case of a general FRW universe with line element

$$ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right) \tag{11}$$

in comoving coordinates  $(t, r, \theta, \varphi)$ , where  $k$  denotes the spacial curvature with  $k = -1, 0, 1$  corresponding to open, flat and closed universe, respectively. In this case, the first Friedmann equation writes:

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}(\rho_m + \rho_\Lambda). \tag{12}$$

According to the formulation of holographic dark energy in non-flat geometry, the cosmological length  $L$  in (4) is considered to be [21]:

$$L \equiv \frac{a(t)}{\sqrt{|k|}} \operatorname{sinn} \left( \frac{\sqrt{|k|} R_h}{a(t)} \right), \tag{13}$$

where

$$\frac{1}{\sqrt{|k|}} \operatorname{sinn}(\sqrt{|k|}y) = \begin{cases} \sin y & k = +1, \\ y & k = 0, \\ \sinh y & k = -1. \end{cases} \quad (14)$$

A straightforward calculation leads to

$$\dot{L} = HL - \operatorname{cosn}\left(\frac{\sqrt{|k|}R_h}{a}\right), \quad (15)$$

where

$$\operatorname{cosn}(\sqrt{|k|}y) = \begin{cases} \cos y & k = +1, \\ 1 & k = 0, \\ \cosh y & k = -1. \end{cases} \quad (16)$$

Repeating the procedure of the previous sub-section and differentiating (4) using (13) and (15) we obtain:

$$\frac{\Omega'_\Lambda}{\Omega_\Lambda^2} = \frac{2}{\Omega_\Lambda} \left( -1 - \frac{\dot{H}}{H^2} + \frac{\sqrt{\Omega_\Lambda}}{c} \operatorname{cosn}(\sqrt{|k|}y) \right). \quad (17)$$

On the other hand, differentiating Friedmann equation (12) we finally obtain

$$2\frac{\dot{H}}{H^2} = -3 - \Omega_k + \Omega_\Lambda + 2\frac{\Omega_\Lambda^{3/2}}{c} \operatorname{cosn}\left(\frac{\sqrt{|k|}R_h}{a}\right) + (1 + \Omega_k - \Omega_\Lambda)\frac{G'}{G}, \quad (18)$$

where we have introduced the curvature density parameter  $\Omega_k \equiv \frac{k}{(aH)^2}$ . Therefore, substituting (18) into (17) we result to

$$\Omega'_\Lambda = \Omega_\Lambda \left[ 1 + \Omega_k - \Omega_\Lambda + \frac{2\sqrt{\Omega_\Lambda}}{c} \operatorname{cosn}\left(\frac{\sqrt{|k|}R_h}{a}\right) (1 - \Omega_\Lambda) \right] - \Omega_\Lambda (1 + \Omega_k - \Omega_\Lambda) \frac{G'}{G}. \quad (19)$$

Expression (19) provides the correction to holographic dark energy differential in non-flat universe, due to the varying nature of  $G$ . Clearly, for  $k=0$  (and thus  $\Omega_k=0$ ) it leads to (9).

### 3. Cosmological implications

Since we have extracted the expressions for  $\Omega'_\Lambda$ , we can calculate  $w(z)$  for small redshifts  $z$ , performing the standard expansions of the literature. In particular, since  $\rho_\Lambda \sim a^{-3(1+w)}$  we acquire

$$\ln \rho_\Lambda = \ln \rho_\Lambda^0 + \frac{d \ln \rho_\Lambda}{d \ln a} \ln a + \frac{1}{2} \frac{d^2 \ln \rho_\Lambda}{d(\ln a)^2} (\ln a)^2 + \dots, \quad (20)$$

where the derivatives are taken at the present time  $a_0 = 1$  (and thus at  $\Omega_\Lambda = \Omega_\Lambda^0$ ). Then,  $w(\ln a)$  is given as

$$w(\ln a) = -1 - \frac{1}{3} \left[ \frac{d \ln \rho_\Lambda}{d \ln a} + \frac{1}{2} \frac{d^2 \ln \rho_\Lambda}{d(\ln a)^2} \ln a \right], \quad (21)$$

up to second order. Since  $\rho_\Lambda = 3H^2 \Omega_\Lambda / (8\pi G) = \Omega_\Lambda \rho_m / \Omega_m = \rho_{m0} \Omega_\Lambda / (1 + \Omega_k - \Omega_\Lambda) a^{-3}$ , the derivatives are easily computed using the obtained expressions for  $\Omega'_\Lambda$ . In addition, we can straightforwardly calculate  $w(z)$ , replacing  $\ln a = -\ln(1+z) \simeq -z$ , which is valid for small redshifts, defining

$$w(z) = -1 - \frac{1}{3} \left( \frac{d \ln \rho_\Lambda}{d \ln a} \right) + \frac{1}{6} \left[ \frac{d^2 \ln \rho_\Lambda}{d(\ln a)^2} \right] z \equiv w_0 + w_1 z. \quad (22)$$

The role of  $G$ -variation will be expressed through the pure number  $G'/G \equiv \Delta_G$ , which will be extracted from observations. In

particular, observations of Hulse–Taylor binary pulsar B1913 + 16 lead to the estimation  $\dot{G}/G \sim 2 \pm 4 \times 10^{-12} \text{ yr}^{-1}$  [25,26], while helio-seismological data provide the bound  $-1.6 \times 10^{-12} \text{ yr}^{-1} < \dot{G}/G < 0$  [27]. Similarly, Type Ia supernova observations [1] give the best upper bound of the variation of  $G$  as  $-10^{-11} \text{ yr}^{-1} \leq \dot{G}/G < 0$  at redshifts  $z \simeq 0.5$  [28], while astereoseismological data from the pulsating white dwarf star G117-B15A lead to  $|\dot{G}/G| \leq 4.10 \times 10^{-11} \text{ yr}^{-1}$  [29]. See also [30] for various bounds on  $\dot{G}/G$  from observational data, noting that all these measurements are valid at relatively low redshifts, i.e.  $z \lesssim 3.5$ .

Since the limits in  $G$ -variation are given for  $\dot{G}/G$  in units  $\text{yr}^{-1}$ , and since  $\dot{G}/G = HG'/G$ , we can estimate  $\Delta_G$  substituting the value of  $H$  in  $\text{yr}^{-1}$ . In the following we will use  $|\dot{G}/G| \lesssim 4.10 \times 10^{-11} \text{ yr}^{-1}$ . Thus, inserting an average estimation for the Hubble parameter  $H \approx \langle H \rangle \approx 6 \times 10^{-11} \text{ yr}^{-1}$  [45], we obtain that  $0 < |\Delta_G| \lesssim 0.07$ . Clearly, this estimation is valid at low redshifts, since only in this range the measurements of  $\dot{G}/G$  and the estimation of the average  $\langle H \rangle$  are valid. However, the restriction to this range is consistent with the  $z$ -expansion of  $w$  considered above.

#### 3.1. Flat FRW geometry

In this case  $\Omega'_\Lambda$  is given by (9), and the aforementioned procedure leads to

$$w_0 = -\frac{1}{3} - \frac{2}{3c} \sqrt{\Omega_\Lambda^0} + \frac{\Delta_G}{3}, \quad (23)$$

$$w_1 = \frac{1}{6c} \sqrt{\Omega_\Lambda^0} (1 - \Omega_\Lambda^0) \left( 1 + \frac{2}{c} \sqrt{\Omega_\Lambda^0} \right) - \frac{(1 - \Omega_\Lambda^0) \sqrt{\Omega_\Lambda^0}}{6c} \Delta_G. \quad (24)$$

These expressions provide  $w_0$  and  $w_1$ , for the holographic dark energy with varying  $G$ , in a flat universe. Obviously, when  $\Delta_G = 0$ , they coincide with those of [13].

In general, apart from the relevant uncertainty in  $\Omega_\Lambda^0$  measurements, we face the problem of the uncertainty in the constant  $c$ . In particular, observational data from type Ia supernovae give the best-fit value  $c = 0.21$  within  $1-\sigma$  error range [16], while those from the X-ray gas mass fraction of galaxy clusters lead to  $c = 0.61$  within  $1-\sigma$  [17]. Similarly, combining data from type Ia supernovae, Cosmic Microwave Background radiation and large scale structure give the best-fit value  $c = 0.91$  within  $1-\sigma$  [18], while combining data from type Ia supernovae, X-ray gas and Baryon Acoustic Oscillation lead to  $c = 0.73$  as a best-fit value within  $1-\sigma$  [19]. However, expressions (23), (24) provide the pure change due to the variation of gravitational constant for given  $c$  and  $\Omega_\Lambda^0$ . For example, and in order to compare with the corresponding result of [13], imposing  $\Omega_\Lambda^0 \approx 0.73$  and  $c = 1$ , and using  $0 < |\Delta_G| < 0.07$  we obtain:

$$w_0 = -0.903_{-0.023}^{+0.023}, \\ w_1 = 0.1041_{-0.0025}^{+0.0025}, \quad (25)$$

where we have neglected uncertainties other than  $G$ -variation. Finally, note that the  $w_0$ -variation due to  $\Delta_G$  is absolute, that is it does not depend on  $c$  and  $\Omega_\Lambda^0$ , while that of  $w_1$  does depend on these parameters. However, the relative variations of  $w_0$ ,  $w_1$  do depend on the  $c$ -value, and they are smaller for smaller  $c$ .

#### 3.2. Non-flat FRW geometry

In this case  $\Omega'_\Lambda$  is given by (19), and the aforementioned procedure leads to

$$w_0 = -\frac{1}{3} - \frac{2}{3c} \sqrt{\Omega_\Lambda^0} \cosn \frac{\sqrt{|k|} R_{h0}}{a_0} + \frac{\Delta_G}{3}, \quad (26)$$

$$w_1 = \frac{\sqrt{\Omega_\Lambda^0}}{6c} \left[ 1 + \Omega_k^0 - \Omega_\Lambda^0 + \frac{2\sqrt{\Omega_\Lambda^0}}{c} \cosn \left( \frac{\sqrt{|k|} R_{h0}}{a_0} \right) (1 - \Omega_\Lambda^0) \right] \times \cosn \left( \frac{\sqrt{|k|} R_{h0}}{a_0} \right) + \frac{\Omega_\Lambda^0}{3c^2} q \left( \frac{\sqrt{|k|} R_{h0}}{a_0} \right) - \frac{\sqrt{\Omega_\Lambda^0}}{6c} (1 + \Omega_k^0 - \Omega_\Lambda^0) \cosn \left( \frac{\sqrt{|k|} R_{h0}}{a_0} \right) \Delta_G. \quad (27)$$

In these expressions,  $\Omega_k^0$  is the present day value of the curvature density parameter, and we have defined

$$q(\sqrt{|k|}y) = \begin{cases} \sin^2 y & k = +1, \\ 0 & k = 0, \\ -\sinh^2 y & k = -1. \end{cases} \quad (28)$$

Finally,  $R_{h0}$  and  $a_0$  are the present values of the corresponding quantities. Clearly, for  $k=0$ , that is for a flat geometry, (26), (27) coincide with (23), (24), respectively.

As we observe, expressions (26), (27), apart from the present values of the parameters  $\Omega_\Lambda^0$ ,  $\Omega_k^0$  contain  $a_0$  and the value of  $R_{h0}$  at present. This last term is present in a non-flat universe, and it is a “non-local” quantity which has to be calculated by an integration (see relations (13) and (5)). However, making use of the holographic nature of dark energy, we can overcome this difficulty. Indeed, from (4) we obtain that  $L_0 = c/(H_0\sqrt{\Omega_\Lambda^0})$ , with  $H_0$  the present value of the Hubble parameter. On the other hand, from (13) we acquire  $R_{h0}/a_0 = \frac{1}{\sqrt{|k|}} \text{sinn}^{-1}(\sqrt{|k|}L_0/a_0)$ . Therefore, we conclude that

$$\frac{R_{h0}}{a_0} = \frac{1}{\sqrt{|k|}} \text{sinn}^{-1} \left( \frac{c\sqrt{|k|}}{a_0 H_0 \sqrt{\Omega_\Lambda^0}} \right) = \frac{1}{\sqrt{|k|}} \text{sinn}^{-1} \left( \frac{c\sqrt{|\Omega_k^0|}}{\sqrt{\Omega_\Lambda^0}} \right), \quad (29)$$

a relation which proves very useful. Substituting into (26), (27) we finally obtain the simple expressions:

$$w_0 = -\frac{1}{3} - \frac{2}{3c} \sqrt{\Omega_\Lambda^0 - c^2 \Omega_k^0} + \frac{\Delta_G}{3}, \quad (30)$$

$$w_1 = \frac{\Omega_k^0}{3} + \frac{1}{6c} \sqrt{\Omega_\Lambda^0 - c^2 \Omega_k^0} \left[ 1 + \Omega_k^0 - \Omega_\Lambda^0 + \frac{2}{c} (1 - \Omega_\Lambda^0) \sqrt{\Omega_\Lambda^0 - c^2 \Omega_k^0} \right] - \frac{1}{6c} \sqrt{\Omega_\Lambda^0 - c^2 \Omega_k^0} (1 + \Omega_k^0 - \Omega_\Lambda^0) \Delta_G. \quad (31)$$

Note that  $w_0$ ,  $w_1$  depend eventually only on  $\Omega_\Lambda^0$ ,  $\Omega_k^0$ ,  $c$  and of course  $\Delta_G$ . Similarly to the previous subsection, in order to give a representative estimation and neglecting uncertainties of other quantities apart from  $G$ -variation, we use  $c=1$ ,  $\Omega_\Lambda^0 \approx 0.73$ ,  $\Omega_k^0 \approx 0.02$ ,  $0 < |\Delta_G| < 0.07$ , obtaining:

$$w_0 = -0.895_{-0.023}^{+0.023},$$

$$w_1 = 0.111_{-0.003}^{+0.003}. \quad (32)$$

Finally, we mention that the relative variations of  $w_0$ ,  $w_1$  depend on the  $c$ -value, and they are smaller for smaller  $c$ .

#### 4. Conclusions

In this work we have investigated the holographic dark energy scenario with a varying gravitational constant, going beyond the simple scenarios of [46]. Imposing flat and non-flat background geometry we have extracted the exact differential equations that determine the evolution of the dark energy density-parameter, where the  $G$ -variation appears as a coefficient in additional terms. Thus, performing a low-redshift expansion of the dark energy equation-of-state parameter  $w(z) \approx w_0 + w_1 z$ , we provide  $w_0$ ,  $w_1$  as functions of  $\Omega_\Lambda^0$ ,  $\Omega_k^0$ , of the holographic dark energy constant  $c$ , and of the  $G$ -variation  $\Delta_G$  (expressions (30), (31)). As expected, the variation of the gravitational constant increases the variation of  $w(z)$ .

In the aforementioned analysis, the  $G$ -variation has been considered as a constant quantity at the cosmological epoch of interest, that is at low redshifts, as it is measured in observations with satisfactory accuracy [25–30]. A step forward would be to consider possible  $G(z)$ -parametrizations [47,48] and extract their effect on  $w(z)$ . However, such parametrizations have a significant amount of arbitrariness, since the present observational data do not allow for such a resolution, and thus we have not performed this extension in the present work.

Finally, we mention that in general, the possible uncertainty of the constant  $c$  can have a larger effect on  $w(z)$  than that of  $G$ -variation. In the above investigation we have just provided the complete expressions, including the correction terms due to the variation of the gravitational constant. One could proceed to a combined observational constraint analysis, allowing for variations and uncertainties in all parameters, as it was partially performed in the specific Brans–Dicke framework in [47]. This extended examination, with not-guaranteed results due to complexity, is under current investigation and it is left for a future publication.

#### References

- [1] A.G. Riess, et al., Supernova Search Team Collaboration, *Astron. J.* 116 (1998) 1009; S. Perlmutter, et al., Supernova Cosmology Project Collaboration, *Astrophys. J.* 517 (1999) 565.
- [2] C.L. Bennett, et al., *Astrophys. J. Suppl.* 148 (2003) 1.
- [3] M. Tegmark, et al., SDSS Collaboration, *Phys. Rev. D* 69 (2004) 103501.
- [4] S.W. Allen, et al., *Mon. Not. R. Astron. Soc.* 353 (2004) 457.
- [5] V. Sahni, A. Starobinsky, *Int. J. Mod. Phys. D* 9 (2000) 373; P.J. Peebles, B. Ratra, *Rev. Mod. Phys.* 75 (2003) 559.
- [6] B. Ratra, P.J.E. Peebles, *Phys. Rev. D* 37 (1988) 3406; C. Wetterich, *Nucl. Phys. B* 302 (1988) 668; A.R. Liddle, R.J. Scherrer, *Phys. Rev. D* 59 (1999) 023509; I. Zlatev, L.M. Wang, P.J. Steinhardt, *Phys. Rev. Lett.* 82 (1999) 896; Z.K. Guo, N. Ohta, Y.Z. Zhang, *Mod. Phys. Lett. A* 22 (2007) 883; S. Dutta, E.N. Saridakis, R.J. Scherrer, *Phys. Rev. D* 79 (2009) 103005, arXiv: 0903.3412 [astro-ph.CO].
- [7] R.R. Caldwell, *Phys. Lett. B* 545 (2002) 23; R.R. Caldwell, M. Kamionkowski, N.N. Weinberg, *Phys. Rev. Lett.* 91 (2003) 071301; S. Nojiri, S.D. Odintsov, *Phys. Lett. B* 562 (2003) 147; V.K. Onemli, R.P. Woodard, *Phys. Rev. D* 70 (2004) 107301; M.R. Setare, J. Sadeghi, A.R. Amani, *Phys. Lett. B* 666 (2008) 288; X.m. Chen, Y.g. Gong, E.N. Saridakis, *JCAP* 0904 (2009) 001; E.N. Saridakis, *Nucl. Phys. B* 819 (2009) 116.
- [8] B. Feng, X.L. Wang, X.M. Zhang, *Phys. Lett. B* 607 (2005) 35; Z.K. Guo, et al., *Phys. Lett. B* 608 (2005) 177; M.-Z. Li, B. Feng, X.-M. Zhang, *JCAP* 0512 (2005) 002; B. Feng, M. Li, Y.-S. Piao, X. Zhang, *Phys. Lett. B* 634 (2006) 101; M.R. Setare, *Phys. Lett. B* 641 (2006) 130; W. Zhao, Y. Zhang, *Phys. Rev. D* 73 (2006) 123509; M.R. Setare, J. Sadeghi, A.R. Amani, *Phys. Lett. B* 660 (2008) 299; J. Sadeghi, M.R. Setare, A. Banijamali, F. Milani, *Phys. Lett. B* 662 (2008) 92;

- M.R. Setare, E.N. Saridakis, Phys. Lett. B 668 (2008) 177;  
M.R. Setare, E.N. Saridakis, JCAP 0809 (2008) 026;  
M.R. Setare, E.N. Saridakis, Int. J. Mod. Phys. D 18 (2009) 549.
- [9] E. Witten, hep-ph/0002297.
- [10] A.G. Cohen, D.B. Kaplan, A.E. Nelson, Phys. Rev. Lett. 82 (1999) 4971.
- [11] G. 't Hooft, gr-qc/9310026;  
L. Susskind, J. Math. Phys. 36 (1995) 6377;  
P. Horava, D. Minic, Phys. Rev. Lett. 85 (2000) 1610;  
S.D. Thomas, Phys. Rev. Lett. 89 (2002) 081301.
- [12] S.D.H. Hsu, Phys. Lett. B 594 (2004) 13.
- [13] M. Li, Phys. Lett. B 603 (2004) 1.
- [14] P. Horava, Phys. Rev. D 79 (2009) 084008;  
G. Calcagni, arXiv:0904.0829 [hep-th];  
H. Lu, J. Mei, C.N. Pope, arXiv:0904.1595 [hep-th];  
C. Charmousis, G. Niz, A. Padilla, P.M. Saffin, arXiv:0905.2579 [hep-th];  
E.N. Saridakis, arXiv:0905.3532 [hep-th];  
X. Gao, Y. Wang, R. Brandenberger, A. Riotto, arXiv:0905.3821 [hep-th];  
M.i. Park, arXiv: 0905.4480 [hep-th];  
Y.F. Cai, E.N. Saridakis, arXiv:0906.1789 [hep-th];  
M. Botta-Cantcheff, N. Grandi, M. Sturla, arXiv:0906.0582 [hep-th].
- [15] R.C. Myers, M.J. Perry, Ann. Phys. 172 (1986) 304;  
P. Kanti, K. Tamvakis, Phys. Rev. D 68 (2003) 024014.
- [16] Q.G. Huang, Y.G. Gong, JCAP 0408 (2004) 006.
- [17] Z. Chang, F.Q. Wu, X. Zhang, Phys. Lett. B 633 (2006) 14.
- [18] X. Zhang, F.Q. Wu, Phys. Rev. D 72 (2005) 043524.
- [19] Q. Wu, Y. Gong, A. Wang, J.S. Alcaniz, Phys. Lett. B 659 (2008) 34;  
Y.Z. Ma, Y. Gong, Eur. Phys. J. C 60 (2009) 303.
- [20] K. Enqvist, S. Hannestad, M.S. Sloth, JCAP 0502 (2005) 004;  
J. Shen, B. Wang, E. Abdalla, R.K. Su, Phys. Lett. B 609 (2005) 200;  
H.C. Kao, W.L. Lee, F.L. Lin, Phys. Rev. D 71 (2005) 123518.
- [21] Q.G. Huang, M. Li, JCAP 0408 (2004) 013.
- [22] M. Ito, Europhys. Lett. 71 (2005) 712;  
K. Enqvist, M.S. Sloth, Phys. Rev. Lett. 93 (2004) 221302;  
Q.G. Huang, M. Li, JCAP 0503 (2005) 001;  
D. Pavon, W. Zimdahl, Phys. Lett. B 628 (2005) 206;  
B. Wang, Y. Gong, E. Abdalla, Phys. Lett. B 624 (2005) 141;  
H. Kim, H.W. Lee, Y.S. Myung, Phys. Lett. B 632 (2006) 605;  
S. Nojiri, S.D. Odintsov, Gen. Relativ. Gravit. 38 (2006) 1285;  
E. Elizalde, S. Nojiri, S.D. Odintsov, P. Wang, Phys. Rev. D 71 (2005) 103504;  
B. Hu, Y. Ling, Phys. Rev. D 73 (2006) 123510;  
H. Li, Z.K. Guo, Y.Z. Zhang, Int. J. Mod. Phys. D 15 (2006) 869;  
M.R. Setare, Phys. Lett. B 642 (2006) 1;  
M.R. Setare, Phys. Lett. B 642 (2006) 421;  
E.N. Saridakis, Phys. Lett. B 660 (2008) 138;  
E.N. Saridakis, JCAP 0804 (2008) 020;  
E.N. Saridakis, Phys. Lett. B 661 (2008) 335.
- [23] L. Amendola, Phys. Rev. D 62 (2000) 043511;  
D. Comelli, M. Pietroni, A. Riotto, Phys. Lett. B 571 (2003) 115;  
M.R. Setare, JCAP 0701 (2007) 023;  
M. Jamil, M.A. Rashid, Eur. Phys. J. C 60 (2009) 141;  
M. Jamil, M.A. Rashid, Eur. Phys. J. C 58 (2008) 111;  
M.R. Setare, E.N. Saridakis, Phys. Lett. B 670 (2008) 1.
- [24] S. D'Innocenti, G. Fiorentini, G.G. Raffelt, B. Ricci, A. Weiss, Astron. Astro-phys. 312 (1996) 345;  
K. Umezū, K. Ichiki, M. Yahiro, Phys. Rev. D 72 (2005) 044010;  
S. Nesseris, L. Perivolaropoulos, Phys. Rev. D 73 (2006) 103511;  
J.P.W. Verbiest, et al., arXiv:astro-ph/0801.2589.
- [25] G.S. Bisnovatyi-Kogan, Int. J. Mod. Phys. D 15 (2006) 1047.
- [26] T. Damour, et al., Phys. Rev. Lett. 61 (1988) 1151.
- [27] D.B. Guenther, Phys. Lett. B 498 (1998) 871.
- [28] E. Gaztanaga, E. Garcia-Berro, J. Isern, E. Bravo, I. Dominguez, Phys. Rev. D 65 (2002) 023506.
- [29] M. Biesiada, B. Malec, Mon. Not. R. Astron. Soc. 350 (2004) 644.
- [30] S. Ray, U. Mukhopadhyay, Int. J. Mod. Phys. D 16 (2007) 1791.
- [31] I. Goldman, Phys. Lett. B 281 (1992) 219.
- [32] M. Jamil, F. Rahaman, M. Kalam, Eur. Phys. J. C 60 (2009) 149.
- [33] O. Bertolami, et al., Phys. Lett. B 311 (1993) 27.
- [34] P.A.M. Dirac, Proc. R. Soc. London A 165 (1938) 199.
- [35] T. Kaluza, Sitz. d. Preuss. Akad. d. Wiss. Physik-Mat. Klasse (1921) 966.
- [36] P.G.O. Freund, Nucl. Phys. B 209 (1982) 146;  
K. Maeda, Class. Quantum Grav. 3 (1986) 233;  
E.W. Kolb, M.J. Perry, T.P. Walker, Phys. Rev. D 33 (1986) 869;  
P. Loré-Aguilar, E. Garcé-Berro, J. Isern, Yu.A. Kubyshin, Class. Quantum Grav. 20 (2003) 3885.
- [37] C.H. Brans, R.H. Dicke, Phys. Rev. 124 (1961) 925.
- [38] P.G. Bergmann, Int. J. Theor. Phys. 1 (1968) 25;  
R.V. Wagoner, Phys. Rev. D 1 (1970) 3209;  
K. Nordtvedt, Astrophys. J. 161 (1970) 1059.
- [39] J.D. Bekenstein, Found. Phys. 16 (1986) 409.
- [40] I.L. Shapiro, J. Sola, JHEP 0202 (2002) 006;  
A. Babić, B. Guberina, R. Horvat, H. Štefančić, Phys. Rev. D 65 (2002) 085002;  
I.L. Shapiro, J. Sola, C. Espana-Bonet, P. Ruiz-Lapuente, Phys. Lett. B 574 (2003) 149;  
B. Guberina, R. Horvat, H. Štefančić, Phys. Rev. D 67 (2003) 083001;  
C. Espana-Bonet, P. Ruiz-Lapuente, I.L. Shapiro, J. Sola, JCAP 0402 (2004) 006.
- [41] M. Reuter, Phys. Rev. D 57 (1998) 971;  
A. Bonnano, M. Reuter, Phys. Rev. D 65 (2002) 043508.
- [42] R. Horvat, Phys. Rev. D 70 (2004) 087301.
- [43] B. Guberina, R. Horvat, H. Nikolic, Phys. Rev. D 72 (2005) 125011.
- [44] Y. Gong, Phys. Rev. D 70 (2004) 064029;  
M.R. Setare, JCAP 0701 (2007) 023.
- [45] Q.J. Zhang, Y.L. Wu, arXiv:0905.1234 [astro-ph.CO].
- [46] B. Guberina, R. Horvat, H. Štefančić, JCAP 0505 (2005) 001;  
B. Guberina, R. Horvat, H. Nikolic, Phys. Lett. B 636 (2006) 80;  
B. Guberina, R. Horvat, H. Nikolic, JCAP 0701 (2007) 012.
- [47] S. Galli, A. Melchiorri, G.F. Smoot, O. Zahn, arXiv:0905.1808 [astro-ph.CO].
- [48] L. Xu, J. Lu, Eur. Phys. J. C 60 (2009) 135;  
L. Xu, J. Lu, W. Li, arXiv:0905.4174 [astro-ph.CO].