Simulating Lane-Changing Dynamics towards Lane-Flow Equilibrium Based on Multi-Lane First Order Traffic Flow Model

Yasuhiro SHIOMI\textsuperscript{a}*, Tomoki TANIGUCHI\textsuperscript{b}, Nobuhiro UNO\textsuperscript{c}, Hiroshi SHIMAMOTO\textsuperscript{d}, Toshiyuki NAKAMURA\textsuperscript{b}

\textsuperscript{a}Dept. of Environmental Systems Eng., Ritsumeikan University, 1-1-1 Nojihigashi, 5258577 Kusatsu, Japan
\textsuperscript{b}Grad. School of Eng., Kyoto University, C-1-2 Kyoto Univ. Katsura, 6158540 Kyoto, Japan
\textsuperscript{c}Grad. School of Management, Kyoto University, Yoshida-honnachi. 6068501 Kyoto, Japan
\textsuperscript{d}Dept. of Civil and Environmental Eng., University of Miyazaki, 1-1 Gakuen Kibanadai Nishi. 8892192 Miyazaki, Japan

Abstract

It is well known that under the condition of high traffic volume lane-flow distribution becomes unbalanced; more traffic tends to use a median lane rather than a middle and outer lane, which causes the deterioration of traffic capacity at bottleneck sections. As intensive development of ITS, active and dynamic lane management has been practically implemented. By employing the technology of ITS, balancing lane-flow distribution is one of the feasible solutions to increase the throughput of bottleneck flow. However, the mechanism of the unbalanced usage among lanes is still unclear and traffic flow models enabling online and network-wide evaluation of dynamic and strategic lane management have not been proposed due to the lack of the method to computing lane-based traffic flow including lane-changing dynamics. This paper developed the multi-lane first order traffic flow model, which depicts the dynamics of lane-changing. It is assumed that each vehicle changes the lane to improve its utility or decrease its disutility, and also that the equilibrium of lane flow distribution is achieved as the condition of stochastic user equilibrium (SUE), where all drivers believe that they cannot improve their utility by changing the lanes. Thus, in the model, lane-changes are represented as the dynamics towards lane-flow equilibrium. As a result of simulating multilane traffic on freeway without any merging and diverging, it is revealed that the proposed model can represent the equilibrium curve of lane-flow distribution, and depict the propagation of traffic congestion at a lane-drop bottleneck section.

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Keywords: lane changing; traffic flow modeling; multi-lane, lane-flow equilibrium; stochastic user equilibrium

* Corresponding name: Yasuhiro SHIOMI Tel.: +80-77-561-5094
E-mail address: shiomi@fc.ritsumei.ac.jp
1. Introduction

It is well known that under the condition of high traffic volume lane-flow distribution becomes unbalanced; more traffic tends to use a median lane rather than a middle and outer lane, which causes the deterioration of traffic capacity at bottleneck sections [Knoop et al., 2010; Wu, 2012; Xing et al., 2014]. As intensive development of ITS, active and dynamic lane management has been practically implemented. By employing the technology of ITS, balancing lane-flow distribution is one of the feasible solutions to increase the throughput of bottleneck flow [Xing et al., 2014]. Besides the unbalanced lane usage, lane traffic management and control should be considered as one of the solutions to improve the efficiency and safety in case of lane regulation under roadworks or incidents and at the merging, diverging and weaving sections. For traffic management to be effective, it is needless to say that a model-based decision support system consisting of traffic state estimation, traffic state prediction and optimization and traffic control measures is essential as mentioned in Yuan et al. (2012). However, due to the lack of the method to computing multilane traffic flow including lane-change dynamics, a model-based decision support system enabling lane-based traffic management to be considered has not been realized.

This paper develops the multi-lane first order traffic flow model, which depicts the dynamics of lane-changing. In the model, we assume that each vehicle changes the lane to improve its utility or decrease its disutility, and also that the equilibrium of lane flow distribution is achieved as the condition of stochastic user equilibrium (SUE), where all drivers believe that they cannot improve their utility by changing the lanes. Thus, in the model, lane-changes are represented as the dynamics towards lane-flow equilibrium. The utility function for a vehicle to choose each lane is defined by only two parameters on the basis of the investigation about lane-changing behaviour done by Knoop et al. (2012) and Shiomi et al. (2013): one is a constant value implying cost breaking the keep-left (or right) principle, and the other one is the average speed depending on the fundamental diagram and the density of the lane. Such parsimony representation will be able to online calibration by using the real time data from conventional loop detectors. To compute the possible solution of multilane traffic under the conservation law of traffic volume, IT principle [Laval and Daganzo, 2006] is applied. Then, in this paper, the reproducibility of lane-flow equilibrium curve on the imaginary ring road and the traffic dynamics at the lane-drop bottleneck is verified.

This paper is organized as follow. In section 2, state-of-the-art of modelling multilane traffic is described. In section 3, the concept of lane-change dynamics and the mathematical representation of lane-flow equilibrium are described. In section 4, the computation methods of multilane traffic flow employing IT principle (Laval and Daganzo, 2006) is overviewed. In section 5, the developed model is verified for the following two cases; reproducibility of lane-flow equilibrium curve on imaginary ring road, and traffic dynamics at lane-drop bottleneck. Finally, we conclude the contribution of the paper and mentioned the recommendation for the future works.

2. State-of-the-art

Considerable scientific attention has been paid on the topic of lane-change behaviour and multi-lane flow modelling during the last two decades. Because lane-change is individual vehicle driving behaviour, that is, whether a vehicle changes its lane totally depends on the decision making which the subject vehicle takes and the situation where the subject vehicle is in, it has been the most straightforward way to apply microscopic modelling [Gipps, 1986; Kita, 1999; Salvucci and Liu, 2002; Webster et al., 2007; Toled et al., 2009; and more]. This approach can consider various conditions and variables which may cause making decision to change lanes. However, due to the computational tasks and complicated model framework, it is not appropriate to apply for online and network-wide freeway traffic evaluation. The other approach is mesoscopic modelling [Shvestsov and Helbing, 1999; Hoogendoorn and Bovy, 2001]. In the approach, gas-kinetic model is applied to depict longitudinal multilane traffic dynamics and lateral movement as well. However, in Shvestsov and Helbing (1999), the proportion of lane changers is exogenously given according with the density. Thus, the motivations behind the lane change behavior are not appropriately considered. In Hoogendoorn and Bovy (2001), the probability of a vehicle changing the lane is estimated by applying discrete choice theory. In this case, however, it is required to calibrate various parameters, so that more precise data is required than conventional loop detectors. Also, it is difficult to employ online and dynamic traffic estimation based on the real time data collection.
From the macroscopic approach, Daganzo (2002a, 2002b) investigated the traffic phenomena on multilane freeway, and proposed a traffic flow theory based on kinematic wave model, in which it is assumed that there are two types of vehicles; slugs, which have lower desired speed and drive on an outer lane, and rabbits, which have higher desired speed and drive on both outer and inside lanes depending on traffic condition. It was proofed that the slugs and rabbits theory could explain the various traffic phenomena. However, the computational method on the basis of this theory to depict multilane traffic has not been developed. Laval and Daganzo (2006) proposed a method to computing multilane traffic flow on the basis of kinematic wave theory, and developed a model to depict the influence of the lane-changers to the traffic flow. This study employed the hybrid approach in which lane changers are computed as particles and considered as moving bottlenecks. It is assumed that the number of lane change vehicles is proportional to the differences of traveling speed among lanes. However, it is apparent that this assumption would not represent the lane-flow equilibrium curve appropriately. Besides, the hybrid approach combining microscopic and macroscopic model, which can be seen in Hong et al. (2010) and Okaue and Okushima (2010), is not feasible in the model-based decision support system. Tang et al. (2009) and Jin (2010) developed macroscopic models depicting lane change traffic which considered the disturbances to the traffic flow caused by lane changes. However, the representation of the models is not in lane-specific manner.

This contribution can be differentiated from the previous studies in the following points; i) the motivation of changing the lane is explicitly considered, and it is treated as utility defined by current macroscopic traffic state, ii) a whole process of lane-changing is computed by macroscopic manner, that is, the extension of kinematic wave theory employing the IT principle, and iii) in the model framework, lane-flow equilibrium curve will be endogenously generated as a result of self-motivated lane changes. The proposed model represents a lane-specific traffic dynamics with parsimony manner. Thus, it is expected that it has high feasibility for online and lane-specific traffic state estimation, prediction and evaluation of dynamic lane control scheme.

3. Concept of lane change and lane-flow equilibrium

3.1. Literature reviews on lane change and lane-flow equilibrium

In this study, we will develop the model depicting lane-changing dynamics and lane-flow equilibrium at a freeway section without any merging and diverging, where all vehicles change the lane to improve their driving circumstances. Namely, mandatory lane-changes heading to off-ramp or coming from on-ramp are not considered.

It is well known that on such section we can observe the specific macroscopic relationship between the total density and the fraction of the lane flow as shown in Figure 1. On 2-lanes sections, more traffic tends to use on the outside lane when traffic density is not so large, while under the presence of higher traffic density than approximately 30 [veh/km/2lanes], more traffic drive on the median lane rather than the other lane. As the density increases, the gap of fraction becomes insignificant. On 3-lane sections, it is more complicated rather than 2-lane sections. First, the fraction of the outside lane is more than the other lanes in the presence of less traffic density than 20 [veh/km], and then traffic on the center lane becomes dominant. In the higher traffic density than 50 [veh/km], the fraction of the median lane becomes the largest and finally the gap of fractions among the lanes get insignificant. This tendency is not special to the observation site shown in Figure 1, but can be observed generally all over the world.
With regard to the mechanism of the lane-flow distribution and its equilibrium condition, Wu (2006) theoretically revealed that the equilibrium curve was achieved as a result of balance between the lane-change demand, that is, the proportion of the following vehicles which are forced to drive less than their desired speed, and the proportion of the available gap in the adjacent lanes. From the empirical aspects, however, Knoop et al. (2012) investigated the relationship between the number of lane-changes and the density of both original and adjacent lanes. It revealed that in the free flow condition the number of lane changes per traffic volume from an outer lane to a median lane and vice versa is not negatively-proportional to the density of the adjacent lane if the density of the original lane is the same level. This fact implies that lane-change behaviors are not fully explained only by the gap acceptance, that is, the proportion of the available gap. According to Shiomi et al. (2013) which investigated lane-change behaviors on 3-lanes section by applying a discrete choice model, in the outside and middle lanes, vehicles tend to remain in the original lane, whereas in the median lane, vehicles tend to change lane to either the middle or outer lane. This fact indicates that drivers basically follow the keep-left (in case of Japan) rule, which also motivates drivers to change the lane or remain on the same lane.

Thus, in this study, it is assumed that:

(i) Drivers are motivated to change the lane to increase the driving speed, though it depends on their desired speed; that is, a driver with high desired speed would change the lane and one with low desired speed would not try to that.

(ii) Basically, drivers would follow the keep-left principle. That is, if the traffic state is same among lanes, a driver would choose the outside lane.

(iii) The demand of lane changes is censored due to the limitation of the available gap on the target lane, which is mutually related to the available capacity of the target lane.

Then, under these assumptions, first order traffic flow model to depict multilane traffic dynamics.

3.2. Definition of utility function and mathematical expression of equilibrium state

Suppose fundamental diagram is defined lane by lane and the average speed of lane \( l \), \( v_l \), is given as

\[ v_l = f_l(k_l) \]

where \( f(l) \) is a fundamental relationship and \( k_l \) is the density on lane \( l \), respectively. A driver would choose a lane which gives him/her more utility or less disutility. As mentioned in the previous section, a driver would be motivated to change the lane to increase his/her driving speed or to follow the keep-left rule. Thus, we defined the cost of a vehicle \( n \) to drive on lane \( l \), \( c_n(k_l) \), as a monotonically increasing function against the density:

\[ c_n(k_l) = \alpha_i + \beta_i \cdot \left(k_light)^{-1} + \varepsilon, \quad \text{(1)} \]

where \( k(t, x) \) is the density on lane \( l \) at time–space point \((t, x)\), \( \alpha_i \) is the disutility to violate the keep-left rule, \( \beta_i \) shows sensitivity to the travel time of a unit of distance, and \( \varepsilon \) is an error term following Weibull distribution,
$W(0, \theta)$, implying the heterogeneity of desired speed and recognition error. Assuming traffic flow is composed of homogenous vehicles in terms of their fundamental diagram and the structure of the cost function, the probability that a vehicle chooses lane $l$ according with the current traffic situation at time $t$ is written as

$$p_l(K) = \exp\left[\frac{-\theta \cdot c_l(k_l)}{\sum_k \exp[-\theta \cdot c_k(k_k)]}\right],$$

where $K(t, x)$ is the total density and written as

$$K(t, x) = \sum_l k_l(t, x).$$

Note that an index $n$ is omitted due to the clear representation. The lane-flow equilibrium condition means the state where each driver believes that he/she can no longer decrease the driving cost by changing the lane, or the situation where even if some vehicle change their lanes, other vehicles would compensate for the change of lane traffic flow by changing the lane immediately and as a result lane flow distribution becomes stable. The equilibrium state is indicated by Eq. (2).

$$p_l^*(K) = \exp\left[\frac{-\theta \cdot c_l(k_l^*)}{\sum_k \exp[-\theta \cdot c_k(k_k^*)]}\right],$$

$$= \frac{k_l^*}{K},$$

where $*$ is the symbol indicating the equilibrium condition.

### 3.3. Expression of lane-change dynamics

The equilibrium condition expressed by Eq. (1) is equivalent to the solution of the optimization problem as

$$\text{min} Z(K) = \sum_k \int_0^{k_k} c_k(\omega) d\omega + \frac{1}{\theta} \sum_k k_k \ln \frac{k_k}{K},$$

subject to

$$K = \sum_k k_k$$

$$k_k \geq 0$$

because the equilibrium condition can be considered as stochastic user equilibrium (SUE) condition, and the cost function is monotonic increasing function with regard to the density. To solve the problem [P-1], the objective function is partially linearized as

$$\text{min} Z(y) = \sum_k y_k c_k(z_k) + \frac{1}{\theta} \sum_k y_k \ln \frac{y_k}{Y},$$

subject to

$$Y = \sum_k y_k$$

$$y_k \geq 0$$

where $z = \{z_k\}$ is a vector of the density on lane $k$ at $(t, x)$. Then, the solution vector $y^*$ is given by calculating KKT condition as

$$y_k^* = Y \cdot \frac{\exp[-\theta \cdot c_k(z_k)]}{\sum_j \exp[-\theta \cdot c_j(z_j)]}.$$  

It is mathematically proven that the operation,
gives a better solution of \([P-1]\) than \(z(t, x)\), where \(W > 1\) [Sheffy; 1984]. This result implies that when the cost of each lane is defined as a monotonic increasing function with regard to the density, the lane-flow distribution gradually approaches the equilibrium condition as vehicles repeatedly change lanes in an ad-hoc manner following the choice probability, Eq.(3). In this model, the process toward the equilibrium represents the dynamics of lane change. In Eq. (4), an adjustment parameter, \(\tau\), is used. It can be interpreted as the same line as \(\tau\) of Laval and Daganzo (2006). Namely, \(\tau\) indicates the number of time step a driver takes to decide and execute a lane change. It relates to the gap availability in the target lane. The higher the density of the target lane is, the longer time to find an available gap takes. Thus, \(\tau\) is considered as such a parameter that relates to traffic condition, and becomes larger when the density is higher.

4. Multilane first-order traffic flow model

4.1. Framework of multilane LWR model

In this study, multilane LWR model developed by Laval and Daganzo (2006) is applied with some modification to compute traffic flow on multilane with lane-changes. The conservation law of multilane traffic is written as

\[
\frac{\partial K_l(t, x)}{\partial t} + \frac{\partial Q_l(t, x)}{\partial x} = \phi_l, \quad l = 1, 2, \ldots, n,
\]

where \(K_l(t, x)\) and \(Q_l(t, x)\) indicate the density and traffic flow on lane \(l\) in position \(x\) at time \(t\), respectively. The non-homogeneous term, \(\phi_l\), in Eq. (5) shows the balance caused by the lane-change vehicles. Thus, this term can be rewritten as

\[
\phi_l = \sum_{l' \neq l} \phi_{l' \rightarrow l} - \sum_{l' \neq l} \phi_{l \rightarrow l'},
\]

where \(\phi_{l' \rightarrow l}\) means the number of vehicles coming from the other lane \((l')\) to the target lane \((l)\).

As mentioned above, it is assumed that fundamental diagram is defined lane by lane. Note that lane-change is caused by the speed differences among lanes even in the free flow condition. Thus, the equation of fundamental diagram proposed by van Lint et al. (2008) is used. It is shown as follow.
\[ V_j(t,x) = f_j(K_j) \]
\[ = \begin{cases} v_{fl} - K_j \cdot \frac{v_{cl} - v_{fl}}{k_{cl}} & \text{if } 0 \leq K_j \leq k_{cl} \\ v_{cl} \cdot k_{cl} \cdot \left(1 - \frac{K_j - k_{cl}}{v_{fl} - k_{fl}}\right) & \text{otherwise} \end{cases} \tag{6} \]
where \( v_{fl}, v_{cl}, k_{cl} \) and \( k_{fl} \) show the free flow speed (km/h), critical speed (km/h), critical density (veh/km) and jam density (veh/km) on lane \( l \), respectively. The example of the fundamental diagram following to Eq. (6) is exhibited as Figure 2.

4.2. Extension to multilane traffic flow

Godunov scheme for single-lane section

To compute the traffic dynamics following the conservation law in Eq. (5), Godunov scheme is applied. Then, Eq. (5) is discretized as
\[ \frac{K_{j+1,l} - K_{jl}}{\Delta t} + \frac{Q_{jl} - Q_{j-1,l}}{\Delta x} = \sum_{l' \neq l} \phi_{jl,l'j} - \sum_{l' \neq l} \phi_{jl,l'j'} \]
where the index \( t \) and \( i \) show time step and cell number, respectively. Following to CFL condition, the time step size \( \Delta t \) and cell length \( \Delta x \) should keep the constrained condition as follow.
\[ \Delta x \geq \max_{\forall l} (v_{fl}) \cdot \Delta t. \]
In the case of single-lane section, the traffic volume transferred from the upper cell \( i \) to the downer cell \( i+1 \), \( A_{ji} \), is given as
\[ A_{ji} = \min(S_{jl} \cdot R_{j,i+1}, (k_{j,i+1} - K_{j,i+1}) \cdot \Delta x) \]
where
\[ S_{jl} = \begin{cases} K_{jl} \cdot v_{jl} \cdot \Delta t & \text{if } 0 \leq K_{jl} \leq k_{jl} \\ k_{jl} \cdot v_{cl} \cdot \Delta t & \text{otherwise} \end{cases} \tag{7} \]
\[ R_{j,i+1} = \begin{cases} k_{cl+1} \cdot v_{cl+1} \cdot \Delta t & \text{if } 0 \leq K_{j,i+1} \leq k_{cl+1} \\ K_{j,i+1} \cdot v_{j,i+1} \cdot \Delta t & \text{otherwise} \end{cases} \]
In Eq. (7), \( S_{jl} \) is the sending function, which means that the traffic demand from the upper cell, and \( R_{j,i+1} \) is the receiving function, which means the supply volume of the downer cell. The transfer volume is limited as the minimum of the traffic demand, supply volume and the physically acceptable number of vehicles in the downer cell. In the following section, the treatment of lane-change vehicles is given.

Definition of the number of the vehicles with desire of lane-change

As mentioned in section 3, lane-change vehicles are generated to improve their driving cost. Given the cost function to each lane in accordance with the density of each cell as Eq.(1), the proportion of the vehicles with the desired to change the lane from \( l \) to \( l' \), in time and place \( (t, x) \), \( p_{j,j',l} \), is defined as Eq. (8).
\[ p_{j,j',l} = \frac{\exp[-\theta \cdot c_{jk} (K_{jl})]}{\sum_k \exp[-\theta \cdot c_{ik} (K_{ik})]} \tag{8} \]
Then, the number of vehicles with desired to change the lane is written in accordance with the sending function \( S_{jl} \) as follow,
\[ L_{jl,l'} = \frac{1}{\tau_{jl}} \cdot S_{jl} \cdot p_{jl,l'}, \]
where $\gamma_{il}$ is the adjustment parameter. It is an unknown parameter depending on the traffic state. It is not able to be observed directly so that should be estimated on the basis of the longitudinal variation of lane flow distribution. Along this line, a feedback estimation method [Wang and Papageorgiou; 2005, and so on] could be applied to determine the parameter. The volume of the traffic with the desired to keep the lane is also defined as

$$M_{til} = S_{til} - \sum_{l' \in \Omega_l} L_{til-l'} ,$$

where $\Omega$ shows the set of lanes which a vehicle can get to within time step $\Delta t$ from the current lane $l$.

**Definition of lane-change vehicles**

Based on the number of the vehicles with desire to keep the lane and the number of the vehicles with desire to change the lane, the adjustment process to determine the transfer volume into the downer cells. IT principle [Laval and Daganzo; 2006] is applied with partial revisions. In this study, it is assumed that there are two criteria to execute a lane-change. First criterion is whether a vehicle can find a space in the target lane, and the other criterion is whether the downstream cell on the target lane can accept the traffic coming from the upstream of the same lane and its adjacent lanes.

Let $H_{ti+1l}$ denote the total desired number flowing into the cell $i+1$ of lane $l$ on time $t$ as

$$H_{ti+1l} = M_{til} + \sum_{l' \in \Omega_l} L_{til-l'} .$$  \hspace{1cm} (9)

For Eq.(9), the first criterion is applied, that is, the desired number of lane-changes is censored according as the acceptable volume on the adjacent cell on the target lane, which makes $H_{ti+1l}$ denoted by

$$H'_{ti+1l} = M_{til} + \gamma_{til} \cdot \sum_{l' \in \Omega_l} L_{til-l'}$$

where

$$\gamma_{il} = \min \left( 1, \frac{\min(R_{til}, (k_{ji} - K_{il})\Delta x)}{H_{ti+1l}} \right)$$

Then, the second criterion is applied. Let $\omega_{il}$ denotes,

$$\omega_{il} = \min \left( 1, \frac{\min(R_{ti+1l}, (k_{ji+1} - K_{ti+1})\Delta x)}{H_{ti+1l}} \right),$$

which defines the possible transfer volume with keeping the lane, $q_{qil}$, and the possible transfer volume with lane change, $\phi_{qil}$, as

$$q_{il} = \omega_{il} \cdot M_{til} ,$$

$$\phi_{qil} = \omega_{il} \cdot \gamma_{il} \cdot L_{qil-l'} ,$$

respectively. Then, the actual traffic volume flowing into the downstream cell $i+1$ on lane $l$, $A_{i+1l}$, is written as

$$A_{i+1l} = q_{il} + \sum_{l' \neq l} \phi_{qil-l'} .$$

Finally, the density of each cell is updated every time step in accordance with

$$K_{i+1l} = K_{il} + \left( A_{i+1l} - q_{il} - \sum_{l' \neq l} \phi_{qil-l'} \right) \cdot \Delta x .$$

**5. Case studies**

In this section, the model is applied to an imaginary ring road for verifying the reproducibility of lane-flow equilibrium, and to a lane-drop bottleneck section for displaying the lane-specific traffic dynamics.

**5.1. Lane-flow equilibrium on ring road**
To verify the reproducibility of lane-flow equilibrium curve, the developed model is applied to an imaginary ring road with 2 lanes and 3 lanes. A ring road can be considered as a homogeneous and infinity length road. To get a convergent solution of \([P^{-1}]\), set the homogenous density to all cells in the ring road as an initial condition, and simulate traffic flow dynamics for 50 time steps with given the adjacent parameter, \(\tau\), increasing value as the calculation processes. Concretely, \(\tau\) is set as equal to the time step counts, so that the lane-changing movements gradually settle down. Then, for various initial density from 1 (veh/km/lane) to 60 (veh/km/lane) by 2 (veh/km/lane), the convergence results are calculated.

Case 1: 2 lanes ring road

The ring road network with 2 lanes as shown in Figure 3 is examined. Note that it is assumed all cell length is same, despite of the appearance, in which the cell length of inner lanes seems to be shorter than outer lanes. The parameter settings for the numerical calculations are set as Table 1. The cost parameters for the outer lane, \(a_1\) and \(b_1\), are set as 0 and 1, respectively, because the differences of the cost between each lane matters to determine the choice probability in Eq. (8) and we considered the outside lane as the base line. The other parameters about the cost function are set by trial and error.

Figure 4 shows the convergence process of lane flow distribution when the initial density is set as 20 (veh/km) per lane. From the figure, it can be seen that at first the fraction of lane flow is the same. At first the oscillation can be observed, but as the time step increases the fraction of lane flow settles down, and finally converged at the point where the median lane has more traffic than the outer lane. Figure 5 shows the relationship between the total density of both lanes and the fraction of lane flow at the convergence. It can be seen that in the low density, most traffic use on the outside lane, and as the density increases the gap between the usages of the outside lane and the median lane becomes diminished. When the total density becomes around 30 (veh/km), the dominant lane changes from the outside lane to the median lane. This tendency is also seen in the observation in Figure 1. Thus, we can conclude the computation results represent the realistic lane usage on 2 lane sections, though it is strongly required to set the parameters used in the simulation by calibration based on the observed data. This model requires only 3 parameters in the case of 2 lane section in addition to the parameters of fundamental diagram, so that it should be not a hard task to estimate the unknown parameters on the basis of the observed data collected from conventional loop detectors.
Table 1. Parameter settings for 2 lanes ring road. The notation, \( l=1 \) means the outer lane and \( l=2 \) shows the median lane.

(a) Cost function for lane choices

<table>
<thead>
<tr>
<th>( \alpha_1 )</th>
<th>( \alpha_2 )</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( \theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.014</td>
<td>1</td>
<td>0.82</td>
<td>1000</td>
</tr>
</tbody>
</table>

(b) Fundamental Diagram

<table>
<thead>
<tr>
<th>( \nu_f1 )</th>
<th>( \nu_f2 )</th>
<th>( \nu_c1 )</th>
<th>( \nu_c2 )</th>
<th>( k_c(\ell=1,2) )</th>
<th>( K_{J\ell}(\ell=1,2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>80 (km/h)</td>
<td>90 (km/h)</td>
<td>90 (km/h)</td>
<td>80 (km/h)</td>
<td>15 (veh/km)</td>
<td>70 (veh/km)</td>
</tr>
</tbody>
</table>

(c) Simulation settings

<table>
<thead>
<tr>
<th>( \Delta t )</th>
<th>Number of cells</th>
<th>( \Delta x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 (sec)</td>
<td>20</td>
<td>222.2 (m)</td>
</tr>
</tbody>
</table>

Fig. 4. Convergence process of the fraction of lane flow on 2 lanes ring road.

Fig. 5. Computed lane-flow equilibrium curve for 2 lanes ring road.

Case 2: 3 lanes ring road

The 2 lanes ring road is extended to 3 lanes. Parameters are set as shown in Table 2. Note that we assume that a vehicle can change the lane to the adjacent lane only within a time step, because the time step size, 10 (sec), is too short to across the whole road width. As same as the case of 2 lanes, the parameters of cost function are set by trial and error, except for \( \alpha_1 \) and \( \beta_1 \).

Figure 6 shows the convergent process when the initial density is set as 20 (veh/km) per a lane. It reveals that the process is rather complicated than the one of 2 lanes case, but finally the fraction of lane flow gets to the
Table 2. Parameter settings for 3 lanes ring road. The notation, \( l = 1, 2, \) and 3 show the outer, middle, and median lane, respectively.

(a) Cost function for lane choices

<table>
<thead>
<tr>
<th>( \alpha_1 )</th>
<th>( \alpha_2 )</th>
<th>( \alpha_3 )</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( \beta_3 )</th>
<th>( \theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.014</td>
<td>0.019</td>
<td>1</td>
<td>0.82</td>
<td>0.81</td>
<td>1000</td>
</tr>
</tbody>
</table>

(b) Fundamental diagram.

<table>
<thead>
<tr>
<th>( v_{f_1} )</th>
<th>( v_{f_2} )</th>
<th>( v_{f_3} )</th>
<th>( v_{c_1} )</th>
<th>( v_{c_2} )</th>
<th>( v_{c_3} )</th>
<th>( k_c ) (( l = 1, 2, 3 ))</th>
<th>( K_0 ) (( l = 1, 2, 3 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>80 (km/h)</td>
<td>90 (km/h)</td>
<td>100 (km/h)</td>
<td>70 (km/h)</td>
<td>80 (km/h)</td>
<td>90 (km/h)</td>
<td>15 (veh/km)</td>
<td>70 (veh/km)</td>
</tr>
</tbody>
</table>

(c) Simulation settings

<table>
<thead>
<tr>
<th>( \Delta t )</th>
<th>Number of cells</th>
<th>( \Delta x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 (sec)</td>
<td>20</td>
<td>277.8 (m)</td>
</tr>
</tbody>
</table>

Fig. 6. Convergence process of the fraction of lane flow on 3 lanes ring road.

Fig. 7. Computed lane-flow equilibrium curve for 3 lanes ring road.

Convergence, showing that less traffic used on the median lane than others and the median gets the highest. Figure 7 represents the relationship between the various total density and fraction of lane flow. It is interestingly noted that
when the density is low the outside lane is dominant, but as the density increases it becomes diminishing. Contrarily, the fraction of the median lane increases and finally gets the most fraction of lane flow. Although there can be found the differences from the observation shown in Figure 1, the simulated lane-flow equilibrium curve can represents the significant features in the fluctuation of the lane flow fraction.

5.2. Traffic dynamics at lane-drop bottleneck section

Traffic dynamics at the bottleneck where the number of lanes reduces from 3 lanes to 2 lanes is simulated in the section. The subject section consists of 80 cells and the 56th cell from the upper boundary is set as a bottleneck. Parameters of cost function and fundamental diagrams, and the simulation settings are given as listed in Table 2 (a), and (b), respectively. With regards to the simulation settings, the time step size is set as 5 (sec), the cell length is set as 138.9 [m], accordingly, and the adjustment parameter, \( \tau \), is set as 5. At the upstream boundary cells, traffic demand is randomly given to each lane, following to the normal distribution with mean 10 (veh/km) and standard deviation 1 (veh/km). At the downstream boundary, it is assumed that vehicles can freely flow out.

Figure 8 shows the simulation results. Note that for the clear representation the simulated results are aggregated in every 5 time steps and 4 longitudinal cells, accordingly, in Figure 8. It can be seen that until 7th time steps there is no congestion queue because high traffic demand have not arrived at the bottleneck, and in this situation the lane flow distribution is almost stable. At 7th step, the congestion queue firstly appears on the median lane. It is interestingly noted that at first the congestion queue propagated to the middle lane and the outer lane, and then propagates towards the upstream. Figure 9 illustrates the proportion of the lane-change vehicles to the number of vehicles flowing out of the cells. In the figure, lane-change, \( 1 \to 2 \), means the lane change from the outer lane to the middle lane, that is, 1, 2, and 3 mean the outer lane, the middle lane, and the median lane, respectively. It can be seen that the proportion also propagates to the downstream in the free flow condition, and to the upstream in the congestion flow condition. It means that this model can represent the propagation of lane-change dynamics along the characteristic wave. Focusing on the bottleneck point, at the 14th cell, the proportion of the lane-change from the median to the middle is the highest, and lane-change from the middle to the median is 0, which is valid. However, at the 13th cell, small proportion of the vehicles changes the lane from the middle to the median. The reason is considered as follow; in the model, it is assumed that drivers do not foresee the traffic condition of the downstream. That is, each driver makes the decision of lane-change according to the current traffic condition, which seems to be unrealistic maneuver. Although this point should be a drawback to be renewed, the developed model can represent the relevant traffic dynamics including lane-changes.

6. Conclusions

A multi-lane, first-order, macroscopic traffic flow model is developed. In the model, it is assumed that a driver changes the lane to improve the utility or cost of driving circumstance. The utility/cost function is composed by a constant value, which indicates the cost to break the keep-left rule, and a coefficient of the inverse of the speed defined by the fundamental diagram, which indicates the sensitivity to the increase of the travel time, and an error term, which implies the heterogeneity of drivers and the limitation of the information about the surrounding traffic situation. The number of lane-change vehicles can be defined by applying random utility theory. It is mathematically proved that lane-flow distribution is gradually converging to the equilibrium condition every time the lane-changes occur. Thus, this model can endogenously represent the lane-flow equilibrium. After rigorously defining the mathematical representation of the proposed model, given the appropriate parameter sets, the reproducibility of the lane-flow equilibrium is validated for two lanes and three lane ring roads. In addition, traffic dynamics including lane-change manoeuvres are simulated for the freeway bottleneck section with lane-drop from 3 lanes to 2 lanes. As a result, it is confirmed that the model can depict the propagation of congestion queue, which is originally appeared in the median lane at the bottleneck, and first propagate to the middle and inner lane, and then propagate to the upstream direction. Also, it is shown that the proportion of lane-change vehicles also propagates along with the characteristic wave. Although the drawbacks of the model were also found, we can conclude that the developed model framework is promising to depict the realistic, lane-specific traffic phenomena, including lane-flow
Fig. 8. Simulated lane-specific traffic dynamics in the section with lane-drop bottleneck.
<table>
<thead>
<tr>
<th>time step</th>
<th>cell</th>
<th>lane change</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>0.11</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.11</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0.11</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
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</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.13</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>0.15</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>0.11</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>0.12</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Fig. 9. Simulated lane-change dynamics: The proportion of lane-change vehicles.
equilibrium. The future works to develop the model are suggested as follows:

(i) Parameter calibrations: This model requires four parameters for a fundamental diagram of each lane, and two parameters for defining cost function of each lane. If the data from a loop detector which is installed lane by lane as usual, the former parameters can be calibrated by using KV plots, and the latter parameters can be also calibrated by the lane-flow distribution.

(ii) An adjustment parameter, \( \tau \): This parameter implies the “intensity” of lane-changes, which should be varied by time and place by place. Thus, to estimate it, feedback estimation algorithms such as KF, EKF, EnKF might be useful.

(iii) Foreseeing behaviors: In the model, it is assumed that a driver would change the lane in an ad-hoc manner only with regard to the surrounding situation. In the real world, however, a driver would have, or be able to obtain some knowledge about the downstream section, for example, “lane-drop 2 miles ahead”, or whatever.

(iv) Mandatory lane-change: This model limitedly considers the lane-change to improve a driver’s utility. Thus, it should be required to model a mandatory lane-change behavior such as towards a off-ramp.

(v) Disturbance caused by lane-changes: Under crowded condition, lane-change might cause disturbances in traffic flow [Tan et al.; 2009, Jin; 2013], which should be considered.

(vi) Multiclass vehicles: The characteristics of lane-change behaviour might be different among the vehicle classes. For example, semi-trailers mostly might use an outer lane and passenger cars might often use a median lane.

(vii) Empirical analysis: To develop the model, it is essential to understand what happens in the real field. Thus, high resolution data collections and empirical analysis is required to develop and validate a model.

Acknowledgements

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References


