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Numerical study of fluid flow on magneto-hydrodynamic mixed convection in a lid driven cavity having a heated circular hollow cylinder

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Abstract

Magneto-hydrodynamic mixed convection in a lid driven cavity along with a heated circular hollow cylinder positioned at the centre of the cavity is studied numerically. The left and right vertical walls are kept at a constant temperature T_c while the top and bottom horizontal walls of the cavity are insulated. The left vertical wall is moving in its own plane of a constant speed while all other walls are fixed. A uniform magnetic field is applied to the horizontal direction normal to the moving wall. A Galerkin weighted residual finite element method with a Newton Raphson iterative algorithm is adopted to solve the governing equations. The computations are carried out for wide ranges of the Hartmann number (*Ha*) and Richardson number (*Ri*). The results are presented in the form of streamlines, isothermal lines and average Nusselt number for the aforementioned parameters. The results show that the aforesaid parameters have noticeable effect on the flow pattern and heat transfer characteristics inside the cavity.

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Keywords: Magneto-hydrodynamic; mixed convection; lid-driven cavity; hollow cylinder; finite element method.

Nomenclature

B_0	magnetic field strength	(x, y)	dimensional coordinates (m)
C_p	Specific heat of fluid at constant pressure	(X, Y)	dimensionless coordinates
g	gravitational acceleration (ms^{-2})	*Greek symbols*	
Ha	Hartmann number	α	thermal diffusivity (m^{2}s^{-1})
k	thermal conductivity of fluid (Wm^{-1}K^{-1})	β	thermal expansion coefficient (K^{-1})
k_s	thermal conductivity of solid (Wm^{-1}K^{-1})	θ	non-dimensional temperature
K	thermal conductivity ratio of the solid and fluid	μ	dynamic viscosity of the fluid (Kg m^{-1}s^{-1})
L	length of the cavity (m)	ν	kinematic viscosity of the fluid (m^{2}s^{-1})
Nu	Nusselt number	σ	electrical conductivity of the fluid (Wm^{-1}K^{-1})
p	dimensional pressure (Nm^{-2})	ρ	density of the fluid (Kg m^{-3})
P	non-dimensional pressure	ψ	stream function
Pr	Prandtl number	*Subscripts*	
Re	Reynolds number	*c*	less heated wall
Ri	Richardson number	*h*	heated wall
T	dimensional temperature (K)	*s*	solid

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u, v	velocity components (ms^{-1})
U, V	non-dimensional velocity components

1. Introduction

Mixed convection is that type of heat transfer in which there is a noteworthy interaction between free and forced convection. Analysis of mixed convection in a lid-driven cavity is relevant to much engineering and environmental applications. These applications include heat exchanger, cooling of electronic equipments, nuclear reactors, solar receiver, thermal storage, chemical processing equipments and drying or geophysics studies, etc. Studies associated with mixed convection in open cavities have received increasing consideration. Literature on the body inserted lid-driven cavity is sparse. Dagtekin and Oztop, 2002 inserted an isothermally heated rectangular block in a lid-driven cavity at different positions to simulate the cooling of electronic equipments. The authors concluded that dimension of the body is the most effective parameter on mixed convection flow. Mamun et al, 2010 performed a numerical study on the effect of a heated hollow cylinder on mixed convection in a ventilated cavity.

Magneto-hydrodynamics (MHD) is that branch of science, which studies the dynamics of electrically conducting fluids in the presence of electromagnetic fields. MHD is usually regarded as a very up to the date subject, because it has many engineering applications such as liquid-metal cooling of nuclear reactors and electromagnetic casting, etc. Rahman et al, 2009 investigated the effect of a heat conducting horizontal circular cylinder on MHD mixed convection in a lid-driven cavity along with joule heating. MHD mixed convection flow in a vertical lid-driven square enclosure, including a heat conducting horizontal circular cylinder with Joule heating was analyzed by Rahman and Alim, 2010. Piazza and Ciofalo, 2002 carried out a numerical investigation on buoyancy-driven magneto-hydrodynamic flow in a liquid-metal filled in a cubic enclosure. The authors found that increasing Hartmann number suppressed the convective motions. Chamkha, 2002 made a study for mixed convection in a square cavity in the presence of magnetic field and an internal heat generation and absorption. He concluded that the flow behavior inside the cavity and heat transfer rate is strongly affected by the magnetic field. Xu et al, 2006 completed an experimental study on natural convection of a molten metal contained in a rectangular enclosure in the presence of an external magnetic field. Oztop et al, 2009 studied the effects of sinusoidal temperature boundary conditions on magnetohydrodynamic buoyancy-induced flow in a non-isothermally heated square enclosure. Sarries et al, 2005 performed a numerical study on unsteady natural convection of an electrically conducting fluid in a laterally and volumetrically heated square cavity under the influence of a magnetic field. Rahman et al, 2010 conducted a numerical study on the conjugate effect of joule heating and magnato-hydrodynamics mixed convection in an obstructed lid-driven square cavity, where the developed mathematical model was solved by employing Galerkin weighted residual method of finite element formulation. Bhuvaneswari et al, 2011 carried out a computational study of convective flow and heat transfer in a cavity in the presence of uniform magnetic field. Rahman et al, 2010 investigated the effect of Reynolds and Prandtl numbers effects on MHD mixed convection in a lid-driven cavity along with joule heating and a centered heat conducting circular block.

As it is clear from the above literature that the combined heat transfer by mixed convection in a lid-driven cavity has been received a great interest in recent years. Nevertheless, to the best knowledge of the authors, combined heat transfer by MHD mixed convection in a lid-driven cavity with heated circular hollow cylinder has received a little attention in the literature. The main purpose of this study is to examine the MHD mixed convection heat transfer and fluid flow and effects of magnetic force in a lid driven cavity for different Richardson number.

2. Physical model

The physical system under study with the system of coordinates is sketched in Fig. 1. The problem deals with a heated circular hollow cylinder with a diameter d and thermal conductivity k_s located at the center of a square enclosure with sides of length L. The two sidewalls are maintained at uniform constant temperatures T_c, while the horizontal top and bottom walls are adiabatic. The left vertical wall of the cavity is allowed to move upward in its own plane at a constant velocity V_0, while the other walls remain stationary. In addition, the gravity acts in the negative y-direction. Moreover, the radiation, pressure work and viscous dissipation are assumed to be negligible and Boussinesq approximation is assumed to be valid. Magnetic force is induced in the horizontal direction to the cavity. All solid boundaries are assumed to be rigid no-slip walls.

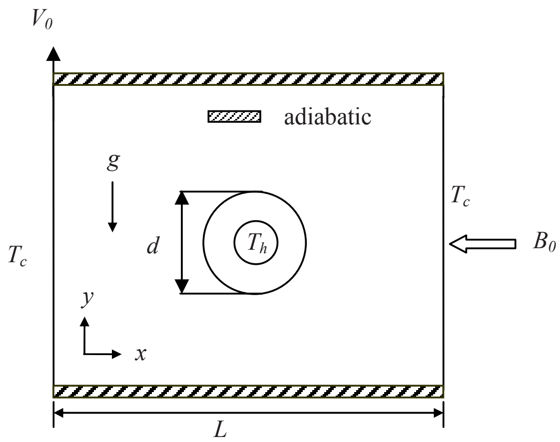

Fig. 1. Schematic of the problem with the domain

3. Mathematical formulation

The governing equations for this investigation are based on the usual balance laws of mass, momentum and energy modified to account for combined buoyancy effects, and hydromagnetic effects. The flow is considered steady, laminar, incompressible and two-dimensional. The variation of fluid properties with temperature has been neglected with the only exception of the buoyancy term, for the Boussinesq approximation has been adopted. The governing equations and the boundary conditions are thrown in the dimensionless form using the following dimensionless variables:

$$X = \frac{x}{L},\ Y = \frac{y}{L},\ U = \frac{u}{V_0},\ V = \frac{v}{V_0},\ P = \frac{p}{\rho V_0^2},\ D = \frac{d}{L},\ \theta = \frac{(T - T_c)}{(T_h - T_c)},\ \theta_s = \frac{(T_s - T_c)}{(T_h - T_c)}$$

Taking into account the above-mentioned assumptions, the non-dimensional governing equations are as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \tag{2}$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + Ri\,\theta - \frac{Ha^2}{Re}V \tag{3}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{Re\,Pr}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \tag{4}$$

For solid region, the energy equation is

$$\frac{\partial^2 \theta_s}{\partial X^2} + \frac{\partial^2 \theta_s}{\partial Y^2} = 0 \tag{5}$$

The non-dimensional parameters that appear in the above formulation are the Reynolds number ($Re = V_0 L/\upsilon$), Prandtl number ($Pr = \nu/\alpha$), Richardson number ($Ri = g\beta\Delta TL/V_0^2$), Hartmann number ($Ha^2 = \frac{\sigma B_0^2 L^2}{\mu}$) and solid fluid thermal conductivity ratio $(K = k_s/k_f)$,

The boundary conditions used in the current study are

At the left vertical lid: $U = 0, V = 1, \theta = 0$

At the right vertical wall: $U = 0, V = 0, \theta = 0$

At the top and bottom walls: $U = 0,\ V = 0,\ \frac{\partial \theta}{\partial N} = 0$

At the outer surface of the hollow cylinder: $\begin{cases} U = 0, V = 0 \\ \left(\frac{\partial \theta}{\partial N}\right)_{fluid} = K\left(\frac{\partial \theta_s}{\partial N}\right)_{solid} \end{cases}$

At the inner surface of the hollow cylinder: $U = 0, V = 0, \theta = 1$. Here N is the non-dimensional distances either along X or Y direction acting normal to the surface. The average Nusselt number at the heated surface of the cylinder based on the dimensionless quantities may be expressed by $Nu = -\frac{1}{\pi}\int_0^{\pi}\frac{\partial \theta}{\partial n}d\varphi$ and the average temperature of the fluid in the cavity is defined by $\theta_{av} = \int \theta \, d\overline{V} / \overline{V}$, where n represents the unit normal vector on the surface of the cylinder and $\overline{V}$ is the cavity volume. The stream function is calculated from its definition as

$U = \frac{\partial \psi}{\partial Y}, V = -\frac{\partial \psi}{\partial X}.$

4. Solution scheme

The governing equations have been solved by using the Galerkin weighted residual finite element technique. The basic unknowns for the governing equations are the velocity components (U, V), the temperature θ and the pressure P. The six node triangular element is used in this work for the development of the finite element equations. All six nodes are associated with velocities as well as temperature; only the corner nodes are associated with pressure. This means that a lower order polynomial is chosen for pressure and, which is satisfied through continuity equation. The velocity component and the temperature distributions and linear interpolation for the pressure distribution according to their highest derivative orders in the differential equations (2) - (5) can be expressed as

$U(X,Y) = N_\beta U_\beta$, $V(X,Y) = N_\beta V_\beta$, $\theta(X,Y) = N_\beta \theta_\beta$, $\theta_s(X,Y) = N_\beta \theta_{s_\beta}$, $P(X,Y) = H_\lambda P_\lambda$, where $\beta = 1, 2, \ldots \ldots, 6$; $\lambda = 1, 2, 3$.

Substituting the element velocity component distributions, the temperature distribution and the pressure distribution from equations (2) - (5) the finite element equations can be written in the form

$$K_{\alpha\beta\gamma^x} U_\beta U_\gamma + K_{\alpha\beta\gamma^y} V_\gamma U_\gamma + M_{\alpha\mu^x} P_\mu + \frac{1}{Re}\left(S_{\alpha\beta^{xx}} + S_{\alpha\beta^{yy}}\right) U_\beta = Q_{\alpha^u} \quad (6)$$

$$K_{\alpha\beta\gamma^x} U_\beta V_\gamma + K_{\alpha\beta\gamma^y} V_\gamma V_\gamma + M_{\alpha\mu^y} P_\mu + \frac{1}{Re}\left(S_{\alpha\beta^{xx}} + S_{\alpha\beta^{yy}}\right) V_\beta - Ri\, K_{\alpha\beta}\theta_\beta = Q_{\alpha^v} \quad (7)$$

$$K_{\alpha\beta\gamma^x} U_\beta \theta_\gamma + K_{\alpha\beta\gamma^y} V_\beta \theta_\gamma + \frac{1}{RePr}\left(S_{\alpha\beta^{xx}} + S_{\alpha\beta^{yy}}\right) \theta_\beta = Q_{\alpha^\theta} \quad (8)$$

$$\left(S_{\alpha\beta^{xx}} + S_{\alpha\beta^{yy}}\right) \theta_\beta = Q_{\alpha^{\theta_s}} \quad (9)$$

where the coefficients in element matrices are in the form of the integrals over the element area and along the element edges S_0 and S_w as

$K_{\alpha\beta^x} = \int_A N_\alpha N_{\beta,x} dA$; $K_{\alpha\beta^y} = \int_A N_\alpha N_{\beta,y} dA$; $K_{\alpha\beta\gamma^x} = \int_A N_\alpha N_\beta N_{\gamma,x} dA$; $K_{\alpha\beta\gamma^y} = \int_A N_\alpha N_\beta N_{\gamma,y} dA$

$K_{\alpha\beta} = \int_A N_\alpha N_\beta dA$; $S_{\alpha\beta^{xx}} = \int_A N_{\alpha,x} N_{\beta,x} dA$; $S_{\alpha\beta^{yy}} = \int_A N_{\alpha,y} N_{\beta,y} dA$; $M_{\alpha\mu^x} = \int_A H_\alpha H_{\mu,x} dA$

$M_{\alpha\mu^y} = \int_A H_\alpha H_{\mu,y} dA$; $Q_{\alpha^u} = \int_{S_0} N_\alpha S_x dS_0$; $Q_{\alpha^v} = \int_{S_0} N_\alpha S_y dS_0$; $Q_{\alpha^\theta} = \int_{S_w} N_\alpha q_{1w} dS_w$; $Q_{\alpha^{\theta s}} = \int_{S_w} N_\alpha q_{2w} dS_w$

The set of non-linear algebraic equations (6) - (9) are solved using reduced integration technique and Newton-Raphson method The convergence of solutions is assumed when the relative error for each variable between consecutive iterations is recorded below the convergence criterion ε such that $\left|\Psi^{n+1} - \Psi^n\right| \le 10^{-4}$, n is number of iteration and Ψ is a function of U, V, θ and θ_s.

5. Findings

A numerical study has been performed to determine the effects of magnetic force on the mixed convection flow of an electrically conducting fluid in a lid-driven cavity. For the intention of discussing the results, the numerical calculations are presented in the form of streamlines, isotherms. With this aim, different parameters such as Reynolds number (*Re*), Prandtl number (*Pr*), Richardson number (*Ri*), Hartmann number (*Ha*) and cylinder diameter (D) are considered. Moreover, Prandtl number, Richardson number and Reynolds number are held fixed at 0.7, 1.0 and 100 respectively.

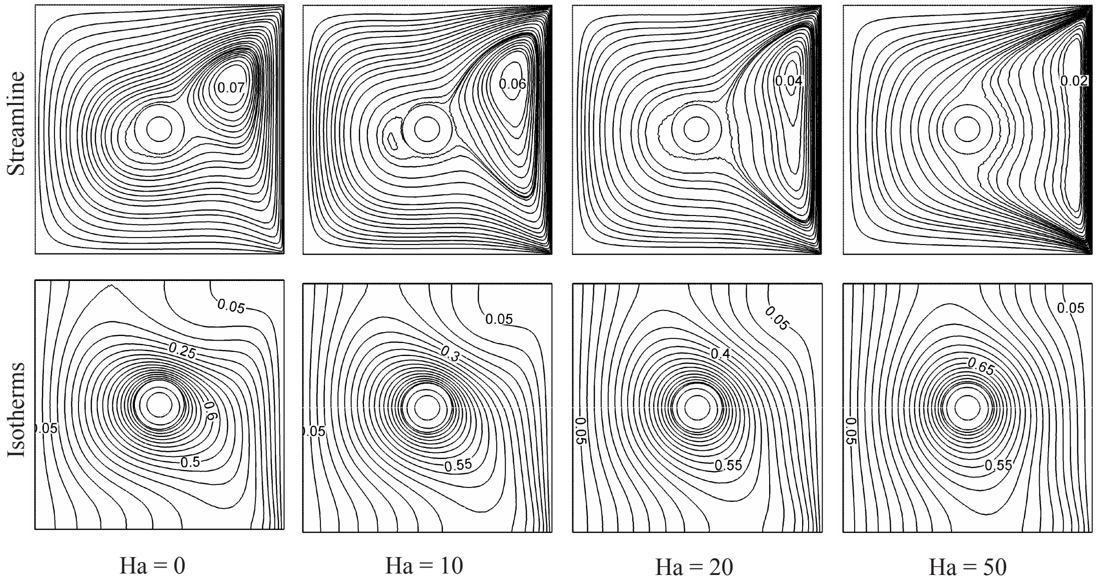

Fig. 2. Effect of Hartmann number on streamlines and isotherms, while Ri = 1.

Figure 2 shows the streamlines and isotherms for different values of Hartmann number Ha = 10, 20, 50 and 100 at Ri = 1. As seen from the first row of this figure, an amount of fluid near the heated hollow cylinder in the cavity is activated so as to create a buoyancy-induced clockwise rotating cell for the lowest value of Ha = 0. It is also observed that the uni-cellular clockwise eddy appears near the right vertical wall, which we call primary eddy, is generated due to the motion of the left vertical wall. As the Hartmann number increases the strength of the rotating cell is reduced and pushed to the right wall of the cavity indicating the establishment of conduction mode of heat transfer. One may notice that the shape of the cell changes from circular to elliptic with the increasing Ha. As the value of Ha increases the core vortices expand vertically. It indicates the reduction of the flow strength of those vortices. The corresponding effect of Hartmann number Ha on the isotherms is depicted in the second row of Fig. 2. The temperature field shows that in the absence of magnetic field the isothermal lines form a thin thermal boundary layer near the vicinity of the heated hollow circular cylinder inside the cavity. In addition, from this figure it can easily be seen that the isotherms are almost parallel to the vertical walls for the highest value of Ha (= 50.0) at the three values of Ri, indicating that most of the heat transfer process is carried out by conduction.

The effect of Hartmann number on average Nusselt number Nu at the hot surface and average temperature θ of the fluid in the cavity with Richardson number is shown in the Fig. 3. The average Nusselt number Nu decreases with increasing Ha for all Ri. Moreover, in absence of magnetic field (Ha = 0), Nu is always higher for all Ri. On the other hand, average temperature θ of the fluid in the cavity increases with increasing Ha for lower values of Ri (= 0.1 and 1).

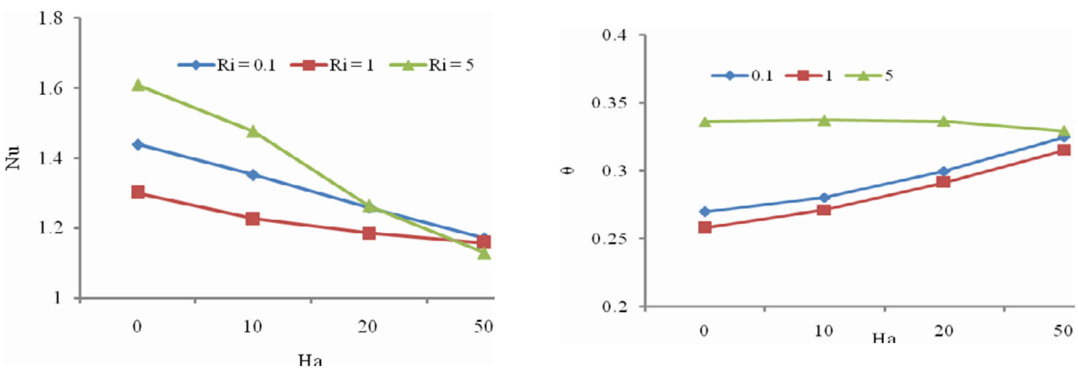

Fig. 3. Effect of Hartmann number Ha on average Nusselt number (left) and average fluid temperature in the cavity (right).

6. Concluding remarks

A computational study is performed to investigate the MHD mixed convection flow in a lid driven enclosure with circular heated hollow cylinder. Results are obtained for wide ranges of Hartmann number *Ha*. The main findings from this work can be listed as follows.

- Flow velocity is reduced with increasing of Hartmann number, and this reduces flow strength and heat transfer. Thus, magnetic field can be a control parameter for heat transfer and fluid flow in lid-driven cavity.
- Thinner thermal boundary layer is observed for lower values of Hartmann numbers.

Higher heat transfer is formed for lower Hartmann number.

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