Physics Letters B 762 (2016) 283–287

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Standard Model Extension and Casimir effect for fermions at finite temperature

A.F. Santos [a,b,*], Faqir C. Khanna [b,c]

[a] Instituto de Física, Universidade Federal de Mato Grosso, 78060-900, Cuiabá, Mato Grosso, Brazil
[b] Department of Physics and Astronomy, University of Victoria, 3800 Finnerty Road, Victoria, BC, Canada
[c] Department of Physics, University of Alberta, T6J 2J1, Edmonton, Alberta, Canada

ARTICLE INFO

Article history:
Received 5 July 2016
Received in revised form 20 September 2016
Accepted 22 September 2016
Available online 28 September 2016
Editor: M. Cvetič

Keywords:
Casimir effect
Standard Model Extension
Finite temperature

ABSTRACT

Lorentz and CPT symmetries are foundations for important processes in particle physics. Recent studies in Standard Model Extension (SME) at high energy indicate that these symmetries may be violated. Modifications in the lagrangian are necessary to achieve a hermitian hamiltonian. The fermion sector of the standard model extension is used to calculate the effects of the Lorentz and CPT violation on the Casimir effect at zero and finite temperature. The Casimir effect and Stefan–Boltzmann law at finite temperature are calculated using the thermo field dynamics formalism.

1. Introduction

Standard Model (SM) has been highly successful in predicting interaction among quarks at energy upto a few TeV. In weak interactions break down of Parity [1,2] and CP symmetry [3] has been observed at low energies. String theory in higher dimensions is possible for particle physics at high energies. Such a theory may have violation of Lorentz and CPT symmetry. At some range of higher energies, can there be a break down invariance properties like Lorentz invariance and CPT symmetry of the SM [4]? Such an extension of the Standard Model (SME) has been applied to several processes in order to get an estimate of the break down of symmetries. Such violations have also been found to occur in loop quantum gravity [5], noncommutative theories [6], spacetimes with a nontrivial topology [7], among others.

The general theory of the SME [8,9] includes the known physics of the SM plus all possible terms that violate Lorentz and CPT symmetry. In addition, the SME is divided into two parts: (i) the minimal version restricted to power counting renormalizable operators and (ii) the nonminimal version which also includes operators of higher dimensions. In this paper our interest is in the fermion sector that is based on a minimal extended Quantum Electrodynamics (EQED) that is part of SME. This EQED involves modifications of the usual QED in both fermion and photon sectors. The relativistic lagrangian that describes fermions in SME does not imply a hermitian hamiltonian. In order to resolve this problem a redefinition of the field is needed and this has been achieved [10]. This will be utilized in the present development. Our aim here is to provide theoretical predictions regarding the quantum vacuum in this EQED. We concentrate on calculating the effects of these modifications on the Casimir force in the fermion sector.

The Casimir effect consists in the calculation of the vacuum energy density of a quantum field in the presence of boundary conditions. H. Casimir [11] was the first to analyze the vacuum fluctuation of the electromagnetic field confined between two conducting parallel plates. The effect was an attractive force between the plates. Sparnaay [12] made the first experimental observation with correct sign and magnitude. Subsequent experiments [13,14] have established this effect to a high degree of accuracy. This phenomenon has been applied to micro- and nanotechnologies [15,16] and superconductors at high temperatures [17,18]. The Casimir effect for fermions at zero and finite temperature also has been investigated [19–21]. This effect for fermions is interesting when the structure of proton in particle physics is considered, in particular for the phenomenological bag model. Quarks and gluons are confined in the bag. In this paper we derive the Casimir effect at finite temperature considering the fermion sector of the EQED of the SME.

* Corresponding author.
E-mail addresses: alesandroferreira@fisica.ufmt.br (A.F. Santos), khannaf@uvic.ca (F.C. Khanna).

http://dx.doi.org/10.1016/j.physletb.2016.09.049

There are three different, but equivalent, formalisms to introduce temperature effects in a quantum field theory. (i) The Matsubara formalism, the imaginary time formalism, [22] which is based on a substitution of time, t, by a complex time, $i\tau$. Since the time variable is exchanged for temperature, this method is a good tool for studying systems at equilibrium. (ii) The closed time path formalism [23] is a real time formalism at finite temperature. This procedure can be used to describe both equilibrium and non-equilibrium phenomena. In addition, leads to a doubling of the degrees of freedom, such that the Green functions are represented by a two dimensional matrix structure. (iii) The Thermo Field Dynamics (TFD) is a real time finite temperature formalism [24–29]. The thermal vacuum, $|0(\beta)\rangle$, belongs to the Fock space $\mathcal{S}_T$ that is a direct product of the original Fock space $\mathcal{S}$ and an independent identical copy of it $\tilde{\mathcal{S}}$ (tilde system). In this formalism the statistical average of an observable $\mathcal{A}$ is expressed as a thermal vacuum expectation value i.e., $\langle\mathcal{A}\rangle = \langle 0(\beta)|\mathcal{A}|0(\beta)\rangle$, where $\beta = \frac{1}{k_B T}$, and T is the temperature and k_B is the Boltzmann constant (we use $k_B = \hbar = c = 1$). The map between the tilde $\tilde{A}_i$ and non-tilde A_i operators is defined by the following tilde (or dual) conjugation rules:

$$\begin{aligned}
(A_i A_j)^{\sim} &= \tilde{A}_i \tilde{A}_j, \\
(cA_i + A_j)^{\sim} &= c^* \tilde{A}_i + \tilde{A}_j, \\
(A_i^{\dagger})^{\sim} &= \tilde{A}_i^{\dagger}, \\
(\tilde{A}_i)^{\sim} &= -\xi A_i,
\end{aligned} \tag{1}$$

with $\xi = -1$ for bosons and $\xi = +1$ for fermions. The temperature effect is implemented in the doubled Fock space by a Bogoliubov transformation which introduces a rotation of the tilde and non-tilde variables. This formalism is useful for systems in equilibrium. For such systems the Bogoliubov transformation is unitary. Here we choose to use the TFD formalism.

This paper is organized as follows. In section 2, the energy-momentum tensor for fermions of the SME is calculated. In section 3, a brief introduction to TFD is presented. In section 4, some applications are developed. The Stefan–Boltzmann law and the Casimir effect at zero and finite temperature are derived. In section 5, some concluding remarks are presented.

2. The energy momentum tensor for the Dirac field of the SME

The Lagrangian for the fermion sector of the extended quantum electrodynamics of the SME is

$$\mathcal{L} = \bar{\psi}\left(i\Gamma^{\mu}\overleftrightarrow{\partial}_{\mu} - M\right)\psi, \tag{2}$$

where

$$\Gamma^{\mu} = \gamma^{\mu} + (c^{\mu\nu} + d^{\mu\nu}\gamma_5)\gamma_{\nu} + e^{\mu} + if^{\mu}\gamma_5 + \frac{1}{2}g^{\kappa\mu\nu}\sigma_{\kappa\nu}, \tag{3}$$

$$M = m + (a^{\mu} + b^{\mu}\gamma_5)\gamma_{\mu} + \frac{1}{2}H^{\mu\nu}\sigma_{\mu\nu}. \tag{4}$$

The parameters in Γ^{μ} are dimensionless while the ones in M have dimension of mass. γ^{μ}, γ_5 and $\sigma_{\kappa\nu}$ denote the Dirac matrices. The coefficients for Lorentz violation are a^{μ}, b^{μ}, $c^{\mu\nu}$, $d^{\mu\nu}$, e^{μ}, f^{μ}, $g^{\kappa\mu\nu}$ and $H^{\mu\nu}$.

The hamiltonian associated with the lagrangian (2) is non-hermitian and corresponds to nonunitary time evolution. This difficulty can be resolved by a spinor redefinition $\psi = A\chi$ in the lagrangian. The quantity A is chosen such that the time-derivative is that of the usual Dirac lagrangian [10]. This spinor redefinition leaves unchanged the physics. Thus the lagrangian becomes

$$\mathcal{L} = \bar{\chi}\left(i\bar{A}\Gamma^{\mu}A\overleftrightarrow{\partial}_{\mu} - \bar{A}MA\right)\chi. \tag{5}$$

Using $\bar{A} = \gamma^0 A^{\dagger}\gamma^0$ and $A^{\dagger}\gamma^0\Gamma^0 A = I$, where I is the unit matrix, this lagrangian contains only time derivative as the usual term, i.e., $i\bar{\chi}\gamma_0\overleftrightarrow{\partial}^0\chi$.

The modified Dirac equation is obtained as

$$\left(i\bar{A}\Gamma^{\mu}A\partial_{\mu} - \bar{A}MA\right)\chi = 0. \tag{6}$$

Using this field equation, the energy-momentum tensor for fermions is given as

$$T^{\mu\nu} = i\bar{\chi}\bar{A}\Gamma^{\mu}A\partial^{\nu}\chi. \tag{7}$$

In order to get the Casimir effect the energy-momentum tensor is written so as to avoid a product of field operators at the same space–time point. Then

$$T^{\mu\nu}(x) = i\bar{A}\Gamma^{\mu}A\partial^{\nu}\lim_{x\to x'}\tau\left[\bar{\chi}(x')\chi(x)\right], \tag{8}$$

where τ is the time ordering operator.

The vacuum average of the energy-momentum tensor is

$$\begin{aligned}
\left\langle T^{\mu\nu}(x)\right\rangle &= \left\langle 0|T^{\mu\nu}(x)|0\right\rangle \\
&= -\lim_{x\to x'}\left\{\bar{A}\Gamma^{\mu}A\partial^{\nu}S(x-x')\right\},
\end{aligned} \tag{9}$$

where the Feynman propagator for the Dirac field [29] is

$$\begin{aligned}
S(x-x') &= -i\left\langle 0|\tau\left[\bar{\chi}(x')\chi(x)\right]|0\right\rangle \\
&= (i\gamma\cdot\partial + m)\,G_0(x-x'),
\end{aligned} \tag{10}$$

with

$$G_0(x-x') = \frac{-i}{(2\pi)^2}\frac{1}{(x-x')^2 + i\xi}, \tag{11}$$

being the propagator of the massless scalar field. To obtain linear order in parameters for Lorentz violation the choice $A = 1 - \frac{1}{2}\gamma^0(\Gamma_0 - \gamma_0)$ and $\bar{A} = 1 - \frac{1}{2}(\Gamma_0 - \gamma_0)\gamma^0$ is considered. Thus for a massless fermionic field the average of the energy momentum tensor becomes

$$\left\langle T^{\mu\nu}(x)\right\rangle = -i\lim_{x\to x'}\{\Gamma\,\partial^{\mu}\partial^{\nu}G_0(x-x')\}, \tag{12}$$

where $\Gamma = \left(1 + \frac{9}{4}\Gamma^i\gamma_i\right)$, with $i = 1, 2, 3$. The Minkowski metric with signature $(+ - - -)$ is used.

The parameters e^i, f^i and g^{kij} in Γ^i are not extractable directly from SME and are taken to be zero or suppressed due to the renormalizibilty and gauge invariance requirements. The parameters c^{ij} and d^{ij} are traceless and symmetric. For simplicity we will consider the case $d^{ij} = 0$. Then the average of the energy-momentum tensor with Lorentz violating term is

$$\left\langle T^{\mu\nu}(x)\right\rangle = -\frac{i}{4}\lim_{x\to x'}\{\Gamma_c\,\partial^{\mu}\partial^{\nu}G_0(x-x')\}, \tag{13}$$

where $\Gamma_c = (31 + 9c_i^i)$, with c_i^i being the parameter that violates Lorentz symmetry. It is important note that the term c_i^i is not the trace of c^{ij}, since eq. (12) yields a term proportional to

$$\begin{aligned}
\Gamma^i\gamma_i &= c^{ij}\gamma_j\gamma_i = c_i^j\gamma_j\gamma^i \\
&= c_1^1\gamma_1\gamma^1 + c_2^2\gamma_2\gamma^2 + c_3^3\gamma_3\gamma^3 = c_i^i,
\end{aligned} \tag{14}$$

where $\gamma_1\gamma^1 = \gamma_2\gamma^2 = \gamma_3\gamma^3 = 1$ is used.

The TFD formalism is used in order to introduce the finite temperature effect.

3. Brief introduction to TFD

TFD consists in the generation of thermal states by doubling the degrees of freedom in a Hilbert space accompanied by the temperature dependent Bogoliubov transformation [29,24–26,30,31]. This doubling is defined by the tilde ($\sim$) conjugation rules, associating each operator in $\mathcal{S}$ to two operators in $\mathcal{S}_T$, where the expanded space is $\mathcal{S}_T = \mathcal{S}\otimes\tilde{\mathcal{S}}$, with $\mathcal{S}$ being the standard Fock space and $\tilde{\mathcal{S}}$ the fictitious space. For an arbitrary fermionic operator F the standard doublet notation is

$$F^a = \left(\begin{array}{c} F^1 \\ F^2 \end{array}\right) = \left(\begin{array}{c} F \\ \tilde{F}^\dagger \end{array}\right), \tag{15}$$

where the physical variables are described by nontilde operators. The tilde operators are auxiliary degrees of freedom which allow accommodation of the thermal properties of the system. A Bogoliubov transformation which corresponds to a rotation in the tilde and non-tilde variables introduces thermal effects. For fermions and using the doublet notation we get

$$\left(\begin{array}{c} b(\alpha) \\ \tilde{b}^\dagger(\alpha) \end{array}\right) = \mathcal{B}(\alpha)\left(\begin{array}{c} b(k) \\ \tilde{b}^\dagger(k) \end{array}\right), \tag{16}$$

where $(b^\dagger, \tilde{b}^\dagger)$ are creation operators, $(b, \tilde{b})$ are destruction operators and $\mathcal{B}(\alpha)$ is the Bogoliubov transformation given as

$$\mathcal{B}(\alpha) = \left(\begin{array}{cc} u(\alpha) & -v(\alpha) \\ v(\alpha) & u(\alpha) \end{array}\right). \tag{17}$$

The quantities $u(\alpha)$ and $v(\alpha)$ are related to the Fermi distribution and are given as

$$v^2(\alpha) = \frac{1}{1+e^{\alpha\omega}}, \qquad u^2(\alpha) = \frac{1}{1+e^{-\alpha\omega}}, \tag{18}$$

such that $v^2(\alpha)+u^2(\alpha)=1$. Here $\omega=\omega(k)$ and $\alpha=\beta$.

Using this formalism the physical α-dependent energy-momentum tensor is defined as

$$\mathcal{T}^{\mu\nu(ab)}(x;\alpha) = \langle T^{\mu\nu(ab)}(x;\alpha)\rangle - \langle T^{\mu\nu(ab)}(x)\rangle. \tag{19}$$

Then

$$\mathcal{T}^{\mu\nu(ab)}(x;\alpha) = -\frac{i}{4}\lim_{x\to x'}\Big\{\Gamma_c\,\partial^\mu\partial^\nu \times \times\Big[G_0^{(ab)}(x-x';\alpha) - G_0^{(ab)}(x-x')\Big]\Big\}, \tag{20}$$

where $a,b=1,2$ and

$$G_0^{(ab)}(x-x') = \int\frac{d^4k}{(2\pi)^4}\times \times e^{-ik(x-x')}G_0^{(ab)}(k), \tag{21}$$

with

$$G_0^{(ab)}(k) = \left(\begin{array}{cc} G_0(k) & 0 \\ 0 & G_0^*(k)\end{array}\right). \tag{22}$$

The α-dependent part of the Green function is

$$G_0^{(ab)}(x-x';\alpha) = \int\frac{d^4k}{(2\pi)^4}\times \times e^{-ik(x-x')}G_0^{(ab)}(k;\alpha), \tag{23}$$

where $G_0^{(ab)}(k;\alpha) = \mathcal{B}^{-1}(\alpha)G_0^{(ab)}(k)\mathcal{B}(\alpha)$. Explicitly, the physical component of $G_0^{(ab)}(k;\alpha)$ is

$$G_0^{(11)}(k;\alpha) \equiv G_0(k;\alpha) = G_0(k) + v^2(\alpha)[G_0^*(k) - G_0(k)]. \tag{24}$$

For fermions the energy-momentum tensor (20) is studied for some choice of the α parameter.

4. Some applications

Here three applications which depend on the choice of the α parameter are studied.

4.1. Stefan–Boltzmann law

Let us consider the generalized Bogoliubov transformation [32] which is written as

$$v^2(k_\alpha;\alpha) = \sum_{s=1}^{d}\sum_{\{\sigma_s\}}2^{s-1}\times \times\sum_{l_{\sigma_1},\ldots,l_{\sigma_s}=1}^{\infty}(-\eta)^{s+\sum_{r=1}^{s}l_{\sigma_r}}\times \times\exp\left[-\sum_{j=1}^{s}\alpha_{\sigma_j}l_{\sigma_j}k^{\sigma_j}\right], \tag{25}$$

where d is the number of compactified dimensions, $\eta=1(-1)$ for fermions (bosons) and $\{\sigma_s\}$ denotes the set of all combinations with s elements.

In this case the choice is $\alpha=(\beta,0,0,0)$ and then

$$v^2(\beta) = \sum_{l=1}^{\infty}(-1)^{l+1}\,e^{-\beta k_0 l}. \tag{26}$$

Using eq. (23) the thermal Green function becomes

$$G_0^{(11)}(x-x';\beta) = G_0(x-x') + \sum_{l=1}^{\infty}(-1)^{l+1} \times\Big[G_0^*(x'-x+i\beta l n_0) - G_0(x-x'-i\beta l n_0)\Big], \tag{27}$$

where $n_0=(1,0,0,0)$. Then the energy-momentum tensor is given as

$$\mathcal{T}^{\mu\nu(11)}(x;\alpha) = \frac{i}{4}\lim_{x\to x'}\sum_{l=1}^{\infty}(-1)^l\,\Gamma_c\,\partial^\mu\partial^\nu\times \times\Big[G_0^*(x'-x+i\beta l n_0) - G_0(x-x'-i\beta l n_0)\Big]. \tag{28}$$

For $\mu=\nu=0$ we obtain the modified Stefan–Boltzmann law

$$\mathcal{T}^{00(11)}(\beta) = \frac{7\pi^2}{60}T^4\left(a+b\,c_i^i\right), \tag{29}$$

where c_i^i is a Lorentz violating term. The constants a and b are defined as $a=\frac{31}{16}$ and $b=\frac{9}{16}$. Thus the lowest-order prediction of the fermions sector of the SME modifies the Stefan–Boltzmann law. The field redefinition changes the Stefan–Boltzmann law by a multiplicative factor.

4.2. Casimir effect at zero temperature

For parallel plates perpendicular to the z direction and separated by a distance a the α parameter is chosen as $\alpha = (0, 0, 0, i2a)$. Then

$$v^2(a) = \sum_{l=1}^{\infty}(-1)^{l+1}\, e^{-i2ak_3 l}, \tag{30}$$

and the energy-momentum tensor becomes

$$\mathcal{T}^{\mu\nu(11)}(x;\alpha) = \frac{i}{4}\lim_{x\to x'}\sum_{l=1}^{\infty}(-1)^l\,\Gamma_c\,\partial^\mu\partial^\nu \times \times\big[G_0^*(x'-x+2alk_3) - G_0(x-x'-2alk_3)\big]. \tag{31}$$

From this equation the Casimir energy and pressure are obtained

$$E(a) = \mathcal{T}^{00(11)}(a) = -\frac{7\pi^2}{2880a^4}\left(a+b\,c_i^i\right),$$
$$P(a) = \mathcal{T}^{33(11)}(a) = -\frac{7\pi^2}{960a^4}\left(a+b\,c_i^i\right), \tag{32}$$

where c_i^i is the Lorentz violating coefficient.

4.3. Casimir effect at finite temperature

In order to analyze the temperature effect in the Casimir effect $\alpha = (\beta, 0, 0, i2a)$ is considered, where the temperature effect and spatial compactification are combined. Then the Bogoliubov transformation, eq. (25), becomes

$$\begin{aligned} v^2(\beta,a) &= v^2(k^0;\beta) + v^2(k^3;a) + 2v^2(k^0;\beta)v^2(k^3;a), \\ &= \sum_{l_0=1}^{\infty}(-1)^{l_0+1}\, e^{-\beta k^0 l_0} + \sum_{l_3=1}^{\infty}(-1)^{l_3+1}\, e^{-i2ak^3 l_3} \\ &\quad + 2\sum_{l_0,l_3=1}^{\infty}(-1)^{l_0+l_3} e^{-\beta k^0 l_0 - i2ak^3 l_3}. \end{aligned} \tag{33}$$

The first term leads to the Stefan Boltzmann law and the second term to the Casimir effect at zero temperature. The Casimir effect at finite temperature is

$$\mathcal{T}^{\mu\nu(11)}(\beta,a) = -\frac{i}{2}\lim_{x\to x'}\sum_{l_0,l_3=1}^{\infty}(-1)^{l_0+l_3} \times \times\,\Gamma_c\,\partial^\mu\partial^\nu\big[G_0^*(x'-x+i\beta l_0 n_0+2alk_3) - G_0(x-x'-i\beta l_0 n_0-2alk_3)\big]. \tag{34}$$

The Casimir energy, $\mathcal{T}^{00(11)}(\beta,a)$, and pressure, $\mathcal{T}^{33(11)}(\beta,a)$, respectively, are given as

$$\mathcal{T}^{00(11)}(\beta,a) = -\frac{8}{\pi^2}\sum_{l_0,l_3=1}^{\infty}(-1)^{l_0+l_3} \times \times\frac{(2al_3)^2-3(\beta l_0)^2}{[(\beta l_0)^2+(2al_3)^2]^3}(a+b\,c_i^i), \tag{35}$$

and

$$\mathcal{T}^{33(11)}(\beta,a) = -\frac{8}{\pi^2}\sum_{l_0,l_3=1}^{\infty}(-1)^{l_0+l_3} \times \times\frac{3(2al_3)^2-(\beta l_0)^2}{[(\beta l_0)^2+(2al_3)^2]^3}(a+b\,c_i^i). \tag{36}$$

Thus the Casimir energy is

$$E(\beta;a) = \Big[\frac{7\pi^2}{60\beta^4} - \frac{7\pi^2}{2880a^4} - \frac{8}{\pi^2}\sum_{l_0,l_3=1}^{\infty}(-1)^{l_0+l_3}\frac{(2al_3)^2-3(\beta l_0)^2}{[(\beta l_0)^2+(2al_3)^2]^3}\Big](a+b\,c_i^i). \tag{37}$$

Note that at low temperatures this energy recovers the Casimir energy at zero temperature, while the high temperature limit is dominated by the positive contribution of the Stefan–Boltzmann term. The Lorentz violating terms emerge at both limits.

The Casimir Pressure is

$$P(\beta;a) = \Big[\frac{7\pi^2}{180\beta^4} - \frac{7\pi^2}{960a^4} - \frac{8}{\pi^2}\sum_{l_0,l_3=1}^{\infty}(-1)^{l_0+l_3}\frac{3(2al_3)^2-(\beta l_0)^2}{[(\beta l_0)^2+(2al_3)^2]^3}\Big](a+b\,c_i^i). \tag{38}$$

For low temperatures the pressure is negative. When temperature increases, a transition to positive pressure happens. It is possible to determine the critical curve of the transition. The point of transition occurs when the pressure vanishes. Then analyzing our result we note that the Lorentz violating term does not modify this transition value.

5. Conclusion

Symmetry, symmetry breaking and physical laws are connected to the description of nature. In string theory it is possible to violate Lorentz and CPT symmetries. The extension of these ideas for SM leads to SME where break down of Lorentz and CPT symmetries is possible. In this paper we use the fermion sector of the SME to calculated the Casimir effect at zero and finite temperature.

The Casimir energy for the electromagnetic and fermions field within the SM at zero and finite temperature is considered and experimentally established. Here our interest is to study SME with Lorentz and CPT violating terms for fermions systems. The energy-momentum tensor for the fermion sector of SME is calculated. Using the TFD formalism the Stefan–Boltzmann law and Casimir energy are obtained at finite temperature. The Casimir energy is found to be $(a+bc_i^i)P$, where P is the standard Casimir pressure, c_i^i is the Lorentz violating parameter and a, b are constants. Final results are multiplied by a constant factor due to the field redefinition. This is necessary to obtain a theory where the hamiltonian is hermitian. Temperature effects contribute to constrain Lorentz violation parameters. Overall the effect of Lorentz and CPT violation on Casimir energy is small.

Acknowledgements

This work by A. F. S. is supported by CNPq projects 476166/2013-6 and 201273/2015-2. We thank Physics Department, University of Victoria for access to facilities.

References

[1] T.D. Lee, C.N. Yang, Phys. Rev. 104 (1956) 254.
[2] C.S. Wu, et al., Phys. Rev. 105 (1957) 1413.

[3] J.H. Christenson, J.W. Cronin, V.L. Fitch, R. Turley, Phys. Rev. Lett. 13 (1964) 138.
[4] V.A. Kostelecky, S. Samuel, Phys. Rev. D 39 (1989) 683;
V.A. Kostelecky, R. Potting, Nucl. Phys. B 359 (1991) 545.
[5] M. Bojowald, H.A. Morales-Tecotl, H. Sahlmann, Phys. Rev. D 71 (2005) 084012.
[6] S.M. Carroll, J.A. Harvey, V.A. Kostelecky, C.D. Lane, T. Okamoto, Phys. Rev. Lett. 87 (2001) 141601.
[7] F.R. Klinkhamer, Nucl. Phys. B 535 (1998) 233.
[8] D. Colladay, V.A. Kostelecky, Phys. Rev. D 55 (1997) 6760, Phys. Rev. D 58 (1998) 116002.
[9] V.A. Kostelecky, Phys. Rev. D 69 (2004) 105009.
[10] V.A. Kostelecky, C.D. Lane, J. Math. Phys. 40 (1999) 6245, arXiv:hep-ph/9909542.
[11] H.G.B. Casimir, Proc. K. Ned. Akad. Wet. 51 (1948) 793.
[12] M.J. Sparnaay, Physica 24 (1958) 751.
[13] S.K. Lamoreaux, Phys. Rev. Lett. 28 (1997) 5.
[14] U. Mohideen, A. Roy, Phys. Rev. Lett. 81 (1998) 21.
[15] M. Bordag, U. Mohideen, V.M. Mostepanenko, Phys. Rep. 353 (2001) 1.
[16] S.K. Lamoreaux, Am. J. Phys. 67 (1999) 850.
[17] M. Bordag, J. Phys. A 39 (2006) 6173.
[18] M.T.D. Orlando, et al., J. Phys. A, Math. Theor. 42 (2009) 025502.
[19] H. Queiroz, J.C. da Silva, F.C. Khanna, J.M.C. Malbouisson, M. Revzen, A.E. Santana, Ann. Phys. 317 (2005) 220.
[20] E. Elizalde, F.C. Santos, A.C. Tort, Int. J. Mod. Phys. A 18 (2003) 1761.
[21] E. Elizalde, M. Bordag, K. Kirsten, J. Phys. A 31 (1998) 1743.
[22] T. Matsubara, Prog. Theor. Phys. 14 (1955) 351.
[23] J. Schwinger, J. Math. Phys. 2 (1961) 407;
J. Schwinger, Lecture Notes of Brandeis, University Summer Institute, 1960.
[24] Y. Takahashi, H. Umezawa, Collect. Phenom. 2 (1975) 55, Int. J. Mod. Phys. B 10 (1996) 1755.
[25] Y. Takahashi, H. Umezawa, H. Matsumoto, Thermofield Dynamics and Condensed States, North-Holland, Amsterdam, 1982.
[26] H. Umezawa, Advanced Field Theory: Micro, Macro and Thermal Physics, AIP, New York, 1993.
[27] A.E. Santana, F.C. Khanna, Phys. Lett. A 203 (1995) 68.
[28] A.E. Santana, F.C. Khanna, H. Chu, C. Chang, Ann. Phys. 249 (1996) 481.
[29] F.C. Khanna, A.P.C. Malbouisson, J.M.C. Malboiusson, A.E. Santana, Thermal Quantum Field Theory: Algebraic Aspects and Applications, World Scientific, Singapore, 2009.
[30] A.E. Santana, A. Matos Neto, J.D.M. Vianna, F.C. Khanna, Physica A 280 (2000) 405.
[31] F.C. Khanna, A.P.C. Malbouisson, J.M.C. Malbouisson, A.E. Santana, Ann. Phys. 324 (2009) 1931.
[32] F.C. Khanna, A.P.C. Malbouisson, J.M.C. Malbouisson, A.E. Santana, Ann. Phys. 326 (2011) 2634.