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Closed Form Formulas for Distributed Circuit Model of Discontinuities in HTS Microstrip Transmission Lines

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Abstract

A distributed circuit model for different kinds of discontinuities in high temperature superconducting (HTS) microstrip transmission lines (TLs), is proposed. In each case, closed form formula for lumped element model is presented based on the configuration of the discontinuity and the characterizations of HTS microstrip TLs. These discontinuities consist of steps in width, open ends, gaps and 90-degree bends. In the case of normal conductor microstrip TLs there are a lot of numerical and analytical equations that can accurately model them, however those formulas are not efficient for HTS TLs. Thus modified relations are extracted utilizing the superconducting characterizations to obtain much more accurate formulas. Additionally temperature dependence of HTS TLs is considered in the relations. Moreover regarding the kinetic inductance in HTS TLs a closed form formula is proposed for characteristic impedance of HTS TLs. Furthermore correction factors based on fringe fields is used to optimize all formulas. Using these formulations can lead to modeling and analysis of some superconducting microwave devices such as resonators, microwave filters, couplers, etc. In contrast to EM analysis, using the distributed circuit model is much easier for analysis of HTS microwave devices. The accuracy of the proposed model is confirmed in comparison with some electromagnetic full-wave simulations. This full analytical approach shows great accuracy in this test case as well.

Keywords: Closed form formulas; Lumped element circuit model; HTS transmission lines; Microstrip lines; Superconducting.

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1. Introduction

High temperature superconducting (HTS) microwave devices are found very important role in recent communication systems. Microwave resonators, filters [1],[2], Duplexers[3], delay lines, antenna [4] and couplers [5] are the most important samples of utilizing HTSs for microwave devices. All of these modules often are implemented in microstrip form. It is clear that these microwave integrated circuits

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(MICs) consist of so many microstrip transmission lines (TLs) that certainly involve some discontinuities. Therefore modeling of discontinuities in microstrip TLs is vital for full analytical analysis and design of superconducting microwave devices. In comparison with numerical EM simulation, analysis of the microstrip devices by circuit modeling is very fast and efficient.

There are some closed form expressions for normal conductor microstrip transmission lines (TLs) discontinuities [6],[7], but these formulas are not efficient in the superconducting case. There is closed form relations for distributed circuit (RLGC) model of superconducting transmission line [8], However there is not any closed form formulas for lumped element modeling of discontinuities in superconducting microstrip circuits. These circuit models can be used in computer aided simulations (for example in the schematic environment of some of the microwave circuit simulators such as ADS or Microwave Office softwares) for very fast and efficient analysis of superconducting microwave devices. Furthermore lumped element circuit model can be extended to use for nonlinear analysis of superconducting microstrip modules.

In this paper we proposed the lumped element model for four kinds of microstrip discontinuities consisting steps in width, open ends, gaps and right-angle bends. To validate the proposed circuit modeling, we analyzed a simple sample structure of superconducting microstrip transmission line (SMTL) by using the lumped element model and compare its result with full-wave electromagnetic (EM) simulation by IE3D (Zeland) software.

2. Circuit Modeling of Discontinuities in the SMTLs

In this section the lumped element circuit modeling for some important discontinuities in the SMTLs is proposed as in Fig 1. Basic concepts of these modeling is derived from well known circuit modeling of discontinuities in normal microstrip TLs in [6],[7], that the values of the lumped elements are modified for the case of superconducting microstrip TLs. Here we proposed these values of lumped element in the circuit modeling. In these equations ‘h’ and ‘t’ are respectively the thickness of substrate and HTS, ‘λ’ is the penetration depth and Z_{C_Sc} is our poposed formulation for the characteristics impedance of SMTLs,

$$Z_{C\_Sc}=\sqrt{Z_{C\_normal}^{2}+\frac{\mu_0}{\varepsilon_0\varepsilon_{re}}\frac{\lambda h}{W^2K^2}\coth(t/\lambda)} \tag{1}$$

where ‘Z_{C_normal}’ is the routine characteristic impedance for normal conductor microstrip TLs and ‘K’ is fringe factor and it can be found with the conformal mapping technique [6] and it can be calculated by,

$$K=\begin{cases}\left(\frac{1}{2\pi}\ln\left(\frac{8h}{W}+\frac{W}{4h}\right)\right)^{-1}\frac{h}{W} & W\le h\\ \left(\frac{W}{h}+2.42-0.44\frac{h}{W}+\left(1-\frac{h}{W}\right)^{6}\right)\frac{h}{W} & W\ge h\end{cases} \tag{2}$$

For steps in width as in Fig 1-a, we have,

$$C = 0.00137h\,\frac{K(W_1)\sqrt{\varepsilon_{re1}}}{Z_{C\_Sc1}}\,(1-\frac{W_2}{W_1})(\frac{\varepsilon_{re1}+0.3}{\varepsilon_{re1}-0.26})(\frac{W_1/h+0.26}{W_1/h+0.8})\ \ (pF);\ \ L_1=\frac{L_{w1}}{L_{w1}+L_{w2}}L,\ \ L_2=\frac{L_{w2}}{L_{w1}+L_{w2}}L \tag{3}$$

$$L_{wi}=\frac{\mu_0 d}{W_iK(W_i)}+\frac{X_s}{\omega W_iK(W_i)}=\frac{\mu_0}{W_iK(W_i)}\{d+\lambda\coth(t/\lambda)\};\ L=0.000987h(1-\frac{L_{w1}}{L_{w2}})^2\ (nH) \tag{4}$$

In case of open ends as in Fig 1-b we can say,

$$\begin{cases}C_p=0.5C_e\\ C_g=0.5C_o-0.25C_e\end{cases} \tag{5}$$

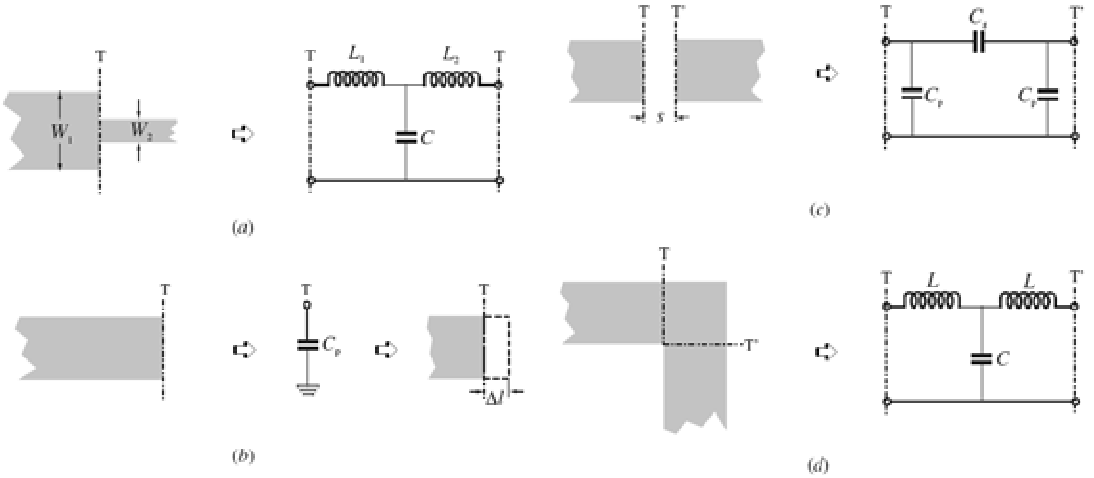


Fig 1. Circuit modeling of a) Step in width b) Open end c) Gap d) Right-angle bend [7]

$$\kappa_1 = 0.434907\frac{\varepsilon_{re}^{0.81}+0.26(W/h)^{0.8544}+0.236}{\varepsilon_{re}^{0.81}-0.189(W/h)^{0.8544}+0.87};\kappa_2 = 1+\frac{(W/h)^{0.371}}{2.35\varepsilon_r+1};\ \kappa_3 = 1+\frac{0.5274\tan^{-1}(0.084(W/h)^{1.9413/\kappa_2})}{\varepsilon_{re}^{0.9236}} \quad (6)$$

$$\kappa_4=1+0.037\tan^{-1}(0.067(W/h)^{1.456})\{6-5\exp(1-\varepsilon_r)\};\ \kappa_5=1-0.218\exp(-7.5W/h) \quad (7)$$

For gaps as in Fig 1-c we have,

$$\begin{cases}C_p=0.5C_e\\ C_g=0.5C_o-0.25C_e\end{cases} \Rightarrow \begin{cases}C_o=WK(\frac{\varepsilon_r}{9.6})^{0.8}(\frac{s}{W})^{m_o}\exp(k_o)\ (pF)\\ C_e=WK(\frac{\varepsilon_r}{9.6})^{0.9}(\frac{s}{W})^{m_e}\exp(k_e)\ (pF)\end{cases};\begin{cases}m_o=\frac{W}{h}(0.619\log(W/h)-0.3853)\\ k_o=4.26-1.453\log(W/h)\end{cases} \quad (8)$$

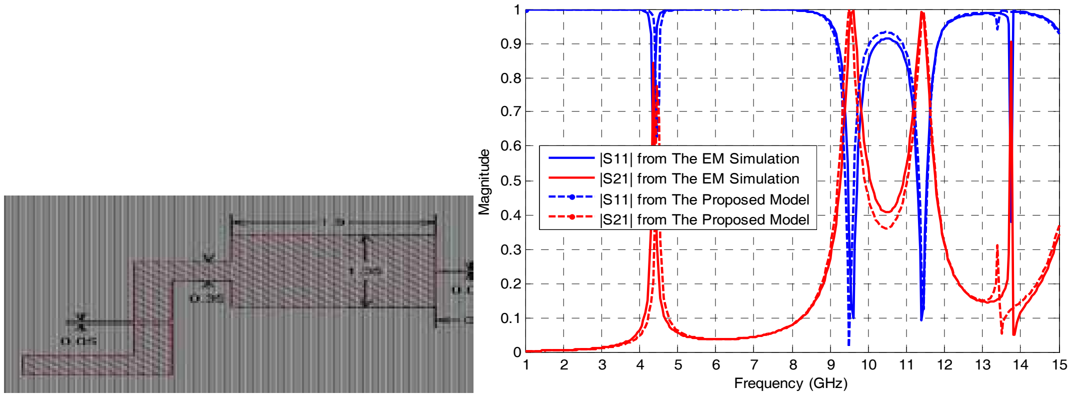
$$\begin{cases}m_e=0.8675\\ k_e=2.043(W/h)^{0.12}\end{cases}\ for\,0.1\le s/W\le 0.3\ ;\begin{cases}m_e=\frac{1.565}{(w/h)^{0.16}}-1\\ k_e=1.97-\frac{0.03}{W/h}\end{cases}\ for\,0.3\le s/W\le 1 \quad (9)$$

Finally the values of lumped elements for right-angle bend as in Fig 1-d can be written as follows,

$$C=WK\begin{cases}(14\varepsilon_r+12.5)\sqrt{W/h}-(1.83\varepsilon_r-2.25)\sqrt{h/W}+\frac{0.02\varepsilon_r h}{W} & for\,W/h<1\\ (9.5\varepsilon_r+1.25)W/h+5.2\varepsilon_r+7 & for\,W/h\ge 1\end{cases}(pF);\ L=100h(4\sqrt{W/h}-4.21)\ (nH) \quad (10)$$

3. Evaluation of the Proposed Model

In this section we are going to evaluate the proposed models with comparison by full-wave electromagnetic (EM) simulation in IE3D (zeland) software that has capability of simulation of superconductor planar structures. For this purpose we consider a simple structure of 450nm YBCO on both sides of a 1mm $LaAlO_3$ substrate. As shown in Fig 2-a this circuit consists of 11 transmission lines, 4 bends, 4 step in widths and 2 gaps, therefore the total transfer matrix of this shape is produced by regular matrix multiplication of 21 transfer matrixes and finally the scattering matrix is inferred from the obtained transfer matrix. All dimensions in Fig 2-a are in mm. Result of the comparison between the EM



(a) (b)

Fig 2. a) Considered structure of SMTL for evaluation of the proposed models b) Comparison of S-parameters between the EM simulation and the proposed model analyzing

simulation and analyzing the mentioned structure at 77K by using the proposed lumped element circuit model is shown in Fig 2-b. As can see in Fig 2-b the fast and efficient method of analyzing the mentioned structure of SMTLs by using the proposed circuit model coincides quiet well with EM simulation results.

4. Conclusion

In this paper we proposed lumped element circuit models for four kinds of discontinuities in HTS microstrip transmission lines. In each case some closed form formulas are presented for the value of all lumped elements in the models. To validate the accuracy of the proposed models, a sample microstrip HTS structure is modeled and analyzed and its modeling results are compared to electromagnetic simulation by IE3D software. This full analytical approach shows great agreement with the results of numerical full wave analysis. Hence these models can be used for analysis of different superconducting microwave devices especially HTS microstrip filters in different temperatures even near critical temperature. This full analytical method, in compare to numerical methods, is very fast and efficient.

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References

[1] G. Tsuzuki, S. Ye, and S. Berkowitz, *IEEE Trans. Microwave Theory Tech.*, No. 50, (2002), 2924.
[2] Li-Min Wang; Wi-Chun Lin; Min-Long Chang; Chiuan-Yu Shiau; Chun-Te Wu, *IEEE Transactions on Applied Superconductivity*, No. 21 , (2011) 551.
[3] Qiang Zhang; Yongbo Bian; Jin Guo; Bin Cui; Jia Wang; Tao Yu; Lu Gao; Yuehui Wang; Chunguang Li; Xueqiang Zhang; Hong Li; Changzheng Gao; Yusheng He, *IEEE Transactions on Applied Superconductivity*, No. 20 , (2010) 2.
[4] Wang, H.Y.; Lancaster, M.J., *IEEE Transactions on Antennas and Propagation*, No. 47 (1989) 829.
[5] Hoffmann, E.; Deppe, F.; Niemczyk, T.; Wirth, T.; Menzel, E. P.; Wild, G.; Huebl, H.; Mariantoni, M.; WeiBl, T.; Lukashenko, A.; Zhuravel, A. P.; Ustinov, A. V.; Marx, A.; Gross, R., Applied Physics Letters, No. 97 ,(2010) 222508.
[6] K.C. Gupta, R. Garg, I. Bahl, P. Bhartia, *Microstrip Lines and Slotlines*, Artech House, Norwood, 1996
[7] Jia-Sheng Hong, M. J. Lancaster, *Microstrip Filters for RF/Microwave Applications*, John Wiley & Sons, Inc., 2001.
[8] M.S. Boutboul, H. Kokabi, M. Pyee, *Physica C* 309(1998) 71.