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ScienceDirect
Procedia CIRP 43 (2016) 262 – 267
www.elsevier.com/locate/procedia

14th CIRP Conference on Computer Aided Tolerancing (CAT)

On Connected Tolerances in Statistical Tolerance-Cost-Optimization of Assemblies with Interrelated Dimension Chains

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Abstract

Identifying a suitable compromise between tight and thus expensive tolerances and wide tolerances that may negatively influence the product quality is a major challenge. This paper focuses on the tolerance-cost-optimization of mechanical assemblies with interrelated dimension chains considering dependencies between the tolerance–cost-relationships. Taking into account interrelated dimension chains the crux is, however, that modifications of a single tolerance can influence several dimension chains as well as the resulting production costs. Based on different existing approaches for the statistical tolerance-cost-optimization, the authors will provide appropriate guidance for the product developer dealing with interrelated dimension chains.

Peer-review under responsibility of the organizing committee of the 14th CIRP Conference on Computer Aided Tolerancing

*Keywords:*tolerance allocation; interrelated tolerance-cost-function; connected dimension chains; modification of tolerance-cost-relation

1. Introduction

Today's products are usually designed in a 3D-CAD environment that can only represent the designer's ideal conception of the parts. In practice, however, these ideal conceptions can never be realized. The ideal product characteristics will never be achieved due to variations during manufacture and assembly as well as varying operating conditions (such as a varying temperature during the use) (Figure 1). At this point, tolerances come into play to restrict the acceptable effects of intrinsic variations during production, assembly and operation.

Despite all gained success in research and development of tolerance engineering, the specification of tolerances of products, assemblies, single parts and even individual features is a tightrope walk because each tighter tolerance enhances the production cost while each wider tolerance might endanger significant quality features of a product [1]. Due to the fact that not only the costs, but also the quality of a product is strongly influenced by the tolerance scheme, tolerance design is one of the most important steps in product development.

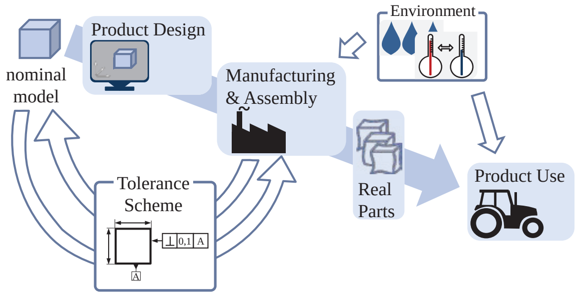

Fig. 1. From the nominal model to the deviation-afflicted real parts

Peer-review under responsibility of the organizing committee of the 14th CIRP Conference on Computer Aided Tolerancing
doi:10.1016/j.procir.2016.02.031

To ensure quality and flawless functionality of products several methods on tolerance analysis have been established in the last decades. Besides more recent approaches on the representation of tolerances such as Deviation Domains [2], T-Maps® [3] or Skin Models [4], vector-chain-based approaches are widespread in academics and industrial practice [1]. All these approaches have in common that relevant quality features are described by so-called "Functional Key Characteristics" (FKCs) [5] which are influenced by varying factors.

Considering the complexity of products the designer has to cope with several different FKCs that are often interrelated. This means that the modification of a single tolerance value influences more than one FKC. This issue is illustrated in Figure 2, where the interrelation of two dimensional chains, each consisting of five rings, is shown.

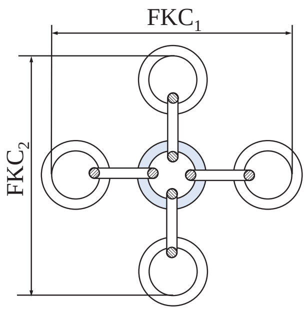

Fig. 2. Interrelated dimension chains

The diameter and roundness of the blue ring influence both FKCs as shown in the drawing

2. Related Work

Specifying a proper tolerance design for each single part of a product effects several departments of a company. Not only production and assembly departments but also the quality management and the rework processes etc. are directly affected by given tolerances. As a consequence, the initial tolerance design of a new product is usually based on previous projects or general tolerances. After the specification of the initial tolerance design a tolerance analysis is usually set up to evaluate the effects of the deviations on certain FKCs. If the functional requirements cannot be fulfilled, the initial tolerance design is changed iteratively until the required scrap rate is under a certain limit. Since tolerances are major cost drivers, finding the "best tolerance design" is essential against the backdrop of technical as well as financial aspects.

Early analytical approaches for tolerance synthesis [6] have been replaced by the first computer-aided approach in 1964 by MOY [7]. The computer-aided approaches were further developed using different algorithms in the 1970s, most famously by SPOTTS and SPECKHART [8], [9]. Even though different tolerance-cost-relations from the 1970s were expanded within the following 20 years by CHASE et al. [10], none of these approaches considers connections of more than one tolerance dimensions.

First, SINGH et al. classifies tolerance chains in [11] and highlights the importance of connections between different dimension chains. According to them, a dimension chain (also called tolerance chain or dimension loop) is an abstract model of an assembly taking geometric relations into account. Thereby, the dimension chain is a sequence of at least two dimensions. The simplest form of a dimension chain is an elementary chain that encounters every end point once. Dimension chains that include a dimension not more than one time are called "simple chains". Apart from elementary and simple chains, each remaining type of dimension chain is called an "interrelated chain" [11].

SINGH made a first approach defining the optimal tolerance design by means of a Genetic Algorithm (GA) to handle the non-linear dependencies. However, their approach is based on a worst-case tolerance analysis. Consequently, this approach is currently not able to handle different probability distributions of tolerances unlike the state-of-the-art Monte-Carlo-based tolerance analysis.

The above and below mentioned approaches assume that the tolerances for each dimension can be allocated arbitrary within each corresponding tolerance zone. Unlike this assumption LÖÖF points out, that usually tolerances can solely be picked from a limited set of discrete values. Based on this assumption a procedure for tolerance-cost-optimization, coupled with commercial CAT Software, using discrete values is presented. Furthermore the possibility for the consideration of general loss functions is provided within this work [12].

GEETHA also worked in a similar field, focusing on the composition of manufacturing costs. Their enhanced tolerance-cost-model considers several additional significant parameters such as costs arising from machine idle times and machine engaged times. A wheel mounting assembly is used to illustrate the optimization using Genetic Algorithm [13].

In Summary, existing research details on the importance of tolerance-cost-optimization of connected dimension chains. Because manufacturing processes often affect more than one dimension of parts, a dependence of different tolerances is inevitable, however currently not considered satisfyingly.

In this paper the authors provide a methodology for the statistical tolerance-cost-optimization for interrelated dimension chains considering connected tolerances. Therefore, the Particle Swarm Optimization (PSO) is used to identify the optimal tolerance design. The practical use of the proposed methodology is detailed for a driving pulley.

3. Tolerance-Cost-Optimization of interrelated Dimension Chains with connected production costs

Finding the ideal tolerance design for each single part of an assembly is a challenging task since even with known tolerance-cost-models it's often not obvious which tolerances should be tightened and/or widened. Therefore, a methodology for a systematic statistical tolerance-cost-optimization is presented in this section. Furthermore, existing tolerance-cost-models are adapted to meet the requirements of connected production costs.

3.1. Methodology

The presented methodology for a systematic tolerance-cost-optimization is shown in Figure 3. It is an extension of the existing method on tolerance allocation of mechanisms by WALTER et al. [1].

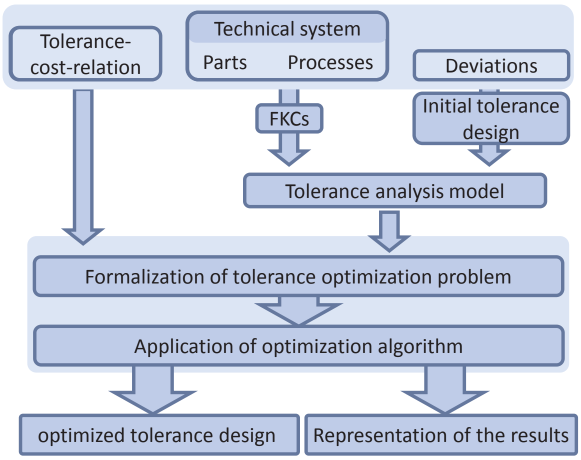

Fig. 3. Methodology: Tolerance-cost-optimization for interrelated dimension chains

Each part or process of a technical system has inherent deviations which can be specified and restricted with an initial tolerance design. In order to ensure the functionality of a technical system certain FKCs corresponding to functional requirements are defined. Commonly, minimum clearance between two parts is considered as a FKC and mathematically described using a dimension chain.

After the formulation of the required dimension chains a tolerance analysis is performed to analyze the effects of geometric deviations caused by manufacturing or assembling on the FKCs. Based on this analysis and the tolerance-cost-relationship for each part or manufacturing process, the tolerance-cost-optimization problem has to be formulated. Usually, this optimization problem focusses on minimizing the resulting manufacturing costs and not violating a certain scrap rate.

To identify the cost-optimal tolerance design a Particle Swarm Optimization (PSO), a metaheuristic optimization algorithm, is used. PSO is not only capable to find the global minima of a certain objective function it can also deal with highly non-linear constraints such as the tolerance-cost-models for manufacturing with alternative process selection.

Inspired by the swarming behavior of biological populations (such as fish flocks or bird swarms), the PSO was invented by KENNEDY et al. in 1995 [14].

PSO imitates the social behavior of a flock of individuals searching for food. A group of n random particles (each represents a valid solution of the optimization problem's constraints) is spread over the defined search space. During each iteration, the particles change their position to approximate the best solution. Therefore, the particles follow the currently best so far achieved solution considering their velocity.

Using the results of the optimization – the tolerance design – can be further adjusted and the results of the optimization process can be used in the product development process.

3.2. Adaption of tolerance-cost-relation on connected dimensions

When a part is produced often several manufacturing steps are done using the same machine and parameter settings. In the example (Figure 4) three shaft shoulders are dimensioned with reference to the end of the shaft. Since the shaft is turned in one manufacturing step all these dimension are connected in terms of accuracy of manufacturing.

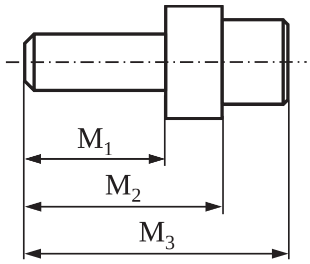

Fig. 4. Connected dimensions of a shaft

Tolerance-cost-relations describe the dependencies between a certain tolerance T_i and the corresponding financial effort in manufacturing costs $C(T_i)$. In this paper the adaption for CHASE's reciprocal model (k = 1) and SPOTTS' reciprocal squared (k = 2) models, according to the following equation, is illustrated:

$$C(T) = A_{fix} + \frac{B_{ind}}{T^k} \qquad (1)$$

Both models (as well as most other models) have in common that the costs are split into fixed manufacturing costs A_{fix} (setup, tooling, material, etc.) and individual costs B_{ind}, representing the production costs of a single dimension with a certain accuracy. Depending on the number of available manufacturing processes there can be several tolerance-cost-relations, each representing a single process or machine. In this case, the process can be chosen depending on the required tolerance of the corresponding cost-model.

A valid tolerance-cost-relation is necessary to formulate the optimization's objective function. Consequently, each part and each process that is used during the production has to be described by a valid tolerance-cost-relation.

Assuming the production of a part with five toleranced dimensions, several operating steps are conducted consecutively on the same machine. Calculating the five corresponding tolerance-cost-relations would lead to five equations, whereas each includes the same fixed costs A_{fix} for setup, material and tooling. As a consequence, the fixed costs would be overrepresented (in this case by factor 5) due to all five operating steps. This fact motivates the authors to extend the tolerance-cost-relation (2) in which the individual costs, $B_{ind,i}$ is divided by the tolerance to the power of the corresponding k_i, are summed up:

$$C(T_i) = A_{fix} + \sum_{i=1}^{n} \frac{B_{ind,i}}{T_i^{k_i}} \quad (2)$$

This tolerance-cost-relation for connected dimensions should be used below to calculate the costs for an assembly with several toleranced dimensions.

4. Case study: Driving pulley

Based on the methodology and the developed cost-model for connected dimensions the tolerance-cost-optimization for a driving pulley (Figure 5) is illustrated.

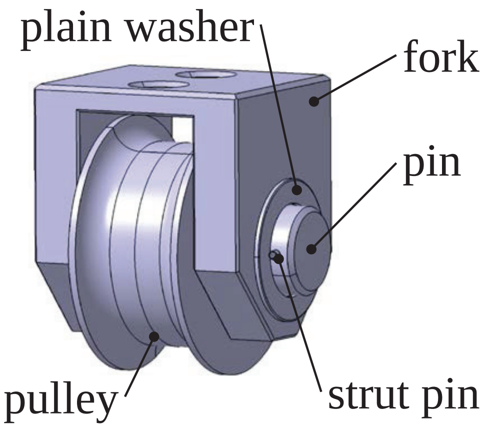

Fig. 5. Driving pulley

The driving pulley consists of a fork, a pulley with a copper bearing, a pin with lubrication nipple, a plain washer and a strut pin. Furthermore two FKCs are defined to ensure the use as proposed:

- The gap between the copper bearing of the pulley and the fork (S_1).
- The gap between plain washer and fork (S_2).

To avoid jamming and to ensure assemblability both interrelated FKCs S_1 and S_2 should be between the lower specification limit of LSL = 0.2 mm and the upper specification limit of USL = 2.0 mm.

4.1. Functional relation of driving pulley

In the sectional view (Figure 6) of the driving pulley, both FKCs and the relevant dimensions are shown.

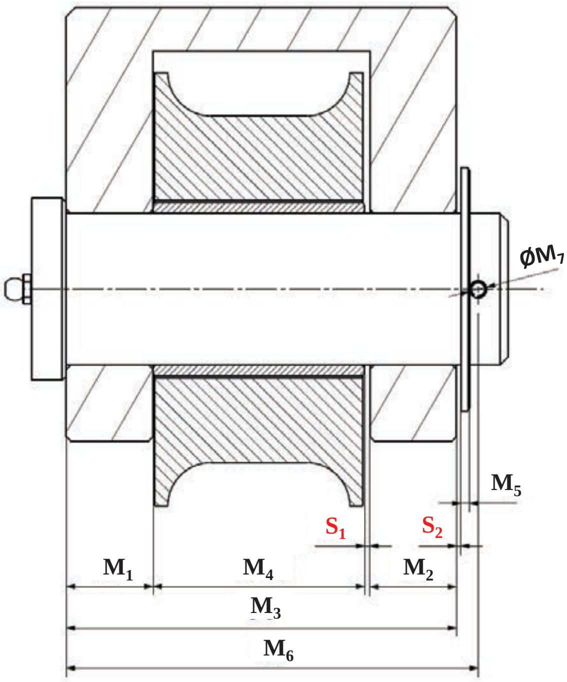

Fig. 6. Driving pulley with interrelated Dimension Chains

The two interrelated FKCs can be described as follows:

$$S_1 = M_3 - M_1 - M_4 - M_2 \quad (3)$$

$$S_2 = M_6 - \frac{1}{2} M_7 - M_5 - M_3 \quad (4)$$

The initial tolerance design, which is based on the predecessor's tolerance design, is detailed in Table 1. Based on the practical experience from the production the tolerance values as well as the corresponding probability distributions are given.

Table 1. Nominal dimensions and corresponding initial tolerances

Dimension	Nominal dimension	Type of distribution	Initial tolerance T_i
M_1	20 mm	normal (±3σ)	±0,1 mm
M_2	20 mm	normal (±3σ)	±0,1 mm
M_3	90 mm	normal (±3σ)	±0,4 mm
M_4	49.5 mm	uniform	±0,2 mm
M_5	2 mm	uniform	±0,1 mm
M_6	95 mm	normal (±3σ)	±0,5 mm
M_7	4 mm	uniform	±0,1 mm

As previously outlined, it is essential to formulate the relation between feasible tolerances and corresponding manufacturing costs. Since cost curves from industrial production are virtually not available, hereafter fictitious costs for all processes are assumed. Taking the manufacturing processes into account, it is obvious that the different dimensions cannot be considered in isolation. Rather it is the case that the feasible accuracy of M_1, M_2 and M_3 are connected. Since the fork is symmetrical, the according tolerances of M_1 and M_2 are identical. Furthermore, the tolerance M_6 of the pin is also connected to the tolerance of the diameter M_7**.**

It is therefore appropriate to use the tolerance-cost-relation for connected dimensions (see section 3.2). The notional fixed and individual costs for all parts are listed in Table 2. The majority of the components can be machined using different machines or processes. Hence, there are different fixed and individual costs listed for those parts.

Table 2. Fixed and individual costs for the parts of the driving pulley

Nominal dimension	A_{fix}	$B_{ind,1}$	k_1	$B_{ind,2}$	k_2
M_1 (process A)		0.40 €*mm	1		
M_2 (process A)	5.20 €	0.10 €*mm	1		
M_3 (process A)				0.70 €*mm	1
M_1 (process B)		0.20 €*mm	1		
M_2 (process B)	6.80 €	0.20 €*mm	1		
M_3 (process B)				0.30 €*mm	1
M_4 (process A)	3.60 €	1.20 €*mm^2	2		
M_4 (process B)	5.40 €	0.80 €*mm	1		
M_5	0.13 €	0.01 €*mm^2	2		
M_6 (process A)	4.20 €	0.40 €*mm	1		
M_7 (process B)				1.20 €*mm	1
M_6 (process B)	5.40 €	0.12 €*mm	1		
M_7 (process B)				0.16 €*mm	1

As previously detailed, the overall costs of connected and non-connected dimensions can be calculated with (2). To evaluate the consequences of the optimization on the scrap rate, a statistical tolerance analysis of the initial tolerance design is performed. Therefore, the probability distributions of each dimension (Table 1) are used for a Monte-Carlo-based tolerance analysis with n = 100,000 samples.

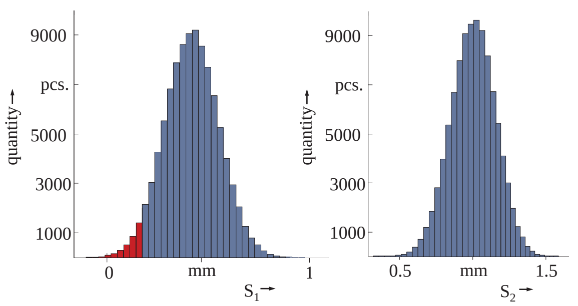

Fig. 7. Resulting Distribution of S_1 (left) and S_2 (right) based on initial tolerance design

As shown in Figure 7, the initial tolerance design is not suitable due to a reject rate of 2.51 % for S_1 and resulting costs of 26.96 €.

4.2. Tolerance-Cost-Optimization of driving pulley

The optimization process aims to minimize the manufacturing costs while not exceeding a permitted number of defects per million. Since the product has two connected dimension chains their individual costs are summed up:

$$\min\left\{C_{S_1}(T_i)+C_{S_2}(T_i)\right\} \tag{5}$$

Whenever the lower (LSL) or upper specification limit (USL) for one FKC is violated, this leads to rejects and thus increases the scrap rate. In this case study 63 defects per million (c = 99.9937 %) which corresponds to a sigma level of ±4σ [15] is acceptable. To calculate the scrap rate the density functions for both FKCs S_1 and S_2 are integrated considering the upper and lower specification limits for each FKC:

$$\int_{LSL_1}^{USL_1}\rho[S_1]dx_1+\int_{LSL_2}^{USL_2}\rho[S_2]dx_2 \geq c \tag{6}$$

To calculate the probability of an event to occur, the probability function has to be integrated. The area enclosed by the probability function $\rho[S_i]$ and the abscissa which is limited by the specification limits represents the propability of a FKC falling within the specification.

The parameter settings for the tolerance-cost-optimization (done in Matlab R2014a using the PSO toolbox of [16]) are listed in Table 3.

4.3. Results and discussion

The optimization completed successfully using the parameter settings listed in Table 3. Besides the population size of the swarm, the definition of the termination criteria is necessary: Hence, 500 iterations can be performed by the algorithm before the algorithm stops without finding a valid solution. Furthermore, the algorithm interrupts, if particles exceed the search space or the cost function is violated. It may occur that no improvement between different generations of the swarm can be achieved. In this case, it's appropriate to define the maximum number of following generations which are nearly identical.

Table 3. Parameter settings in the PSO Toolbox

Parameter	Value
Population size	20
Tolerance on the objective function violation in €	1e-3
Termination tolerance on the constraint violation in -	1e-6
Maximum number of (nearly) identical generations	100
Maximum number of iterations	500

The performed tolerance-cost-optimization of the driving pulley finally successfully finished after 100 generations. the identified optimal tolerance design (detailed in Table 4) goes hand in hand with decreasing manufacturing costs by ~17 % from 26.96 € to 22.43 €. Moreover, the required scrap rate for both FKCs of 0.0063 % can be guaranteed with this optimal tolerance design.

Table 4. Initial and optimal tolerances

Dimension	Kind of Distribution	Initial Tolerance	Optimized Tolerances
M1	normal (±3σ)	±0,1 mm	±0,43 mm
M2	normal (±3σ)	±0,1 mm	±0,43 mm
M3	normal (±3σ)	±0,4 mm	±0,27 mm
M4	uniform	±0,2 mm	±0,53 mm
M5	uniform	±0,1 mm	±0,20 mm
M6	normal (±3σ)	±0,5 mm	±0,99 mm
M7	uniform	±0,1 mm	±0,20 mm

The case study illustrates that connected dimensions are relatively common on machined parts. Furthermore, it was shown that the tolerance-cost-relation for connected dimensions can be adapted in an appropriate way and integrated in statistical tolerance-cost-optimization of assemblies.

5. Summary and Conclusion

The determination of tolerances requires knowledge not only about the feasible accuracy of every manufacturing process, but also about the tolerance-related costs for each part and process. In this paper an enhanced tolerance-cost-model for connected tolerances was presented. Furthermore, these models were integrated into the tolerance allocation. Its use was illustrated for an exemplary tolerance-cost-optimization on a driving pulley with interrelated dimension chains. Besides this specific application, a general methodology for tolerance-cost-optimization was presented and discussed. In contrast to existing approaches this general methodology provides the possibility to optimize systems with or without connected dimension chains whereas connected tolerances are considered in a new way.

Nevertheless there are future challenges arising. Two potential fields of research should be highlighted:

There is no up-to-date information on tolerance-cost-relations for different processes. Although it can be assumed that the methodology is suitable for tolerance-cost-functions from industrial practice, it would be crucial to validate the methodology with real data.

Furthermore, it is attractive to consider more (different) adjustment possibilities for the optimization. In the presented case study, only the tolerance values are considered. Desired mean shifts for each dimension as well as restrictions of the distributions are currently not taken into account.

In conclusion, the utility of the presented methodology for the optimization of connected tolerances with interrelated dimension chains was shown and could therefore be extended and adapted to more complex problems in future activities.

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