



## Could saturation effects be visible in a future electron–ion collider?

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### ABSTRACT

We expect to observe parton saturation in a future electron–ion collider. In this Letter we discuss this expectation in more detail considering two different models which are in good agreement with the existing experimental data on nuclear structure functions. In particular, we study the predictions of saturation effects in electron–ion collisions at high energies, using a generalization for nuclear targets of the b-CGC model, which describes the  $ep$  HERA quite well. We estimate the total, longitudinal and charm structure functions in the dipole picture and compare them with the predictions obtained using collinear factorization and modern sets of nuclear parton distributions. Our results show that inclusive observables are not very useful in the search for saturation effects. In the small  $x$  region they are very difficult to disentangle from the predictions of the collinear approaches. This happens mainly because of the large uncertainties in the determination of the nuclear parton distribution functions. On the other hand, our results indicate that the contribution of diffractive processes to the total cross section is about 20% at large  $A$  and small  $Q^2$ , allowing for a detailed study of diffractive observables. The study of diffractive processes becomes essential to observe parton saturation.

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The search for signatures of parton saturation effects has been subject of an active research in the last years (for recent reviews see, e.g. [1]). It has been observed that HERA data in the small  $x$  and low  $Q^2$  region can be successfully described with saturation models [2–6]. Moreover, the experimentally measured total cross sections present the property of geometric scaling [7], which appears naturally in the framework of the color glass condensate (CGC) formalism. This property was considered as an evidence of saturation. However, very recently [8] it has been shown that geometric scaling may also be derived from the DGLAP equations. Another prediction which follows naturally from saturation models is the suppression of high  $p_T$  hadron yields at forward rapidities in  $dAu$  collisions [9–13]. This suppression has been observed at RHIC [14]. However, also in this case, alternative explanations [15,16], not based on saturation physics, could account for the experimental data. Taken together, these results provide some evidence for saturation at HERA and RHIC. However, more definite conclusions are not possible due to the small value of the saturation scale in the kinematical range of HERA and due to the complexity of  $dAu$  collisions, where we need to consider the substructure of the projectile and the target, as well as the fragmentation of the produced partons. So far, other models (without saturation included)

are able to describe the same set of data (see e.g. Refs. [17,18]). In order to discriminate between these different models and test the CGC physics, it would be very important to consider an alternative search. To this purpose, the future electron–nucleus colliders are ideal [19–22], because they can probably determine whether parton distributions saturate or not.

After the eRHIC was proposed [20], it became crucial to have some quantitative estimates of the impact of saturation effects on observables. Some of these estimates can be found in [21,23–25]. In particular, in [23] we calculated several inclusive observables ( $F_2^A$ ,  $F_L^A$ ,  $F_2^{c,A}$  and their logarithmic slopes), while in [24] the behavior of the diffractive structure function  $F_2^{D(3)}$  and the diffractive cross section  $\sigma^{\text{diff}}$  were studied in detail. An interesting conclusion from Ref. [24] was related to the growth of  $R_\sigma = \sigma^{\text{diff}}/\sigma^{\text{tot}}$  with the atomic number of the target, especially in the small  $x$  and low  $Q^2$  region. This ratio could be as large as 0.3–0.4. This is very large compared to the corresponding ratio in  $ep$  collisions which is of the order of 0.10–0.15. Such a large value of  $R_\sigma$  was anticipated long ago in Ref. [26] but the investigation of this enhancement was not further developed. Literally, our result implies that in 30–40% of the  $eA$  collisions the nucleus escapes intact! This phenomenon is so spectacular that it deserves further investigation.

Our goal in this Letter is twofold. Firstly, to improve our previous estimates for the inclusive observables at  $eA$  colliders considering a more realistic model for the nuclear dipole cross section which allows us to describe the scarce experimental data on nu-

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clear structure functions. This can give us more confidence about the behavior predicted by the saturation model for  $F_2^A$ ,  $F_L^A$  and  $F_2^{c,A}$  at the small  $x$ . Secondly, to compare these predictions with those obtained using collinear factorization and the parametrizations of the nuclear parton distributions. These improved calculations should allow us to give a partial answer to the question posed in the title of the Letter.

In order to study the behavior of the observables of deep inelastic scattering (DIS) at small  $x$  and to include saturation effects, it is useful to describe the photon-hadron scattering in the dipole frame, in which most of the energy is carried by the hadron, while the photon has just enough energy to dissociate into a quark-antiquark pair before the scattering. This description contrasts with the usual description in the infinite momentum frame of the hadron, based on collinear factorization, where the photon scatters a sea quark, which is typically emitted by the small- $x$  gluons in the hadron. The QCD description of DIS at small  $x$  can be interpreted as a two-step process [27]. The virtual photon (emitted by the incident electron) splits into a  $q\bar{q}$  dipole, with transverse separation  $\mathbf{r}$ , which subsequently interacts with the target. In terms of virtual photon-target cross sections  $\sigma_{T,L}$  for the transversely and longitudinally polarized photons, the nuclear  $F_2$  structure function is given by  $F_2^A(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}}\sigma_{tot}$ , with [27]

$$\sigma_{tot} = \sigma_T + \sigma_L \quad \text{and} \\ \sigma_{T,L} = \int d^2\mathbf{r} dz |\Psi_{T,L}(\mathbf{r}, z, Q^2)|^2 \sigma_{dip}^A(x, \mathbf{r}), \quad (1)$$

where  $\Psi_{T,L}$  is the light-cone wave function of the virtual photon and  $\sigma_{dip}^A$  is the dipole nucleus cross section describing the interaction of the  $q\bar{q}$  dipole with the nucleus target. In Eq. (1)  $\mathbf{r}$  is the transverse separation of the  $q\bar{q}$  pair and  $z$  is the photon momentum fraction carried by the quark (for details see e.g. Ref. [28]). Moreover, the nuclear longitudinal structure function is given by  $F_L^A(x, Q^2) = (Q^2/4\pi^2\alpha_{em})\sigma_L$  and the charm component of the nuclear structure function  $F_2^{c,A}(x, Q^2)$  is obtained directly from Eq. (1) isolating the charm flavor and considering  $m_c = 1.5$  GeV.

The main input for the calculations of inclusive observables in the dipole picture is  $\sigma_{dip}^A(x, \mathbf{r})$  which is determined by the QCD dynamics at small  $x$ . In the eikonal approximation it is given by

$$\sigma_{dip}^A(x, \mathbf{r}) = 2 \int d^2\mathbf{b} \mathcal{N}^A(x, \mathbf{r}, \mathbf{b}), \quad (2)$$

where  $\mathcal{N}^A(x, \mathbf{r}, \mathbf{b})$  is the forward scattering amplitude for a dipole with size  $\mathbf{r}$  and impact parameter  $\mathbf{b}$  which encodes all the information about the hadronic scattering, and thus about the non-linear and quantum effects in the hadron wave function. It can be obtained by solving the BK (JIMWLK) evolution equation in the rapidity  $Y \equiv \ln(1/x)$  [1]. Many groups have studied the numerical solution of the BK equation [29], but several improvements are still necessary before using the solution in the calculation of observables. In particular, one needs to include the next-to-leading order corrections into the evolution equation and perform a global analysis of all small  $x$  data. It is a program in progress (for recent results see [30]). In the meantime it is necessary to use phenomenological models for  $\mathcal{N}^A$  which capture the most essential properties of the solution.

In [23,24] we assumed that the impact parameter dependence of  $\mathcal{N}^A$  can be factorized as  $\mathcal{N}^A(x, \mathbf{r}, \mathbf{b}) = \mathcal{N}^A(x, \mathbf{r})S(\mathbf{b})$  and proposed a generalization for the nuclear case of the IIM model [5] which was, at that time, the most sophisticated one. In other words, we have disregarded the impact parameter dependence of the scattering amplitude and assumed that the dipole nucleus scattering amplitude is related to the dipole proton one by a simple modification in the saturation scale:  $Q_{s,p}^2(x) \rightarrow Q_{s,A}^2 = A^{\frac{1}{3}} \times$

$Q_{s,p}^2(x)$ . This naive model is useful to obtain some idea about the magnitude of the saturation effects in  $eA$  processes in comparison to the linear case. However, in order to get more reliable predictions we should use a phenomenological model which describes the current scarce experimental data on the nuclear structure function as well as includes the impact parameter dependence in the dipole nucleus cross section. A model which satisfies these requirements was proposed some years ago in Ref. [31]. In this model the forward dipole-nucleus amplitude was parametrized as follows

$$\mathcal{N}^A(x, \mathbf{r}, \mathbf{b}) = 1 - \exp\left[-\frac{1}{2}AT_A(\mathbf{b})\sigma_{dip}^p(x, \mathbf{r}^2)\right], \quad (3)$$

where  $T_A(\mathbf{b})$  is the nuclear profile function, which is obtained from a 3-parameter Fermi distribution for the nuclear density normalized to unity (for details see, e.g., Ref. [32]). The above equation, based on the Glauber-Gribov formalism [33], sums up all the multiple elastic rescattering diagrams of the  $q\bar{q}$  pair and is justified for large coherence length, where the transverse separation  $r$  of partons in the multiparton Fock state of the photon becomes a conserved quantity, i.e. the size of the pair  $r$  becomes eigenvalue of the scattering matrix. It is important to emphasize that for very small values of  $x$ , other diagrams beyond the multiple Pomeron exchange considered here should contribute (e.g. Pomeron loops) and a more general approach for the high density (saturation) regime must be considered. However, we believe that this approach allows us to estimate the magnitude of the high density effects in the RHIC and LHC kinematic range.

In [31] the author has assumed that  $\sigma_{dip}^p$  was given by the GBW model. However, in the last years an intense activity in the area resulted in more sophisticated dipole proton cross sections, which had more theoretical constraints and which were able to give a better description of the more recent HERA data [10–13,34,35]. In what follows we will use the b-CGC model proposed in Ref. [34], which improves the IIM model [5] with the inclusion of the impact parameter dependence in the dipole proton cross sections. The parameters of this model were recently fitted to describe the current HERA data [35]. Following [34] we have

$$\sigma_{dip}^p(x, \mathbf{r}^2) \equiv \int d^2\bar{\mathbf{b}} \frac{d\sigma_{dip}^p}{d^2\bar{\mathbf{b}}}, \quad (4)$$

where

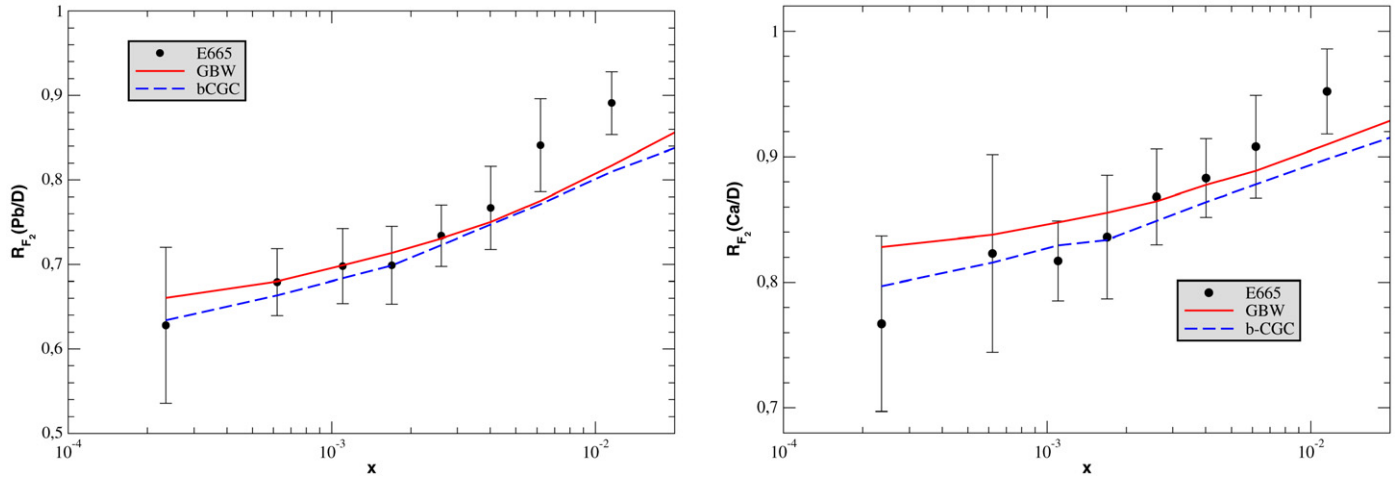
$$\frac{d\sigma_{dip}^p}{d^2\bar{\mathbf{b}}} = 2\mathcal{N}^p(x, \mathbf{r}, \bar{\mathbf{b}}) = 2 \times \begin{cases} \mathcal{N}_0 \left(\frac{rQ_{s,p}}{2}\right)^{2(\gamma_s + \frac{\ln(2/rQ_{s,p})}{\kappa\lambda Y})}, & rQ_{s,p} \leq 2, \\ 1 - \exp^{-a\ln^2(brQ_{s,p})}, & rQ_{s,p} > 2, \end{cases} \quad (5)$$

with  $Y = \ln(1/x)$  and  $\kappa = \chi''(\gamma_s)/\chi'(\gamma_s)$ , where  $\chi$  is the LO BFKL characteristic function. The coefficients  $a$  and  $b$  are determined uniquely from the condition that  $\mathcal{N}^p(x, \mathbf{r})$  and its derivative with respect to  $rQ_s$  are continuous at  $rQ_s = 2$ . In this model, the proton saturation scale  $Q_{s,p}$  now depends on the impact parameter

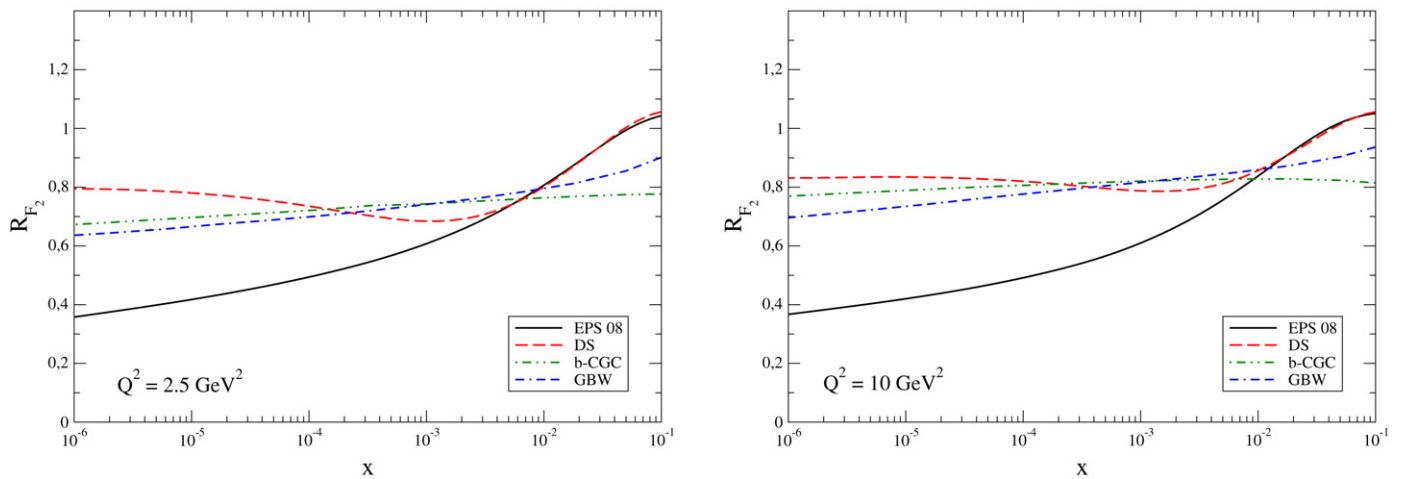
$$Q_{s,p} \equiv Q_{s,p}(x, \bar{\mathbf{b}}) = \left(\frac{x_0}{x}\right)^{\frac{\lambda}{2}} \left[\exp\left(-\frac{\bar{b}^2}{2B_{CGC}}\right)\right]^{\frac{1}{2\gamma_s}}. \quad (6)$$

The parameter  $B_{CGC}$  was adjusted to give a good description of the  $t$ -dependence of exclusive  $J/\psi$  photoproduction. Moreover the factors  $\mathcal{N}_0$  and  $\gamma_s$  were taken to be free. In this way a very good description of  $F_2$  data was obtained. The parameter set which is going to be used here is the one presented in the second line of Table II of [35]:  $\gamma_s = 0.46$ ,  $B_{CGC} = 7.5 \text{ GeV}^{-2}$ ,  $\mathcal{N}_0 = 0.558$ ,  $x_0 = 1.84 \times 10^{-6}$  and  $\lambda = 0.119$ .

In Fig. 1 we compare with the E665 data [36] the predictions for the ratio  $R_{F_2} = 2F_2^A/AF_2^p$  obtained using the b-CGC model as input in our calculations. It is worth recalling that if the nucleus were a mere superposition of nucleons, this ratio and also the



**Fig. 1.** Ratio  $R_{F_2} \equiv 2F_2^A/AF_2^D$  for  $A = \text{Pb}$  and  $A = \text{Ca}$ . Although joined by a line the results of the saturation models have been computed for the  $(\langle x \rangle, \langle Q^2 \rangle)$  of the data points from the E665 Collaboration [36].



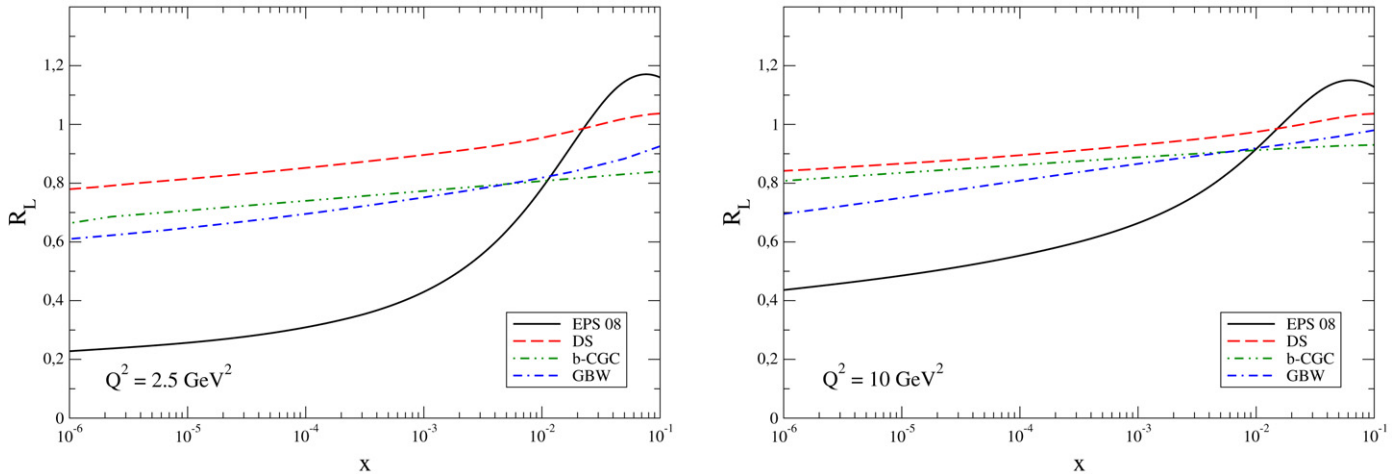
**Fig. 2.** Comparison between the predictions for  $R_{F_2}$  of the saturation models (b-CGC and GBW) and the collinear ones (DS and EPS08) for two different values of  $Q^2$  and  $A = \text{Pb}$ .

equivalent ones constructed for  $F_L$  and  $F_2^C$  would be equal to one. For comparison we also show the predictions obtained using the GBW model. As expected, both models fail to describe the large  $x$  region. However, both models describe quite well the scarce experimental data in the region of small values of  $x$  and low  $Q^2$ . The basic difference between the predictions occur in the normalization of the ratio, with the b-CGC model predicting a large nuclear effect at small  $x$ . It is important to emphasize that after the choice of the model for the dipole proton cross section our predictions are parameter free. Since our model for the dipole nuclear dipole cross section, Eq. (3), describes reasonably well the experimental data for the nuclear structure function, we feel confident to compare its predictions with those obtained using the collinear factorization as well as extending it to the calculation of the longitudinal and charm structure functions.

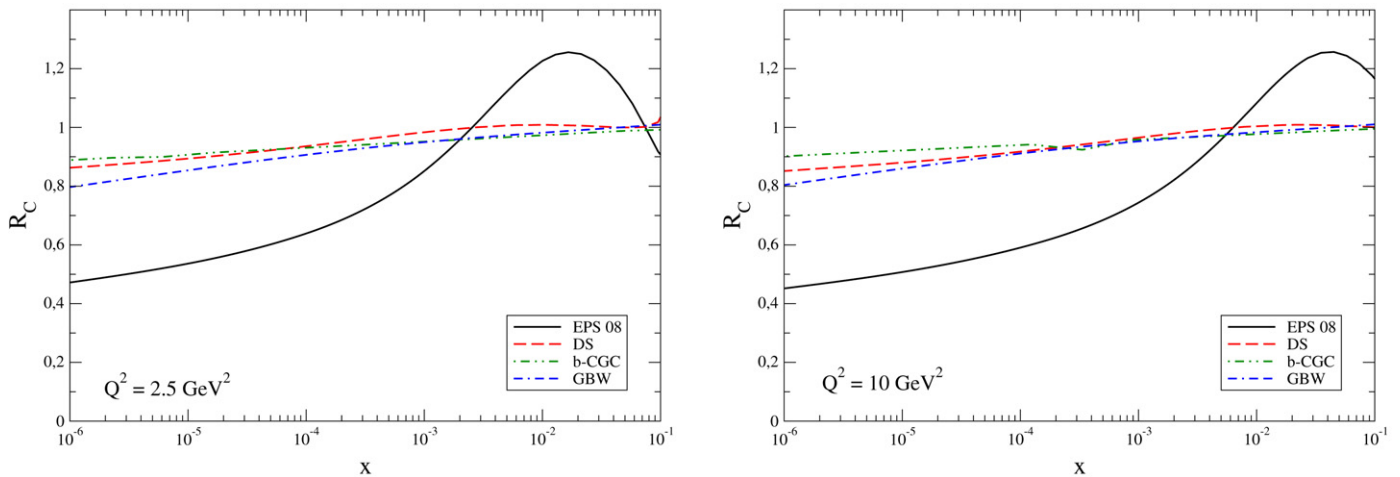
In Fig. 2 we present our predictions for the  $x$  dependence of the ratio  $R_{F_2}$  for two different values of the photon virtuality  $Q^2$ . The two saturation models predict a similar behavior in the small  $x$  region, yielding values of  $\approx 0.7$  for  $x = 10^{-4}$  and  $Q^2 = 2.5 \text{ GeV}^2$ . This value grows to  $\approx 0.8$  for  $Q^2 = 10 \text{ GeV}^2$ . For comparison we also show the predictions obtained using the collinear factorization and nuclear parton distributions resulting from the global analysis of the nuclear experimental data using the DGLAP evolution equation (for a recent discussion see [37]). Here we consider two different sets of nuclear parton distributions: the DS [38] and

EPS 08 [39] nuclear parametrizations. They represent a lower and an upper bound for the magnitude of the nuclear effects, respectively. In particular, in the analysis of Ref. [39] the authors include data on hadron production in the forward region at RHIC [14]. As a result the amount of nuclear shadowing is considerably larger than the one obtained in other parametrizations. It is important to emphasize that the collinear predictions *do not* include dynamical effects associated to non-linear (saturation) physics, since they are based on the linear DGLAP dynamics. Therefore, the comparison between the saturation and collinear predictions could, in principle, tell us if the observable considered can be used to discriminate between linear and non-linear dynamics. The results shown in Fig. 2 demonstrate that this is not the case of the nuclear structure function. Although the predictions of the two collinear models are similar for large  $x$  ( $\geq 10^{-2}$ ), where there exist experimental data, the predicted behavior at small  $x$  is very distinct. This difference is a consequence of the choice of different experimental data used in the global analysis and of the assumptions related to the behavior of the nuclear gluon density [37]. The difference between the collinear predictions is so large at small  $x$  that it is not possible to extract any information about the presence or not of new QCD dynamical effects from the study of the nuclear structure function.

Let us now consider the behavior of the nuclear longitudinal structure function. It is believed that this observable can be used to constrain the QCD dynamics at small  $x$  in  $ep$  collisions at HERA



**Fig. 3.** Comparison between the predictions for  $R_L \equiv 2F_L^A/AF_L^D$  of the saturation models (b-CGC and GBW) and the collinear ones (DS and EPS08) for two different values of  $Q^2$  and  $A = \text{Pb}$ .



**Fig. 4.** Comparison between the predictions for  $R_C \equiv 2F_2^{c,A}/AF_2^{c,D}$  of the saturation models (b-CGC and GBW) and the collinear one (DS and EPS08) for two different values of  $Q^2$  and  $A = \text{Pb}$ .

(see, e.g. [40]). One of our goals is to verify if this assumption is also valid in  $eA$  collisions. Our results are shown in Fig. 3. The predictions of saturation models are, as in the case of  $R_{F_2}$ , contained within the uncertainty range of the collinear models. On the other hand, the difference between the two collinear models is bigger. In this case the results for large  $x$  are distinct too. This is due to the strong dependence of  $F_L^A$  on the nuclear gluon distribution, which is very different in the two models considered [37].

Finally, we show in Fig. 4 our results for the ratio  $R_C \equiv 2F_2^{c,A}/AF_2^{c,D}$ , which is determined by the charm component of the nuclear structure function. In this case the saturation model predictions and the DS collinear ones are similar, while the EPS08 one is very distinct. Similarly to the  $R_L$  case, this behavior is associated to the large magnitude of nuclear effects present in the nuclear gluon distribution predicted by the EPS08 parametrization, which has the strongest shadowing. We did not include in the plots the somewhat older but very well-known EKS parametrization. Its predictions for  $R_{F_2}$ ,  $R_L$  and  $R_C$  would always lie between the GBW and EPS08 curves. So we can say that, even neglecting the EPS08 parametrization (which is the most extreme), the predictions of saturation models overlap with those from collinear models.

All the results obtained so far rely on the validity of Eq. (1). This formula can be derived from perturbative QCD in leading logarithm approximation. This was done in the pioneering works by Nikolaev and Zakharov [27] and also by Mueller [41]. These works

introduced the correspondence between the  $k_T$  factorization pQCD approach (based on unintegrated gluon distribution functions) and the color dipole approach (based on dipole-target cross sections). Later, it was shown in [44] that in next to leading order there are differences between the two approaches. In order to access the uncertainties in each approach separately and the differences between them we would need complete and extensive NLO calculations, which are not yet available (see related discussion in [45]). In the dipole approach there are some recent papers [46] where a NLO analysis is performed. Our curves obtained with the collinear factorization approach are leading order results. In order to go beyond LO we would need to know, at the NLO level, the nuclear parton distribution functions (nPDFs), the DGLAP evolution equations and the expressions for  $F_2$ ,  $F_L$  and  $F_{2c}$ . There are already some NLO analyses of the nPDFs. In the one performed by de Florian and Sassot [38] the authors conclude that the NLO corrections change the LO results by 20% or less. This is smaller than the main uncertainties, which are caused by different choices of the data sets to be fitted. In particular, at low  $x$  it is crucial to decide if one includes or not the RHIC data on forward hadron production in deuteron-gold collisions. If one includes these data, the amount of nuclear shadowing is much larger. By comparing two extreme distributions, DS and EPS (which have the weakest and the strongest nuclear shadowing respectively) which differ from each other by a factor two in the low  $x$  domain, we are already taking into ac-

count the major source of uncertainties in the collinear approach. In view of our results, a NLO analysis would be important for a corroboration, but its absence does not prevent us from drawing some conclusions. In fact, since all the curves tend to be close and even to overlap each other, our conclusion is that these observables,  $F_2$ ,  $F_L$  and  $F_{2c}$ , are not very efficient for discriminating the non-linear from the linear physics. In view of the strength of the corrections (where they are available) it seems very unlikely that a NLO calculation would generate a large separation between the linear and non-linear results. In other words, the error bars associated with all our curves would not change our conclusion, namely that they are too close to be interesting from the phenomenological point of view.

Let us summarize our results for inclusive observables  $F_2^A$ ,  $F_L^A$  and  $F_2^{c,A}$ , which should be measured in the first run of a future electron–ion collider. The conclusion that we can draw from the previous figures is that the two dipole models yield similar predictions for the structure functions and they fall inside the range of predictions of the EPS and DS parametrizations. The latter differ among each other because of the large freedom inherent to global data analysis. Although these observables are sensitive to saturation effects, as shown in [23] by the comparison between the predictions of the full (linear + non-linear contributions) dipole models and their corresponding purely linear versions, it is not yet possible to draw any firm conclusion concerning the QCD dynamics from inclusive quantities. This is a pessimistic partial answer for the question posed on the title, which implies that in order to discriminate the saturation effects we should consider less inclusive observables.

An alternative can be the study of the logarithmic  $Q^2$  and  $1/x$  slopes of the structure functions, as already discussed in [23,42]. As pointed out in [42], the logarithmic  $Q^2$  slope of  $F_2^A$  can be useful to address the boundary between the linear and saturation regimes, due to the  $A$  dependence present in the nuclear saturation scale (see also [23]). Another possibility is the study of observables measured in diffractive deep inelastic scattering (DDIS), since the total diffractive cross section is much more sensitive to large-size dipoles than the inclusive one [2,24]. Basically, the saturation effects screen large-size dipole (soft) contributions, so that a fairly large fraction of the cross section is hard and hence eligible for a perturbative treatment. This was the main motivation of Ref. [24], where we have computed observable quantities like  $R_\sigma = \sigma_{\text{diff}}/\sigma_{\text{tot}}$  and  $F_2^{D(3)}$  in the dipole picture using a naive saturation model. One of the main conclusions was related to the growth of  $R_\sigma$  with the atomic number of the target, especially in the small  $x$  and low  $Q^2$  region, where the ratio could be as large as 0.3–0.4. Although we postpone for a future publication a detailed analysis of the diffractive observables using the b-GCC model discussed in this Letter, it is important to verify if the diffractive contribution for the total cross section is still large when estimated using a more realistic saturation model. The total diffractive cross section is given by:

$$\sigma_{\text{diff}} = \sigma_L^D + \sigma_T^D \quad \text{and} \\ \sigma_{L,T}^D = \frac{1}{4} \int d^2\mathbf{r} dz |\Psi_{T,L}(\mathbf{r}, z, Q^2)|^2 \int d^2\mathbf{b} \left( \frac{d\sigma_{\text{dip}}^A}{d^2\mathbf{b}} \right)^2, \quad (7)$$

where  $d\sigma_{\text{dip}}^A/d^2\mathbf{b} = 2\mathcal{N}^A(x, \mathbf{r}, \mathbf{b})$ , with  $\mathcal{N}^A$  given by Eq. (3).

The above equation is taken from [21] and [22]. In these works the authors make an adaptation of the usual formula for the diffractive cross section in the dipole approach to the case where the impact parameter is explicitly included. In this case one can trade the usual integration on the momentum transfer variable  $t$  by the integration on the impact parameter. The standard formula can be found in many papers and its derivation is in Section 11 of the textbook [28]. It is based on the Good–Walker formalism for

high energy diffraction. Recalling Eqs. (2) and (3) we realize that, in (7) we first make the average over the matter distribution in the nucleus and then take the square of the resulting expression. This corresponds to considering the particular case of coherent diffraction off the nucleus, also called “no break up” diffraction. In this process the nucleus stays completely intact and is deflected to small angles. As shown in [22], in this case the cross section falls off very rapidly with  $t$ . Measuring the intact recoil nucleus at very small  $t$  in a future electron ion collider is challenging. On the other hand, in these collisions it is also possible to break up a nucleus into color neutral constituents without filling the rapidity gap between the  $q\bar{q}$  dipole and the nuclear fragmentation region. This is called incoherent diffraction or “break up” diffraction. The distinction between coherent and incoherent diffraction was also discussed in early works, such as [48] and [49]. As one would expect, the “break up” processes have larger cross sections than the “no break up” ones, as shown in [21] and [22]. The inclusion of “break up” processes would enhance the ratios in Fig. 5. According to the mentioned works, the difference between the two kinds of process is larger for larger  $Q^2$ , larger  $x$  and lighter nuclei, i.e., the region where saturation effects are less important. Since we are interested primarily in the saturation region and since we wish to compare our present results with previous ones [24] (with no impact parameter dependence) obtained for the “no break up” processes, we shall use only Eq. (7) and leave a more complete analysis (with break up processes) for the future.

The ratio  $R_\sigma$  has been measured in  $e$ – $p$  diffractive scattering by the ZEUS Collaboration [43]. They measure DIS events with rapidity gaps. From this they have a procedure to extract the total diffractive cross section, which involves some assumptions. All this is described in Section 8 of [43]. In the case of proton targets, it is observed experimentally a very similar energy dependence of the inclusive diffractive and the total cross section. The saturation models provides a simple explanation for this finding [2,47]. In contrast, to explain this aspect of the data using a description based on the collinear factorization is non-trivial. In this case the energy dependence of the inclusive and diffractive cross sections is controlled by the  $x$  dependence of the ordinary and the diffractive parton densities, which is not predicted by the theory. Therefore we keep using the dipole formalism, assuming that it is valid in  $e$ – $p$  collisions (yielding quantities which can be measured) and extrapolate it to nuclear targets. We expect that, repeating the same experimental analysis for DIS processes with nuclear targets, the ratio  $R_\sigma$  can be obtained, which can be compared with our predictions.

In Fig. 5 we present our predictions for the ratio  $R_\sigma$  as a function of  $x$  for two values of  $Q^2$  and the atomic number considering the b-CGC and GBW models. In comparison with our previous results [24], we can observe that the inclusion of the impact parameter in the saturation model reduces the ratio in approximately 30%. However, the values predicted are still large, especially for small  $Q^2$  and large  $A$ . Our new predictions imply that the ratio for  $e$ Pb collisions is a factor two larger than observed in  $ep$  collisions at HERA. Consequently, the study of diffractive processes should be an easy task in an  $eA$  collider.

It would be very interesting to compare our predictions for  $R_\sigma$  with similar ones based on other approaches. Surprisingly, there are no such predictions in the literature. The only explicit calculations of  $R_\sigma$  were presented in [2] and [47] and only for the dipole approach with the inclusion of saturation effects in the dipole cross section. For DIS off nuclei there are some recent papers [21,22,25], which also use the dipole formalism and obtain similar results. The other approach to nuclear diffractive DIS is the one based on diffractive nuclear parton distribution functions and leading twist collinear factorization, as, for example, in [48]. However, in this approach the focus is rather on determining the

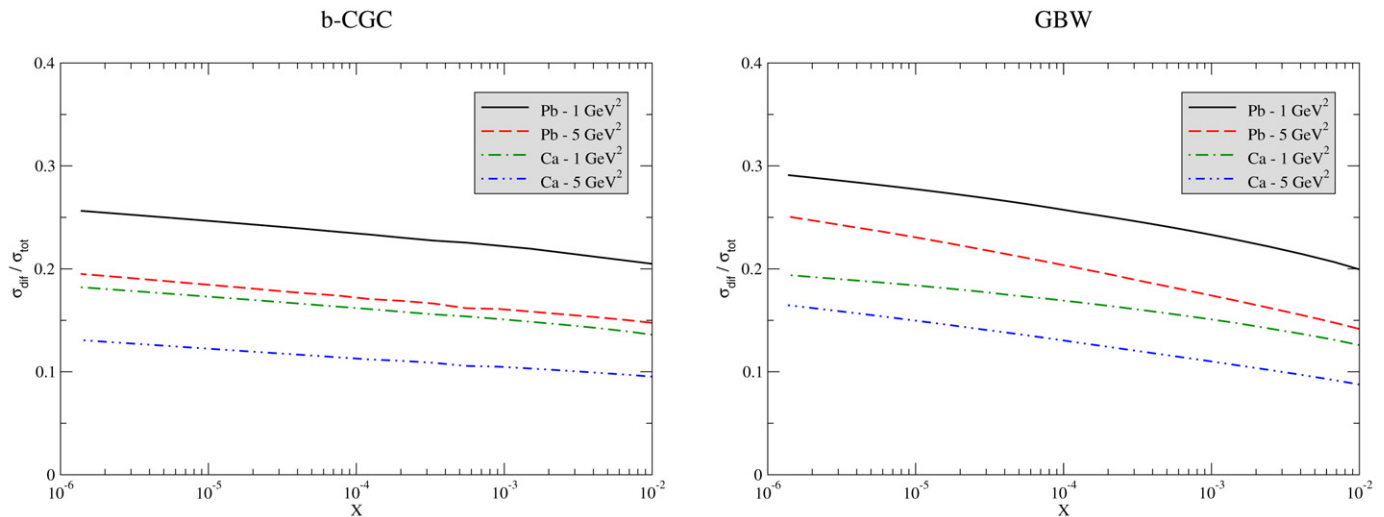


Fig. 5. Ratio of diffractive to total cross sections,  $R_\sigma$ , as a function of  $x$  with the b-CGC (left) and GBW (right) models. Long dashed and solid lines are for Pb targets at  $Q^2 = 5.0$  and  $1.0 \text{ GeV}^2$ , respectively. Dot-dot-dash and dot-dash lines are the same for Ca targets.

diffractive nPDF's and using them to make predictions for other diffractive processes, like vector meson or heavy quark diffractive production. Although the ratio  $R_\sigma$  is not calculated in [48], the authors find that the probability of diffraction in the quark (gluon) channel reaches about 30% (40%) at large nuclei. It is important to emphasize that the use of the leading twist formula for diffractive processes can be inadequate, as discussed in [50], which proposed a modified expression which includes an estimate of power-suppressed effects. Moreover, from theoretical considerations, it is clear that for sufficiently large parton densities, dynamics beyond what can be described by leading-twist factorization and linear DGLAP evolution must become important.

As a summary, in this Letter we have studied the predictions of saturation physics for electron–ion collisions at high energies, using a generalization for nuclear targets of the b-CGC model which describes the  $ep$  HERA quite well. We have estimated the nuclear structure function  $F_2^A(x, Q^2)$ , as well as the longitudinal and charm contributions and compared with the predictions obtained using collinear factorization and distinct sets of nuclear parton distributions. The basic idea is to compare the predictions from non-linear and linear QCD dynamics and verify if the experimental analysis of these observables in the future electron–ion collider could reveal the presence of saturation physics, as well as constrain the behavior of the saturation scale. Our results indicate that the inclusive observables are not adequate for this purpose due to the large uncertainty present in the collinear predictions at small  $x$ . On the other hand, it is expected that the collinear factorization formalism fails to describe diffractive  $eA$  processes, while the saturation formalism remains valid. The study of diffractive processes might help to observe the saturation physics. In this Letter we have estimated the contribution of these processes for the total cross section and demonstrated that it is about 20% at large  $A$  and small  $Q^2$ , allowing for a detailed study of diffractive observables. Our results motivate a more detailed study of the diffractive structure function as well as the lepton production of vector mesons in  $eA$  processes. These studies are fundamental in order to get a definitive answer for the question posed on the title.

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## References

- [1] E. Iancu, R. Venugopalan, hep-ph/0303204; H. Weigert, Prog. Part. Nucl. Phys. 55 (2005) 461; J. Jalilian-Marian, Y.V. Kovchegov, Prog. Part. Nucl. Phys. 56 (2006) 104.
- [2] K. Golec-Biernat, M. Wüsthoff, Phys. Rev. D 59 (1999) 014017; K. Golec-Biernat, M. Wüsthoff, Phys. Rev. D 60 (1999) 114023.
- [3] J. Bartels, K. Golec-Biernat, H. Kowalski, Phys. Rev. D 66 (2002) 014001.
- [4] H. Kowalski, D. Teaney, Phys. Rev. D 68 (2003) 114005.
- [5] E. Iancu, K. Itakura, S. Munier, Phys. Lett. B 590 (2004) 199.
- [6] J.R. Forshaw, G. Shaw, JHEP 0412 (2004) 052.
- [7] A.M. Staśta, K. Golec-Biernat, J. Kwieciński, Phys. Rev. Lett. 86 (2001) 596; V.P. Gonçalves, M.V.T. Machado, Phys. Rev. Lett. 91 (2003) 202002; N. Armesto, C.A. Salgado, U.A. Wiedemann, Phys. Rev. Lett. 94 (2005) 022002; C. Marquet, L. Schoeffel, Phys. Lett. B 639 (2006) 471.
- [8] F. Caola, S. Forte, Phys. Rev. Lett. 101 (2008) 022001.
- [9] J. Jalilian-Marian, Nucl. Phys. A 748 (2005) 664.
- [10] A. Dumitriu, A. Hayashigaki, J. Jalilian-Marian, Nucl. Phys. A 765 (2006) 464; A. Dumitriu, A. Hayashigaki, J. Jalilian-Marian, Nucl. Phys. A 770 (2006) 57.
- [11] D. Kharzeev, Y.V. Kovchegov, K. Tuchin, Phys. Lett. B 599 (2004) 23.
- [12] V.P. Gonçalves, M.S. Kugeratski, M.V.T. Machado, F.S. Navarra, Phys. Lett. B 643 (2006) 273.
- [13] D. Boer, A. Utermann, E. Wessels, Phys. Rev. D 77 (2008) 054014.
- [14] I. Arsene, et al., BRAHMS Collaboration, Phys. Rev. Lett. 91 (2003) 072305; I. Arsene, et al., BRAHMS Collaboration, Phys. Rev. Lett. 93 (2004) 242303; I. Arsene, et al., BRAHMS Collaboration, Phys. Rev. Lett. 94 (2005) 032301.
- [15] D.E. Kahana, S.H. Kahana, J. Phys. G 35 (2008) 025102.
- [16] R. Vogt, Phys. Rev. C 70 (2004) 064902.
- [17] J.R. Forshaw, G. Kerley, G. Shaw, Phys. Rev. D 60 (1999) 074012; J.R. Forshaw, G.R. Kerley, G. Shaw, Nucl. Phys. A 675 (2000) 80C.
- [18] R.C. Hwa, C.B. Yang, R.J. Fries, Phys. Rev. C 71 (2005) 024902; J.w. Qiu, I. Vitev, Phys. Lett. B 632 (2006) 507; B.Z. Kopeliovich, J. Nemchik, I.K. Potashnikova, M.B. Johnson, I. Schmidt, Phys. Rev. C 72 (2005) 054606.
- [19] R. Venugopalan, AIP Conf. Proc. 588 (2001) 121, hep-ph/0102087.
- [20] A. Deshpande, R. Milner, R. Venugopalan, W. Vogelsang, Annu. Rev. Nucl. Part. Sci. 55 (2005) 165.
- [21] H. Kowalski, T. Lappi, R. Venugopalan, Phys. Rev. Lett. 100 (2008) 022303.
- [22] H. Kowalski, T. Lappi, C. Marquet, R. Venugopalan, Phys. Rev. C 78 (2008) 045201.
- [23] M.S. Kugeratski, V.P. Gonçalves, F.S. Navarra, Eur. Phys. J. C 46 (2006) 465.
- [24] M.S. Kugeratski, V.P. Gonçalves, F.S. Navarra, Eur. Phys. J. C 46 (2006) 413.
- [25] N.N. Nikolaev, W. Schafer, B.G. Zakharov, V.R. Zoller, JETP Lett. 84 (2007) 537.
- [26] N.N. Nikolaev, B.G. Zakharov, V.R. Zoller, Z. Phys. A 351 (1995) 435.
- [27] N.N. Nikolaev, B.G. Zakharov, Phys. Lett. B 332 (1994) 184; N.N. Nikolaev, B.G. Zakharov, Z. Phys. C 49 (1991) 607; N.N. Nikolaev, B.G. Zakharov, Z. Phys. C 53 (1992) 331; N.N. Nikolaev, B.G. Zakharov, Z. Phys. C 64 (1994) 631.
- [28] V. Barone, E. Predazzi, High-Energy Particle Diffraction, Springer-Verlag, Berlin, Heidelberg, 2002.
- [29] M.A. Braun, Eur. Phys. J. C 16 (2000) 337; N. Armesto, M.A. Braun, Eur. Phys. J. C 20 (2001) 517; M.A. Kimber, J. Kwieciński, A.D. Martin, Phys. Lett. B 508 (2001) 58;

- E. Levin, M. Lublinsky, Nucl. Phys. A 696 (2001) 833;  
M. Lublinsky, Eur. Phys. J. C 21 (2001) 513;  
M. Lublinsky, E. Gotsman, E. Levin, U. Maor, Nucl. Phys. A 696 (2001) 851;  
K. Golec-Biernat, L. Motyka, A.M. Stasto, Phys. Rev. D 65 (2002) 074037;  
K. Golec-Biernat, A.M. Stasto, Nucl. Phys. B 668 (2003) 345;  
K. Rummukainen, H. Weigert, Nucl. Phys. A 739 (2004) 183;  
E. Gotsman, M. Kozlov, E. Levin, U. Maor, E. Naftali, Nucl. Phys. A 742 (2004) 55;  
K. Kutak, A.M. Stasto, Eur. Phys. J. C 41 (2005) 343;  
G. Chachamis, M. Lublinsky, A. Sabio Vera, Nucl. Phys. A 748 (2005) 649;  
T. Ikeda, L. McLerran, Nucl. Phys. A 756 (2005) 385;  
C. Marquet, G. Soyez, Nucl. Phys. A 760 (2005) 208;  
J.L. Albacete, N. Armesto, J.G. Milhano, C.A. Salgado, U.A. Wiedemann, Phys. Rev. D 71 (2005) 014003;  
R. Enberg, K. Golec-Biernat, S. Munier, Phys. Rev. D 72 (2005) 074021.
- [30] J.L. Albacete, Phys. Rev. Lett. 99 (2007) 262301.  
[31] N. Armesto, Eur. Phys. J. C 26 (2002) 35.  
[32] V.P. Goncalves, M.V.T. Machado, Eur. Phys. J. C 30 (2003) 387.  
[33] V.N. Gribov, Sov. Phys. JETP 29 (1969) 483;  
V.N. Gribov, Sov. Phys. JETP 30 (1970) 709.  
[34] H. Kowalski, L. Motyka, G. Watt, Phys. Rev. D 74 (2006) 074016.  
[35] G. Watt, H. Kowalski, Phys. Rev. D 78 (2008) 014016.  
[36] E665 Collaboration, M.R. Adams, et al., Z. Phys. C 67 (1995) 403.  
[37] E.R. Cazaroto, F. Carvalho, V.P. Goncalves, F.S. Navarra, Phys. Lett. B 669 (2008) 331.  
[38] D. de Florian, R. Sassot, Phys. Rev. D 69 (2004) 074028.  
[39] K.J. Eskola, H. Paukkunen, C.A. Salgado, K.J. Eskola, H. Paukkunen, C.A. Salgado, JHEP 0807 (2008) 102.  
[40] V.P. Goncalves, M.V.T. Machado, Eur. Phys. J. C 37 (2004) 299;  
M.V.T. Machado, Eur. Phys. J. C 47 (2006) 365.  
[41] A. Mueller, Nucl. Phys. B 415 (1994) 115.  
[42] V.P. Goncalves, Phys. Lett. B 495 (2000) 303.  
[43] For a recent review see, S. Chekanov, et al., Nucl. Phys. B 800 (2008) 1.  
[44] A. Bialas, H. Navelet, R.B. Peshanski, Nucl. Phys. B 593 (2001) 438.  
[45] R.S. Thorne, Phys. Rev. D 71 (2005) 054024.  
[46] J.L. Albacete, Y.V. Kovchegov, Phys. Rev. D 75 (2007) 125021;  
I. Balitsky, Phys. Rev. D 75 (2007) 014001;  
Y.V. Kovchegov, H. Weigert, Nucl. Phys. A 784 (2007) 188.  
[47] E. Levin, M. Lublinsky, Phys. Lett. B 521 (2001) 233.  
[48] L. Frankfurt, V. Guzey, M. Strikman, Phys. Lett. B 586 (2004) 41.  
[49] Y.V. Kovchegov, L. McLerran, Phys. Rev. D 60 (1999) 054025.  
[50] A.D. Martin, M.G. Ryskin, G. Watt, Eur. Phys. J. C 44 (2005) 69.