

 $Lh_c$ 

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## ABSTRACT

The production cross-section of  $h_c$ , the  $^1P_1$  charmonium state, can be predicted in Non-Relativistic QCD (NRQCD) using heavy-quark symmetry. We show that at the Large Hadron Collider a large cross-section for this resonance is predicted and it should be possible to look for the  $h_c$  through its decay into  $J/\psi + \pi$  even with the statistics that will be achieved within a few months of run-time at the LHC.

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Non-Relativistic QCD (NRQCD) [1] is an effective theory obtained from QCD useful for understanding the physics of quarkonia. In this effective description, states of momenta much larger than the heavy quark mass,  $m$  are excluded from the QCD Lagrangian and new interaction terms are added to account for this exclusion. A crucial parameter is the relative velocity,  $v$ , of the quarks bound in a quarkonium state in terms of which the quarkonium state is expanded into Fock-components. It turns out that the  $Q\bar{Q}$  states appear in either colour-singlet or colour-octet configurations in this expansion where the colour-octet configuration evolves non-perturbatively into a physical colour-singlet state. The cross-section for the production of a quarkonium  $H$  takes on the following factorised form:

$$\sigma(H) = \sum_{n=\{\alpha,S,L,J\}} \frac{F_n}{m^{d_n-4}} \langle O_\alpha^H(^{2S+1}L_J) \rangle \quad (1)$$

where  $F_n$ 's are the short-distance coefficients, calculable in a perturbation theory in  $\alpha_s$ , and  $O_n$  are operators of naive dimension  $d_n$ , describing the long-distance physics. The  $Q\bar{Q}$  pair produced in the short-distance process has a separation of a scale much smaller than  $1/m$  which is pointlike on the scale of the quarkonium wavefunction, which is of order  $1/(mv)$ . The non-perturbative factor  $\langle O_n^H \rangle$  is proportional to the probability for a pointlike  $Q\bar{Q}$  pair in the state  $n$  to form a bound state  $H$ . The factorisation of the short-distance and long-distance parts of the cross-section guarantees the momentum-independence of the non-perturbative terms. These can be, therefore, obtained from one experiment at a given energy and used to compute the cross-section of the quarkonium state in a different experimental setting.

Before this effective theory approach was developed, the production of quarkonia was sought to be understood in terms of the colour-singlet model [2,3]. While at lower energies this model was seen to provide an adequate description of the data, it was seen [4] in the phenomenology of large- $p_T$   $P$ -wave charmonium pro-

duction at the Tevatron [5] that colour-octet operators are very significant. Processes involving  $P$ -wave quarkonia do not have a consistent description in terms of colour singlet operators alone [6]. Surprisingly, when data on direct  $J/\psi$  production and on  $\psi'$  production from the CDF experiment at the Tevatron was analysed, it was seen that it was necessary to include the colour-octet contributions for phenomenological reasons [7], even though in the case of the  $S$ -waves the octet contributions are sub-leading in  $v$ . With the inclusion of the colour-octet contributions the full set of charmonium production data from the CDF could be described albeit at the inclusion of non-calculable long-distance matrix elements [8, 9]. It was only the shape of the  $p_T$ -distributions and not the absolute normalisations that was a prediction of NRQCD. Consequently, independent tests of NRQCD were necessary and several such proposals were made [10–16]. However, many of these proposals are not for large- $p_T$  quarkonium production and while they may be of some phenomenological interest they do not provide a rigorous test of NRQCD because the NRQCD factorisation formula holds strictly only at large- $p_T$ . For a very comprehensive review of  $J/\psi$  production at the Tevatron and the related theory, see Ref. [17].

One interesting test of NRQCD comes from the study of the polarisation of  $J/\psi$ 's at large- $p_T$  [18]. The production of large- $p_T$   $J/\psi$ 's proceeds primarily from the fragmentation of single gluons and the  $Q\bar{Q}$  pair produced in the fragmentation process inherits the transverse polarisation of the gluon. The heavy-quark symmetry of NRQCD then comes into play in protecting this transverse polarisation in the non-perturbative evolution of the  $Q\bar{Q}$  pair into a  $J/\psi$ . The large- $p_T$   $J/\psi$  is, therefore, strongly transversely polarised. This is not true at even moderately low  $p_T$  where the  $J/\psi$  is essentially unpolarised. The  $p_T$  dependence of the polarisation is, therefore, a very good test of the theory [19].

The CDF experiment has measured the  $p_T$ -dependence of the polarisation and they find no evidence for any transverse polarisation at large  $p_T$  [20]. Given the success of NRQCD in explaining the production cross-sections, this failure with respect to predicting the polarisation is, indeed, a shock. It may well be that the successful prediction of the production cross-sections of the var-

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ious resonances was fortuitous and that the effective theory is missing out on some aspect of the physics of quarkonium formation. It could be that the mass of the charm quark is not large enough to be treated in NRQCD. On the other hand, polarisation measurements are usually fraught with problems and it may well be that the problem is elsewhere. Finally, the problem may well have to do with the theoretical uncertainties in the prediction of polarisation. For example, the colour-singlet channel predicts the polarisation of the  $J/\psi$  to be longitudinal. So any effect that could substantially increase the colour-singlet contribution could change the full predictions of polarisation quite drastically. To this end, a modified colour-singlet model with the production of  $J/\psi$ 's initiated by a scattering of a gluon with a Reggeized gluon has been considered [21] but parts of the diffractive amplitudes involved in this calculation are not easily calculable. A more direct approach would be to study the effect of higher-order QCD corrections. These could substantially modify the theoretical expectations regarding polarisation. Recent work [22] on NLO corrections to both the colour-singlet and colour-octet channels in the production of  $J/\psi$  suggest that even these are not enough to understand the polarisation data. The situation is somewhat different in the case of  $\Upsilon$  production [23] where the colour-singlet contribution, enhanced by NLO and a part of the NNLO corrections, seems to be able to account for the data from Tevatron. For reviews of the current status of these calculations and their experimental consequences, see Refs. [24,25].

In this situation, it is worthwhile looking for other tests of NRQCD which successfully navigate between low- $p_T$  and polarisation. Such a suggestion had been made years ago in the context of charmonium production at Tevatron [26]: the production of  $h_c$ , the  $^1P_1$  charmonium state. In NRQCD, this state is produced in the colour-singlet mode and through the production of an intermediate octet  $^1S_0$  state. The non-perturbative matrix element for the transition of this octet state to the physical  $^1P_1$  state can be inferred from other non-perturbative parameters fixed at the Tevatron. This is a consequence of the heavy-quark symmetry of NRQCD. Consequently, one can predict the rate for  $h_c$  production in NRQCD. In this Letter, we investigate this prediction in the context of the Large Hadron Collider (LHC). One channel which may be suitable for the detection of the  $h_c$  is its decay into a  $J/\psi + \pi$ . The decay branching fraction for  $h_c \rightarrow J/\psi + \pi$  has been estimated from spectroscopy.

It may be argued that the measurement of the other charmonium resonances like the  $J/\psi$ ,  $\chi$ 's and the  $\psi'$  will already provide the tests of the NRQCD factorisation formula. The non-perturbative parameters have been determined at the Tevatron and the factorisation formula implies that these are not momentum-dependent. So it should be possible to predict the cross-sections for these resonances at the LHC and check for the validity of NRQCD. While this is true, it must be remembered that for several years quarkonium production has also been studied in terms of a phenomenological model known as the semi-local duality model or the colour-evaporation model [27]. In this model, it is assumed that the open-charm cross-section integrated over the region between  $2m$  and the open charm threshold should be equal to the sum of the resonance cross-sections. The resonance cross-section is then some fraction of the open charm cross-section integrated over this mass range. The fraction is unknown a priori but is fixed by comparing to the data – it is the analog of the non-perturbative parameter that appears in NRQCD computations of the cross-section. This approach is seen to provide a reasonable description of the data from Tevatron [28,29]. However, it must be borne in mind that the separation into perturbative and non-perturbative parts in this model is not rigorously provided by a factorisation formula as in NRQCD and, consequently, the fractions (non-perturbative parameters) that are determined by fitting to Tevatron data are not guaranteed to be

energy-independent. If the energy dependence of these parameters is large, then it will not be possible to use the semi-local duality approach in any predictive way at the LHC. However, it may so happen that, in actual practice, the energy dependence of the fractions turns out to be small in which case semi-local duality will be able to predict the resonance cross-sections at the LHC as well as NRQCD can. But these predictions, in the semi-local duality approach can be made for only those resonances which have been measured at the Tevatron. It is not possible to predict the cross-section for particles which have not been detected at the Tevatron within this model approach. The search for  $h_c$  at the LHC is, therefore, important in establishing NRQCD as the correct theory of quarkonium production.

It is also worth emphasising that the  $h_c$  had eluded experiments for a long time and it is only recently that its existence has been verified in  $e^+e^-$  experiments at the CLEO [30]. The LHC is expected to produce this resonance copiously and it may provide a study of this resonance in various decay channels and may help understand its properties.

At the LHC, the production of the  $h_c$  proceeds through the following partonic subprocesses:

$$\begin{aligned} g + g &\rightarrow {}^1P_1^{[1]} + g, \\ g + g &\rightarrow {}^1S_0^{[8]} + g, \\ q(\bar{q}) + g &\rightarrow {}^1S_0^{[8]} + q(\bar{q}), \\ q + \bar{q} &\rightarrow {}^1S_0^{[8]} + g. \end{aligned} \quad (2)$$

The large- $p_T$  hadronic production cross-section is given as

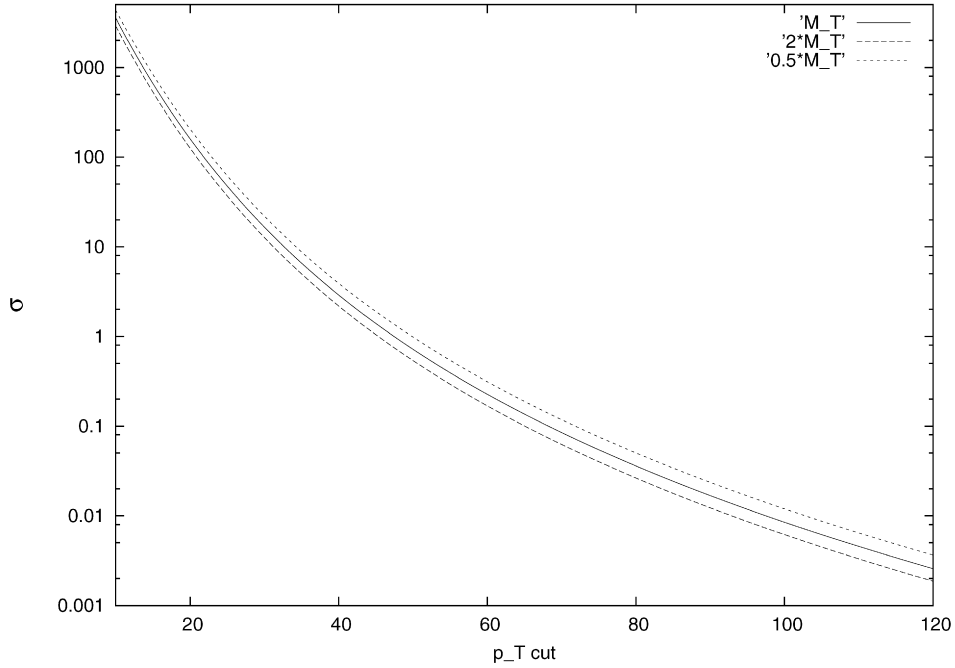
$$\begin{aligned} \frac{d\sigma}{dp_T}(pp \rightarrow h_c X) &= \sum \int dy \int dx_1 x_1 G_{a/p}(x_1) x_2 G_{b/p}(x_2) \frac{4p_T}{2x_1 - \bar{x}_T e^y} \\ &\times \frac{d\hat{\sigma}}{d\hat{t}}(ab \rightarrow Q\bar{Q}[{}^{2S+1}L_J]d) \langle 0 | \mathcal{O}_{1,8}^{h_c} ({}^{2S+1}L_J) | 0 \rangle. \end{aligned} \quad (3)$$

In the above expression, the sum runs not only over all the partons  $a, b, d$  contributing to the subprocesses but also over the different subprocesses which yield either the relevant singlet or octet  $c\bar{c}$  pair in the final state;  $G_{a/p}$  and  $G_{b/p}$  are the distributions of partons  $a$  and  $b$  in the hadrons with momentum fractions  $x_1$  and  $x_2$ , respectively. The expressions for the singlet and the octet subprocess cross-sections,  $d\hat{\sigma}/d\hat{t}$ , are given in Refs. [31] and [9], respectively.

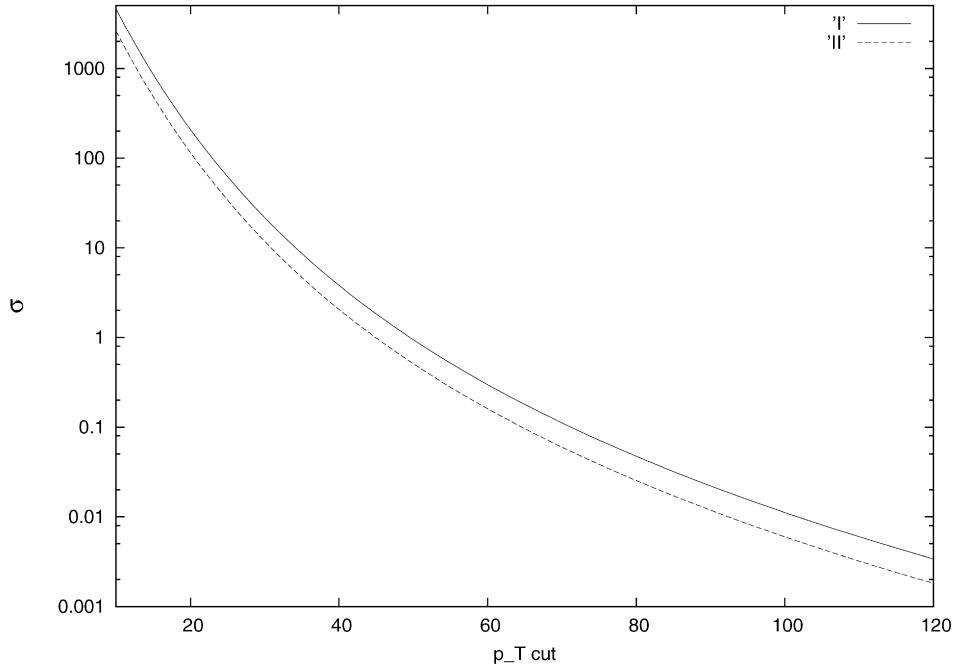
The  ${}^1S_0^{[8]} \rightarrow h_c$  is mediated by a gluon emission in an  $E1$  transition. To fully determine the production rate we need the colour-singlet matrix element for the  ${}^1P_1$  state  $\langle \mathcal{O}_1^{h_c} ({}^1P_1) \rangle$  and the value for the colour-octet matrix element that takes the octet  ${}^1S_0$  state to a  $h_c$ ,  $\langle \mathcal{O}_8^{h_c} ({}^1S_0) \rangle$ . The colour-singlet matrix element is related to the derivative of the wavefunction of at the origin by

$$\langle \mathcal{O}_1^{h_c} ({}^1P_1) \rangle = \frac{27}{2\pi} |R'(0)|^2. \quad (4)$$

The Tevatron data on  $\chi_c$  production fixes [9] the colour-octet matrix element which specifies the transition of a  ${}^3S_1$  octet state into a  ${}^3P_J$  state. We would expect from heavy-quark spin symmetry of the NRQCD Lagrangian that the matrix-element for  ${}^1S_0^{[8]} \rightarrow h_c$  should be equal in the heavy quark limit to that for  ${}^3S_1^{[8]} \rightarrow \chi_{c1}$ . This is because the essential difference between these transitions comes through the magnetic quantum number so that the corrections to this equality will be of  $O(v^2) \sim 30\%$ . For the derivative of the wave-function we use a similar argument to fix it to be the same as for the  $\chi_c$  states.



**Fig. 1.** The cross-section (in nb) for  $h_c$  production as a function of  $p_T$  cut for different choices of QCD scale.

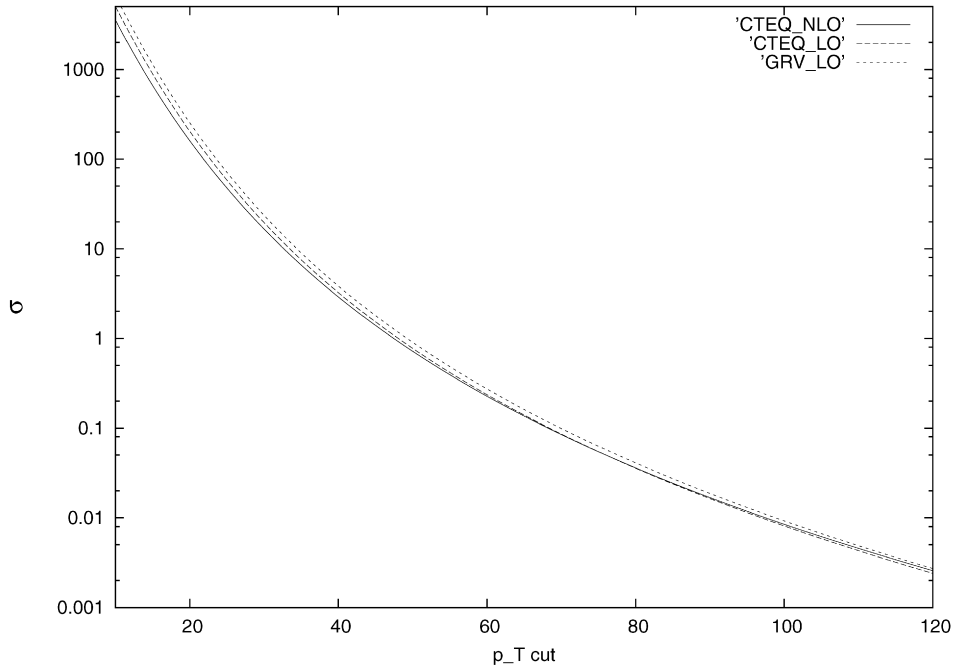


**Fig. 2.** The cross-section (in nb) for  $h_c$  production as a function of  $p_T$  cut for the range of allowed values of the octet matrix element.

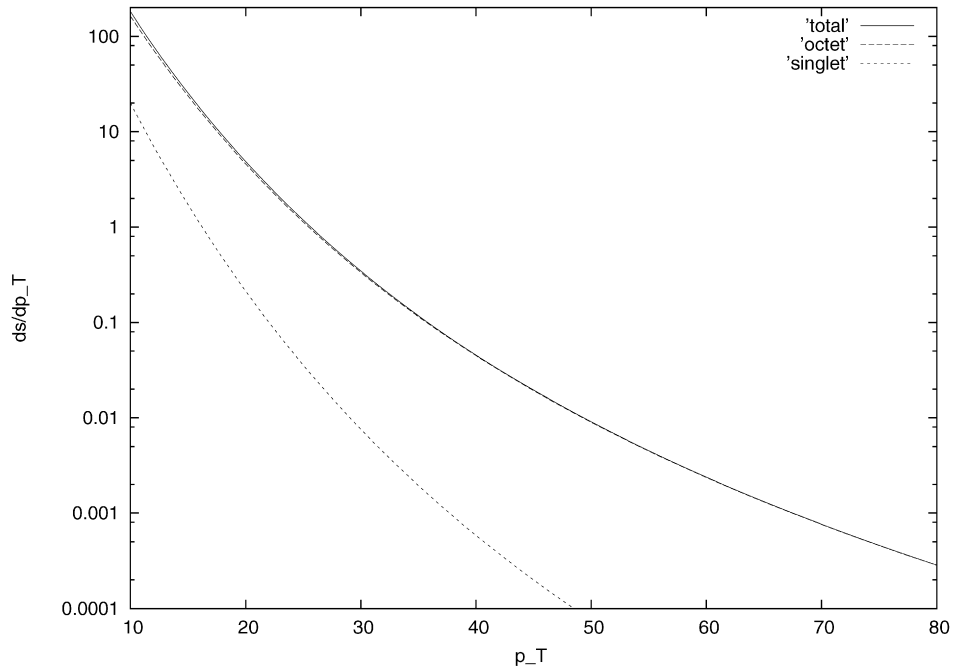
With these inputs, we have computed the cross-section for  $h_c$  production in  $pp$  collisions at the LHC ( $\sqrt{s} = 14$  TeV). We have computed the cross-section integrated over  $p_T$  with a lower  $p_T$  cut. In Fig. 1, we present the results for the  $p_T$ -integrated cross-section as a function of the  $p_T$ -cut for three different choices of the QCD scale:  $Q = M_T/2$ ,  $M_T$  and  $2M_T$ . We have used the CTEQ 4M parton densities [32]. The cross-section has been folded in with the branching ratio of the  $^1P_1$  state into  $J/\psi + \pi$  and the  $J/\psi \rightarrow l^+l^-$ , where  $l = e$  or  $\mu$ . We have integrated over the rapidity interval  $-2.0 \leq y \leq 2.0$ . For the singlet matrix element, we use the value extracted from  $\chi_c$  decays, which is  $\langle \mathcal{O}_1^{h_c}(^1P_1) \rangle = 0.32$  [33] and for the octet matrix element we have  $\langle \mathcal{O}_8^{h_c}(^1S_0) \rangle = 0.0098$  [9]. With these inputs, we find that the cross-section for  $h_c$  production

(folded in with the decay fraction into a  $J/\psi$  and  $\pi$ , which we take to be 0.5% [34] and a 6% leptonic decay branching fraction of the  $J/\psi$ ) is large enough to have a substantial number of events with the statistics that will be acquired in the first few months of LHC running. For example, for a lower  $p_T$ -cut of about 40 GeV, the integrated cross-section is about a couple of nb. Even with a modest luminosity of about  $10 \text{ pb}^{-1}$ , one can expect about 20,000 events. This number increases substantially by lowering the  $p_T$ -cut or, of course, with larger values of luminosity.

Varying the QCD scale between the largest and the smallest values that it can take results in a variation in the cross-section which is about a factor of 2. While the results for the cross-section for  $h_c$  production in Fig. 1 show the variation with respect to QCD scale



**Fig. 3.** The cross-section (in nb) for  $h_c$  production as a function of  $p_T$  cut for different parton distribution sets.



**Fig. 4.** The  $p_T$  distributions for  $h_c$  production (in nb/GeV).

inputs, in Fig. 2 we display the uncertainty in the cross-section coming from varying the value of the octet matrix element. We expect a 30% variation about the central value of 0.0098 for the octet matrix element. The two curves in Fig. 2 correspond to the upper and lower values that the octet matrix element can take. In Fig. 2 the QCD scale is taken to be  $M_T$ . The variation in the cross-section due to the change in the octet matrix element is about 60%.

In Fig. 3, we show the  $p_T$ -integrated cross-section choosing different parton density sets. In addition to the CTEQ 4M densities used earlier, we use the LO CTEQ [32] and GRV densities [35]. It is only at low values of  $p_T$  that a sizeable change in the cross-section due to the variation of the parton density inputs can be seen and even at a  $p_T$  value of 10 GeV the variation is not more than about

30%. The decay branching fraction of  $h_c$  into a  $J/\psi + \pi$  could be as large as 1% [34], and if we use this instead of the 0.5% used in the above calculations we could have a production cross-section which is twice as large.

The  $p_T$  distribution  $Bd\sigma/dp_T$  is shown in Fig. 4. We have plotted the octet and the singlet contributions separately. We find that, over a whole range of large  $p_T$ , the singlet contribution is negligible and that the  $h_c$  is produced almost exclusively from the colour-octet channel.

It is important to note that while the cross-section for the production of  $h_c$  is large, the detection of the  $h_c$  in the  $J/\psi + \pi$  mode is by no means easy. The main problem is trying to resolve the two-photon decay of the pion from single-photon backgrounds.

The reconstruction of high-energy pions was already a problem at the Tevatron but the CDF experiment managed eventually to have a nice signal for it. In fact, in the CDF search for the  $\chi_c$  through its decay into a  $J/\psi + \gamma$  final state the  $h_c$  was considered as a possible background. A double-Gaussian fit trying to simultaneously fit the  $\chi$  and the  $h_c$  resonance in the  $18 \text{ pb}^{-1}$  data analysed by the CDF experiment resulted in  $1109 \pm 91 \chi_c$  events and  $136 \pm 102 h_c$  events [36]. The large error in the  $h_c$  sample meant that there was no statistically significant signal for the  $h_c$  in the  $18 \text{ pb}^{-1}$  data but there was hope that subsequent high-statistics data from the Tevatron would yield a signal. Unfortunately, these analyses have not been carried out. At the LHC, the  $\pi^0$  reconstruction is likely to be the main problem again but with the large cross-section and luminosity that will be available even an efficiency of a few percent will not affect the statistical significance of the  $h_c$  signal.

We would like to conclude by making the following points:

- In spite of the success that we have had in understanding charmonium production at the Tevatron using NRQCD, we still need to have independent tests of this effective theory because the colour-octet parameters, and consequently the normalisations of the cross-sections of the various charmonium resonances, are not given by the theory but only fixed by fitting to the data.
- Polarisation predictions for  $J/\psi$  and  $\psi'$  at large- $p_T$ , considered to be good tests of NRQCD, disagree violently with what is measured by the CDF experiment at the Tevatron.
- The production of  $h_c$  in NRQCD is a good test of the theory because: (i) it is a prediction for large- $p_T$  production where NRQCD factorisation is expected to hold, (ii) the cross-section can be predicted because the relevant colour-octet parameter can be inferred from octet parameters measured in  $\chi_c$  production at the Tevatron and using heavy-quark symmetry and, (iii) the cross-section is very large at the LHC and should lead to detection of the resonance, in spite of the problems in reconstructing the  $J/\psi + \pi^0$  decay of the  $h_c$ . Moreover, the cross-section measurement is much simpler than measuring the polarisation of the charmonium state.
- Such a prediction for the cross-section of  $h_c$  production can not be made in the alternative approach to quarkonium production, viz., the semi-local duality model [27–29].

In conclusion, even with the statistics accumulated with a few months of LHC running the charmonium resonance,  $h_c$ , can not only be detected but its properties can be studied in detail. We have presented predictions of NRQCD for the production cross-section of the  $h_c$  and so the study of this state at the LHC will help test NRQCD independently and provide us more understanding of the physics of quarkonium formation.

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