



N^3 entropy of M5 branes from dielectric effect

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Abstract

We observe that the N^3 entropy behavior of near-extremal M5 branes can be reproduced from SYM side with the role of Myers' terms. We start by generalizing the Klebanov–Tseytlin (KT) supergravity solution that displays the N^3 entropy behavior. The new feature of the general solution is visibility of the “internal” degrees of the M5 branes, i.e., the M0 branes and the M2 branes. With the rationale provided by the supergravity analysis, we consider a D0 brane quantum mechanical setup with Myers' terms. Using localization technique, we show that the leading N^3 behavior of the free energy comes from the “classical contribution” with the rest subleading.

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1. Introduction

One of the recent lessons on the mechanism behind AdS/CFT – that was born in the birthplace of D-brane physics [1] – is that it may be far more dynamical in nature than might have originally been conceived. Although the complete first-principle derivation of the correspondence is still missing, indications exist that such dynamical nature would be the key to the derivation. One aspect of the dynamical nature is (conjectured) generation of D-brane curvature out of the non-geometric theories (SYM/open string) that one starts with. It was proposed in [2] that open string *quantum effects* may engineer the curvature around the host branes. Transition between an open

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string and a closed string is another dynamical aspect, which should also play a crucial role [3] in the first-principle derivation of AdS/CFT.

In the original formulation of AdS/CFT, it was SYM, the massless modes of the open string, that received attention after a “decoupling” limit. It has been increasingly clear to our view, however, that it is the *full-fledged* open string theory that is required for reproduction of the results of the dual closed string in general. Although this would be at odds with the strongest (or most radical) form of the AdS/CFT, it should not be entirely surprising since the correspondence would then be understood as a generalization of open–closed string type duality [3]. In one further step, we point out a possibility below which would take the open string to a much elevated status. This possibility is that the role of the open string is more fundamental than might have been expected: *an open string* setup may be required to reproduce certain *near-extremal supergravity* results.¹

A large amount of evidence has been accumulated over the years, especially in the case of AdS₅/CFT₄. Nevertheless, we note in this paper that there could be bulk physics associated with stringy effects that may not be suitably described by pure SYM even after non-perturbative effects are taken into account. (“Pure” SYM means SYM without an extra effect such as the dielectric effect.) Non-extremality of certain supergravity solutions may reflect massive open stringy effect [5,6], and potentially be an example. More specifically, the entropy of such a configuration will reflect stringy effects, and may take more than pure SYM on the dual side. Naturally one may wonder whether/how such effects can be seen from the SYM side.

In this paper, we take the entropy of various near-extremal M5 brane configurations; there have been suggestions that 5D maximal SYM may adequately describe the M5 brane dynamics [7,8]. The N^3 behavior was observed in [9] in the field theory anomaly context. The authors of [10] considered the quantum mechanical model that results from reducing the 5D SYM to time dimension. (See [11–13] for related discussions.) They studied the model’s degrees of freedom, and computed a certain index of the model. 5D SYM theory is the low energy limit of D4 branes, and, as such, a UV completion is required for a proper description of the D4 dynamics. For example, it potentially has an issue with renormalizability. In this work, we largely set this issue aside because we will take an alternative route.

It is natural to believe that the Kaluza–Klein modes (see, e.g., [10]) and the self-dual string (see, e.g., [14,15]) should be responsible for the N^3 behavior of M5 branes. (They are “instanton particles” and “little” strings in the IIA setup respectively.) We propose below using the IIA setup that it is the dielectric effect [16,17] in a D0/D4 system that is responsible for the leading N^3 entropy behavior. A D0/D4 system can be studied by the quantum mechanical lagrangian that results from reducing the $\mathcal{N} = 1$ 6D gauge theory whose field contents are vector multiplet, adjoint hypermultiplet and fundamental hypermultiplet, to time dimension. The Myers’ terms enter in the standard manner. The vacuum structure was studied in the Myers’ paper [17]; we will show that the non-commuting solution is responsible for the N^3 behavior.

To establish the entropy correspondence between supergravity and open string/SYM, it is necessary to understand the mapping between various branches of moduli space in these theories: physics of a given *branch* of supergravity moduli space must be matched with physics of the corresponding *branch* of SYM. In other words, AdS/CFT mappings of two theories should be made *branch by branch*. Once the dielectric effect is taken into account, the minimum energy configuration is a non-commuting solution. One must consider the non-commutative branch (as

¹ An open string ideal gas model was used to reproduce the N^2 entropy behavior of a near-extremal D3 brane configuration [4]. In the present case, it is the closed string interaction terms, i.e., the Myers’ terms, that are essential for reproduction of N^3 entropy.

opposed to, e.g., the branch that is associated with a commuting solution) because that would correspond to the supergravity moduli branch under consideration here.²

An important point to be made in the following sections is how to justify the use of the one-dimensional SYM. The 5D SYM has generalized instanton-type solutions with various moduli. It will be argued that as a result of the branch mapping between SYM and supergravity, one must consider point-like soliton solutions. With that argument, we will consider the reduced action, i.e., D0 action, since the D0 brane would correspond to the point-like “instanton” solutions,³ a narrower than otherwise branch of the SYM moduli space.

One of the interesting aspects of D-brane physics is that it admits various “dual” descriptions. Let us illustrate this “duality” taking the present example, a D0/D4 system. There are two ways to describe the system within the field theory technique, the D4-based description and the D0-based description. (Interesting discussions can be found in [20] and [21] on related matters.) In the D4-based description, one uses 5D SYM, and D0 branes are realized as its soliton solutions, the “instantonic particles”. If the 5D SYM theory were complete, one would integrate out the perturbative degrees of freedom, and get the D0 action at an intermediate stage of computing the partition function. One would then evaluate the resulting action further to complete the partition function. However, since 5D SYM is not (known to be) complete (see [22] for recent progress), one should rely on the full open string techniques. In the worldsheet setup of the open string, it is not entirely clear how to integrate out the perturbative brane degrees of freedom. (One way would be to use field theory techniques including all the massive modes of the open string. However, this procedure would be impossible to implement.) Therefore, technical reasons alone force one to turn to the fundamental description of D0 branes, D0-based description of D0/D4 system (more on this later).

One thing that needs to be understood in the D0 based description is how D0/D4 system is different from pure D0 system. In other words, the blow-up type solution is there even for a purely D0 brane system. What differentiates D0/D4 system from pure D0 system in the D0 based description? The answer to this question may lie in the peculiar string theoretic manner that the dielectric effect enters as we will point out later.

The near-extremal limit of supergravity should correspond to keeping the next leading order terms in α' . Therefore, one should take the corresponding step in the SYM side; in general, it is expected that Myers’ terms will be an infinite series in a derivative expansion. Taking the near-extremal limit in the supergravity configuration should correspond to taking the leading terms out of the infinitely many Myers’ terms.

The organization of the rest of the paper is as follows. In Section 2, after briefly reviewing the Klebanov–Tseytlin (KT) solution [23], we construct a class of IIA (or 11D) supergravity solutions of a D0/D4 type that display N^3 behavior by putting together ingredients in literature. The solutions can be viewed as a generalization of the Klebanov–Tseytlin solution. We observe that D2 branes are present through the Myers-type effect. We take the existence of such solutions as an indication that the Myers’ effect should be included in the gauge theory description as well. In Section 3, we set the stage for Section 4 in which we analyze the entropy of the D0/D2/D4 system in a D0 brane-based description. We determine in our convention the 5D SYM lagrangian with the Myers term, a system that yields a quantum mechanical model with the D4 brane effect and Myers’ effect incorporated. The action is reduced to time dimension. With the quantum me-

² The whole issue is tied with the limitations of our tools (both in open string/SYM and supergravity) that do not, in general, allow one to choose the degrees of freedom appropriate for different branches “in a continuous manner”.

³ See [18] and [19] for reviews of instanton.

chanical lagrangian ready, we compute the entropy of the system through localization technique [24,25]. To that end, we first recall facts about the residual supersymmetry of the system. After noting that the partition function consists of several parts, it is shown that the N^3 behavior comes from the “classical part”⁴ with the other parts yielding subleading contributions. In Section 5, we discuss some of the subtle issues, and comment on future directions.

2. Rationale provided by supergravity

Although the KK modes and the self-dual strings are likely to be responsible for the N^3 behavior, their presence in the near-extremal solutions [26,27,23] is not evident. There might exist a more general class of solutions with the same N^3 behavior. It is expected that they would have the following characteristics: reduction to the KT solution in some limit, and the more evident presence of the KK modes and the self-dual strings (or M2 branes).

In this section, we confirm this expectation by explicit construction of a class of near-extremal solutions of D0/D4 with the N^3 entropy behavior. They indeed reduce to the KT solution in a certain limit, and provide rationale for incorporating the Myers’ term in the open string/SYM analysis in the following sections. Presumably, the near-extremal M5 solution of [23] could be viewed as describing M5 branes with the lower dimensional M-branes completely dissolved.

One can construct a solution that describes a KKW/M2/M5 by boosting an M2/M5 solution [28–30] along, say, the x_3 direction. (Details will be presented in [31].) The resulting solution has the following form

$$ds_{11}^2 = (H\tilde{H})^{1/3} [H^{-1}(-K^{-1}f dt^2 + dx_1^2 + dx_2^2) + \tilde{H}^{-1}(K d\hat{x}_3^2 + dx_4^2 + dx_5^2) + f^{-1} dr^2 + r^2 d\Omega_4^2], \tag{1}$$

where

$$\begin{aligned} H &= 1 + N \frac{h_0^3}{r^3}, & \tilde{H} &= \sin^2 \zeta + H \cos^2 \zeta, & f &= 1 - \frac{\mu^3}{r^3}, \\ K &= 1 + N_0 \frac{k_0^3}{r^3}, & d\hat{x}_3^2 &= [dx_3 + (K^{-1} - 1) dt]^2, \end{aligned} \tag{2}$$

and

$$\hat{F}_4 = \frac{1}{2} \cos \zeta * dH + \frac{1}{2} \sin \zeta dH^{-1} \epsilon_3 + \frac{3}{2} \sin 2\zeta H^{-2} dH \bar{\epsilon}_3. \tag{3}$$

ϵ_3 and $\bar{\epsilon}_3$ are volume forms on \mathbb{M}^3 and \mathbb{E}^3 parameterized respectively by (t, x_1, x_2) and (x_3, x_4, x_5) ; $*$ is the Hodge dual of \mathbb{E}^5 that is transverse to the M5 branes.

This solution is a generalization of the boosted solution of [29,30]. In $Nh_0^3 \ll 1, N_0k_0^3 \gg 1$ limit, the solution (1) can be viewed as a stack of N_0 black M5-branes. This should be a manifestation of the Myers’ effect, since, in the supergravity context, the dielectric effect should manifest itself as a dissolution of lower dimensional branes into higher dimensional ones. One can show that the solution exhibits the $S \sim N^3 T^5$ entropy behavior in the near-extremal limit.

Since we will use IIA setup in the following sections, let us reduce the solution to the corresponding IIA configuration. The resulting configuration will have the same near-extremal entropy

⁴ The Myers terms come from open strings coupling to a closed string. Therefore, it can effectively be view as a loop effect from the standpoint of the open strings.

behavior $S \sim N^3 T^5$. Dimensional reduction of (1) along the x_3 direction in which KK waves travel leads to the D0/D2/D4 interpolating solution, D2/D4 part of which was constructed in [32] in the string frame. In the Einstein frame,

$$ds_{11}^2 = e^{-\phi/6} ds_{10}^2 + e^{4\phi/3} (dx_3 + A)^2, \quad (4)$$

reducing along x_3 , one gets

$$ds_{10}^2 = H^{3/8} \tilde{H}^{1/4} K^{-3/8} [H^{-1} (-K^{-1} f dt^2 + dx_1^2 + dx_2^2) + \tilde{H}^{-1} (dx_4^2 + dx_5^2) + f^{-1} dr^2 + r^2 d\Omega_4^2], \quad (5)$$

$$e^\phi = H^{1/4} \tilde{H}^{1/2} K^{-3/4}, \quad A_{[1]} = (K^{-1} - 1) dt, \quad (6)$$

and, on account of $\hat{F}_4 = F_4 + F_3 \wedge (dx_3 + A)$,

$$F_4 = \frac{1}{2} \cos \zeta * dH + \frac{1}{2} \sin \zeta dH^{-1} \epsilon_3, \quad F_3 = \frac{3}{2} \sin 2\zeta H^{-2} dH \bar{\epsilon}_2. \quad (7)$$

ϵ_3 and $\bar{\epsilon}_2$ are the (t, x_1, x_2) and (x_3, x_4) coordinate volume forms respectively; $*$ still the Hodge dual in the transversal \mathbb{E}^5 . Setting $K = 1$ in (5)–(7) leads to a D2/D4 interpolating solution that generalizes the solution found in [32].

3. Open string/SYM setup

Above, we have considered the entropy of the Klebanov–Tseytlin solution and its generalization. In this section, we review open string derivation of the corresponding low energy SYM action, focusing on the standard cubic Myers’ terms. The Myers’ terms will play a central role in reproducing the N^3 entropy from the SYM side, and the precise form of the SYM action with the Myers’ term will be given within our convention. The action will preserve part of the supersymmetry. The residual supersymmetry will be used in the next section in which the partition function is evaluated using localization technique, a very convenient tool for evaluating the full partition function.

In the D0-based description, the presence of D4 branes are realized in part through the sector that comes from dimensional reduction of the $\mathcal{N} = 1$ 6D SYM fundamental hypermultiplet. (We will have more on this in Section 4.) The RR gauge field $C^{(3)}$ enters as a background in the effective field theory level. One of the key issues is the residual supersymmetry of the Myers’ term since residual susy is essential for employing localization technique. In the following subsection, we recall a few things about the residual supersymmetry of the 5D SYM with the Myers’ terms added, and reduce the system to time dimension. With these tasks completed, we will be ready to compute the partition function in Section 4.

3.1. Myers’ terms, supersymmetry and reduction

It is convenient to start in six dimensions, and reduce the system to five dimensions. The 6D theory has $(1, 1)$ supersymmetry. In terms of $(0, 1)$ supersymmetry, the $(1, 1)$ vector multiplet splits into an $(0, 1)$ vector multiplet, $(A_{\hat{\mu}}, \chi)$, and a $(0, 1)$ hypermultiplet in the adjoint, (ζ, Z) . In addition, the system contains the fundamental hypermultiplet, (w, μ) [33].

The precise form of the low energy 5D SYM action that reproduces the corresponding open string results can be determined by going through the standard procedure. In our convention, it is given by

$$\begin{aligned}
 \int \mathcal{L} = \int d^5x \text{Tr} & \left[-\frac{1}{4} F_{\hat{\mu}\hat{\nu}} F^{\hat{\mu}\hat{\nu}} + \frac{1}{2} \sum_{i=1}^3 \mathcal{D}_i^2 - i \bar{\chi}_{\dot{\alpha}} \Gamma^{\hat{\mu}} D_{\hat{\mu}} \chi^{\dot{\alpha}} - (D_{\hat{\mu}} Z^{\dot{\alpha}})^{\dagger} D_{\hat{\mu}} Z^{\dot{\alpha}} \right. \\
 & - i \bar{\zeta} \Gamma^{\hat{\mu}} D_{\hat{\mu}} \zeta + \bar{Z}_{\dot{\alpha}} \sum_{c=1}^3 (\tau^c \mathcal{D}_c)^{\dot{\alpha}}{}_{\dot{\beta}} Z^{\dot{\beta}} - Z_{\dot{\alpha}} \sum_{i=1}^3 (\tau^i \mathcal{D}_i)^{\dot{\alpha}}{}_{\dot{\beta}} \bar{Z}^{\dot{\beta}} \\
 & \left. + 4i ([\bar{\zeta}, Z^{\dot{\beta}}]_{\epsilon_{\dot{\alpha}\dot{\beta}} \chi^{\dot{\alpha}}} - \bar{\chi}_{\dot{\alpha}} [\zeta, \bar{Z}_{\dot{\beta}}]_{\epsilon^{\dot{\alpha}\dot{\beta}}}) \right] \\
 & + \epsilon^{\dot{\alpha}\dot{\beta}} (D_{\hat{\mu}} \bar{w}_{\dot{\alpha}})_a (D^{\hat{\mu}} w_{\dot{\beta}})^a - \bar{w}_{\dot{\alpha}a} \sum_{i=1}^3 (\tau^i)^{\dot{\alpha}}{}_{\dot{\beta}} (D_i)^a{}_b w^{\dot{\beta}b} - i \bar{\mu}_a \Gamma^{\hat{\mu}} (D_{\hat{\mu}} \mu)^a \\
 & + 2i [\bar{\mu}_a (\chi^{\dot{\alpha}})^a{}_b \epsilon_{\dot{\alpha}\dot{\beta}} (w^{\dot{\beta}})^b + (\bar{w}_{\dot{\alpha}})_a \epsilon^{\dot{\alpha}\dot{\beta}} (\bar{\chi}_{\dot{\beta}})^a{}_b \mu^b] \tag{8}
 \end{aligned}$$

where

$$\hat{\mu} = (\mu, 5). \tag{9}$$

For technical reasons, the action is written in the 6D notation. Other than minor conventional differences, this action can be easily deduced from the results that were obtained in [34] which was based on earlier work of [35]. Again, for technical and convenience reasons, we start with the 5D SYM above, and reduce it to time dimension instead of directly considering a D0 system from the beginning.

One feature of the susy transformation of (8)⁵ is worth noting: the gauge multiplet fields transform within themselves (whereas the hypermultiplet transformations involve the gauge fields). This feature of the susy transformations will be used in the next section where localization is employed to evaluate the partition function.

Let us (implicitly) reduce the action (8) to time dimension, and add the Myers terms⁶

$$-\frac{i}{3} f g_{YM} \epsilon_{pqr} \phi^p \phi^q \phi^r \tag{10}$$

where f is a constant. For now, we will be implicit about the ranges of the indices (p, q, r) . They will become clear in Section 4.2. The resulting action is

$$\begin{aligned}
 \int \mathcal{L}_{D0/D4} = \int dt \text{Tr} & \left[-\frac{1}{4} F_{\hat{\mu}\hat{\nu}} F^{\hat{\mu}\hat{\nu}} + \frac{1}{2} \sum_{i=1}^3 \mathcal{D}_i^2 - i \bar{\chi}_{\dot{\alpha}} \Gamma^{\hat{\mu}} D_{\hat{\mu}} \chi^{\dot{\alpha}} - (D_{\hat{\mu}} Z^{\dot{\alpha}})^{\dagger} D_{\hat{\mu}} Z^{\dot{\alpha}} \right. \\
 & - i \bar{\zeta} \Gamma^{\hat{\mu}} D_{\hat{\mu}} \zeta + \bar{Z}_{\dot{\alpha}} \sum_{c=1}^3 (\tau^c \mathcal{D}_c)^{\dot{\alpha}}{}_{\dot{\beta}} Z^{\dot{\beta}} - Z_{\dot{\alpha}} \sum_{i=1}^3 (\tau^i \mathcal{D}_i)^{\dot{\alpha}}{}_{\dot{\beta}} \bar{Z}^{\dot{\beta}} \\
 & \left. + 4i ([\bar{\zeta}, Z^{\dot{\beta}}]_{\epsilon_{\dot{\alpha}\dot{\beta}} \chi^{\dot{\alpha}}} - \bar{\chi}_{\dot{\alpha}} [\zeta, \bar{Z}_{\dot{\beta}}]_{\epsilon^{\dot{\alpha}\dot{\beta}}}) - \frac{2}{3} i f g_{YM} \epsilon^{pqr} \phi_p \phi_q \phi_r \right]
 \end{aligned}$$

⁵ The supersymmetry transformations can be found in, e.g., [18] and [34].

⁶ For supersymmetry in the presence of the Myers terms, see [36] and [37].

$$\begin{aligned}
& + \epsilon^{\dot{\alpha}\dot{\beta}} (D_{\hat{\mu}} \bar{w}_{\dot{\alpha}})_a (D^{\hat{\mu}} w_{\dot{\beta}})^a - \bar{w}_{\dot{\alpha}a} \sum_{i=1}^3 (\tau^i)^{\dot{\alpha}}_{\dot{\beta}} (\mathcal{D}_i)^a_b w^{\dot{\beta}b} \\
& - i \bar{\mu}_a \Gamma^{\hat{\mu}} (D_{\hat{\mu}} \mu)^a + 2i [\bar{\mu}_a (\chi^{\dot{\alpha}})^a_b \epsilon_{\dot{\alpha}\dot{\beta}} (w^{\dot{\beta}})^b + (\bar{w}_{\dot{\alpha}})_a \epsilon^{\dot{\alpha}\dot{\beta}} (\bar{\chi}_{\dot{\beta}})^a_b \mu^b]. \quad (11)
\end{aligned}$$

We record the potential part of the fields in the adjoint representation for later use:

$$\begin{aligned}
& - \text{Tr} \left(\frac{1}{4} [\phi_{\hat{\mu}}, \phi_{\hat{\nu}}]^2 + \frac{1}{2} \sum_{c=1}^3 \mathcal{D}_c^2 + \frac{1}{2} [\phi_{\hat{\mu}}, \phi_m]^2 \right. \\
& \left. + \frac{i}{2} \sum_{c=1}^3 \mathcal{D}_c \eta_{cmn} [\phi^m, \phi^n] - \frac{i}{3} f g_{YM} \epsilon_{pqr} \phi^p \phi^q \phi^r \right) \quad (12)
\end{aligned}$$

where we have rewritten the Z -fields in terms of ϕ 's [34]. η_{cmn} is the 't Hooft symbol whose explicit form can be found, e.g., in [35]. The index m (with $\hat{\mu} = 5$) represents the directions transverse to the D4 branes. Note that the coefficient of the Myers' terms have an extra power of the gauge coupling. This is because they come from the coupling between open string states and a closed string state. As well-known, the closed string coupling goes $g_c \sim g_0^2 \sim g_{YM}^2$. The extra factor of g_{YM} will be important for the entropy computation later.

4. Entropy analysis with Myers' term

With the stage set in the previous sections, we evaluate here the free energy of the quantum mechanical system, (11). There are several essential ingredients in the analysis; strictly speaking, some of them are assumptions. First of all, the D0-based setup itself is assumed to properly describe the D0/D4 system with the dielectric effect taken into account. The second ingredient is the aforementioned issue of the *moduli branch matching* between SYM and supergravity. In the present context, we take this to imply that the SYM branch corresponding to the supergravity branch under consideration should be associated with $w_0 = 0$, where w_0 denotes the vev of w .⁷ The reason is that the KT supergravity solution has the lower dimensional objects completely dissolved: it should correspond to a situation where the solitons become point-like. The third ingredient is localization technique, and it is the main topic of this section: we employ localization technique to evaluate the partition function, and thereby derive the N^3 behavior. Towards the end of this section, we comment on the difference between a D0/D4 and a D(-1)/D3 system, pondering the peculiar way in which the Myers' terms arise for Dp branes with $p \neq 0$. An additional effect of D4 branes in the D0-based description will be commented on at the end of this section as well.

⁷ In principle, the path integral over the quantum fluctuations of the fundamental hypermultiplet must be considered. However, localization technique renders this step unnecessary, as we will see shortly.

4.1. Entropy of D0/D4 via localization

As recalled in the previous section, the dielectric effect preserves part of the supersymmetry. With the supersymmetry partially preserved, one can rely on localization technique⁸ to compute the free energy. More specifically, consider adding a localizing term QS_L to the action (11),

$$\int D\Phi \exp\left(\frac{i}{g_{YM}^2}S + i\frac{t}{g_{YM}^2}QS_L\right) = \exp\left(\frac{i}{g_{YM}^2}\mathcal{F}\right) \tag{13}$$

where Φ is a collective symbol for the fields, t is a localization parameter and Q is a nilpotent operator that is constructed basically out of the residual supersymmetry transformations. The functional S_L represents an appropriately chosen localizing term; it is possible [36,37] to choose S_L such that the QS_L becomes the action for the adjoint fields ((15) below).

In the path integral,⁹ there are two contributions: the scalar vevs and the one-loop contributions. The scalar vevs are determined by minimizing the potential with the Myers' terms. In general, there could be some parameters over which a matrix theory type path integral would have to be further performed. We will note that there is no modulus in the minimum solution of [17], therefore, this step is not required. As usual, the action will be expanded around the scalar vevs. The one-loop contribution can be evaluated by the saddle point method.

The fields in the adjoint representation can be re-expressed using the 10D notation [34]. Combining it with the fundamental hypermultiplet part of the action, one gets

$$\begin{aligned} \mathcal{L} = & \text{Tr}\left(-\frac{1}{4}F_{MN}F^{MN} - \frac{i}{2}\bar{\lambda}\Gamma^M D_M\lambda\right) - \frac{1}{3}ifg_{YM}\epsilon_{pqr}\phi^p\phi^q\phi^r \\ & + \epsilon^{\dot{\alpha}\dot{\beta}}(D_{\hat{\mu}}\bar{w}_{\dot{\alpha}})(D^{\hat{\mu}}w_{\dot{\beta}}) - (\bar{w}_{\dot{\alpha}})\sum_{c=1}^3(\tau^c)^{\dot{\alpha}}_{\dot{\beta}}(\mathcal{D}_c)(w^{\dot{\beta}}) \\ & - i\bar{\mu}\Gamma^{\hat{\mu}}(D_{\hat{\mu}}\mu) + 2i[\bar{\mu}(\chi^{\dot{\alpha}})\epsilon_{\dot{\alpha}\dot{\beta}}(w^{\dot{\beta}})^b{}_u + (\bar{w}_{\dot{\alpha}})\epsilon^{\dot{\alpha}\dot{\beta}}(\bar{\chi}_{\dot{\beta}})\mu]. \end{aligned} \tag{14}$$

The relationship between gauge fermionic terms in (8) and λ -terms in (14) can be found in [34].

As we have noted in the previous section, the supersymmetry transformations of the gauge multiplet fields do not involve any hypermultiplet fields. This implies that the hypermultiplet fields will contribute to the partition function only classically but not through one-loop or soliton sector. Also because of the aforementioned branch matching that implies $w_0 = 0$, one can entirely focus on the adjoint multiplet part of the lagrangian, choosing it as QS_L . Let us focus on the adjoint fields,

$$tQS_L = t\mathcal{L}_{adjoint} = t\text{Tr}\left(-\frac{1}{4}F_{MN}F^{MN} - \frac{i}{2}\bar{\lambda}\Gamma^M D_M\lambda\right) - \frac{i}{3}tfg_{YM}\epsilon_{pqr}\phi^p\phi^q\phi^r \tag{15}$$

where t is a localization parameter.

⁸ In [37], it was shown that the supercharge they considered is nilpotent on gauge invariant quantities. Presumably, once one adds the gauge fixing term it would make (as in [25]) the sum of the supercharge and the BRST charge nilpotent on all quantities, not only on gauge invariant quantities.

⁹ The supergravity solutions found in Section 2 are black brane solutions with finite Hawking temperature. This suggests, although not without subtlety, a possibility that it may be a finite temperature field theory that needs to be employed. We expect the finite temperature to preserve the N^3 behavior. The constant f that appears in (10) may be related to the temperature. We postpone the precise relevance and effects of finite temperature for near-future research.

Using this orientation, let us now work with (11). With the choice of the branch $w_0 = 0$, the bosonic part of the potential is given by (11) which we quote here,

$$V = -\text{Tr} \left(\frac{1}{4} [\phi_{\hat{\mu}}, \phi_{\hat{\nu}}]^2 + \frac{1}{2} \sum_{c=1}^3 \mathcal{D}_c^2 + \frac{1}{2} [\phi_{\hat{\mu}}, \phi_m]^2 + \frac{i}{2} \sum_{c=1}^3 \mathcal{D}_c \eta_{cmn} [\phi^m, \phi^n] \right) + \frac{i}{3} f g_{YM} \epsilon_{pqr} \phi^p \phi^q \phi^r. \quad (16)$$

The \mathcal{D} -field equation of (12) is

$$2\mathcal{D}_c + i\eta_{cmn} [\phi^m, \phi^n] = 0. \quad (17)$$

Substituting this into (16), the potential becomes

$$V = -\text{Tr} \left(\frac{1}{4} [\phi^M, \phi^N]^2 - \frac{i}{3} f g_{YM} \epsilon_{pqr} \phi^p \phi^q \phi^r \right) \quad (18)$$

where M, N are 10D indices. This potential was analyzed by Myers in [17]. Upon substituting the vacuum solution that represents a non-commutative configuration, the potential yields

$$V = -\frac{f^4 g_{YM}^4}{32} N(N^2 - 1). \quad (19)$$

The entropy is obtained by taking a derivative of this result with respect to the SYM coupling, and it displays the leading N^3 behavior.

Finally, let us turn to the fluctuation part and expand (15) around the non-commuting solution just reviewed. In the saddle point method, we keep only up to (and including) the quadratic terms,

$$\mathcal{L}_{adjoint,2nd} = \text{Tr} \left(-\frac{1}{4} F_{MN} F^{MN} - \frac{i}{2} \bar{\lambda} \Gamma^M D_M \lambda \right) \Big|_{\phi \rightarrow \phi + \phi_0, 2nd} + \frac{i}{3} f g_{YM} \epsilon_{pqr} \text{Tr} \phi^p \phi^q \phi^r \Big|_{\phi \rightarrow \phi + \phi_0, 2nd}. \quad (20)$$

The path integral that one should evaluate is

$$\int dA d\phi d\lambda e^{\int -\frac{1}{4} \text{Tr}(F_{0,mn} F_{0,mn}) + \frac{i}{3} f g_{YM} \epsilon_{pqr} \text{Tr} \phi_0^p \phi_0^q \phi_0^r} e^{-\frac{1}{4} \text{Tr} f_{\mu\nu} f^{\mu\nu}} e^{\int -\frac{i}{2} \bar{\lambda} \Gamma^M (\partial_M - i A_0^M) \lambda} \times \exp \left[\int -\frac{1}{2} \text{Tr} \partial_\mu \phi_m \partial^\mu \phi^m + i f g_{YM} \epsilon_{pqr} \text{Tr} \phi_0^r \phi^p \phi^q - \frac{1}{2} \text{Tr}[\phi_{0,m}, \phi_{0,n}][\phi^m, \phi^n] - \frac{1}{2} \text{Tr}[\phi_{0,m}, \phi_n][\phi^m, \phi^{0n}] - \frac{1}{2} \text{Tr}[\phi_{0,m}, \phi_n][\phi^{0,m}, \phi^n] \right] \quad (21)$$

where $f_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$. Note that the scalar part and the gauge part decouple. The remaining task is to show that the rest of the integration does not change the leading N^3 behavior of the classical contribution.

The leading N behavior should come from the scalar part of the path integral,

$$\int d\phi \exp \left(\frac{1}{g_{YM}^2} \int -\frac{1}{2} \text{Tr} \partial_\mu \phi_m \partial^\mu \phi^m + i f g_{YM} \epsilon_{pqr} \text{Tr} \phi_0^r \phi^p \phi^q - \frac{1}{2} \text{Tr}[\phi_{0m}, \phi_{0n}][\phi^m, \phi^n] + \frac{1}{2} \text{Tr}[\phi_{0m}, \phi_n][\phi^{0n}, \phi^m] - \frac{1}{2} \text{Tr}[\phi_{0m}, \phi_n][\phi^{0m}, \phi^n] \right)$$

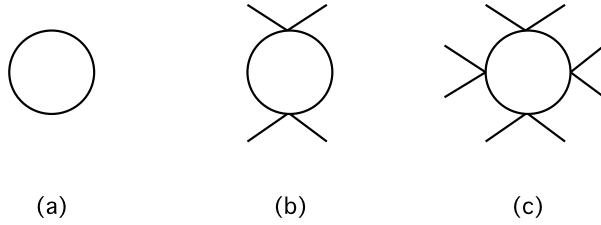


Fig. 1. One-loop diagrams.

$$\begin{aligned}
 &= \int d\phi \exp\left(\frac{1}{g_{YM}^2} \int -\frac{1}{2} \partial_\mu (\phi_m)_{s_1 t_1} \partial^\mu (\phi^m)_{t_1 s_1} \right. \\
 &\quad \left. + \frac{f^2(N^2 - 1)}{16} \delta_{t_1 t_2} (\phi_n)_{t_1 s_2} (\phi_n)_{s_2 t_2} + V_{\phi^2} \right) \tag{22}
 \end{aligned}$$

where we have defined

$$\begin{aligned}
 V_{\phi^2} \equiv & -\frac{1}{2} ([\phi_{0m}, \phi_{0n}]_{s_1 s_2} [\phi_{ms_2 t_2} \phi_{nt_2 s_1} - \phi_{ns_2 t_2} \phi_{mt_2 s_1}]) \\
 & + \frac{1}{2} (\phi_{0ms_1 t_1} \phi_{0ns_2 t_2} \phi_{nt_1 s_2} \phi_{mt_2 s_1} - \phi_{0ms_1 t_1} \phi_{0nt_2 s_1} \phi_{nt_1 s_2} \phi_{ms_2 t_2} \\
 & - \phi_{0mt_1 s_2} \phi_{0ns_2 t_2} \phi_{ns_1 t_1} \phi_{mt_2 s_1} + \phi_{0mt_1 s_2} \phi_{0nt_2 s_1} \phi_{ns_1 t_1} \phi_{ms_2 t_2}) \\
 & - \frac{1}{2} (\phi_{0ms_1 t_1} \phi_{0ms_2 t_2} \phi_{nt_1 s_2} \phi_{nt_2 s_1} + \phi_{0mt_1 s_2} \phi_{0mt_2 s_1} \phi_{ns_1 t_1} \phi_{ns_2 t_2}) \\
 & + i f g_{YM} \epsilon_{pqr} (\phi_0^r)_{s_1 t_1} \phi_{t_1 s_2}^p \phi_{s_2 s_1}^q. \tag{23}
 \end{aligned}$$

It should be possible to fully evaluate the integral since the exponent is of quartic order in ϕ . For our purpose it is sufficient to evaluate it perturbatively. Let us view the first line as the propagator terms and the rest as vertices. The propagator is given by

$$\langle \phi_{ms_1 t_1}(x) \phi_{ns_2 t_2}(y) \rangle = g_{YM}^2 \delta_{mn} \left(\delta_{s_1 t_2} \delta_{s_2 t_1} - \frac{1}{N} \delta_{s_1 t_1} \delta_{s_2 t_2} \right) \Delta_{xy, N}, \tag{24}$$

$$\Delta_{xy, N} \equiv \int \frac{dq}{2\pi i} \frac{e^{iq(x-y)}}{Nq^2 - \frac{f^2}{16} N(N^2 - 1)}. \tag{25}$$

One factor of N in the denominator of (25) could be rescaled away to be in line with the usual convention; with the rescaling, the discussion below would still be valid since all the vertices would get an extra factor of $\frac{1}{N}$ as well. In the computation below, we only keep the $\delta_{s_1 t_2} \delta_{s_2 t_1}$ piece since we are interested in the leading N behavior. The first few Feynman diagrams that need to be evaluated are given in Fig. 1.

For an illustration, let us consider the diagram (b). We will now show that the leading terms have N^3 behavior. Also, it is not difficult to see from power-counting that the higher order diagrams are at most of N -cubic order. The diagram (b) corresponds to two insertions of V_{ϕ^2} . When squared, the leading behavior comes from the terms in the second bracket in (23); they yield

$$\frac{f^4}{2048} (2N^2 + N)(N^2 - 1)^2 \int dx dy \Delta_{xy, N}^2. \tag{26}$$

The integral yields $N^{-\frac{9}{2}}$ making the overall leading power $N^{\frac{3}{2}}$. The leading N behavior of the diagrams with more insertions of V_{ϕ^2} can be deduced as follows. With more insertions, the

number of the propagators increases as well. At the final stage of the evaluation, one would compute

$$(N\text{-factors from vertices}) \int dp \frac{1}{[Np^2 - \frac{f^2}{16}N(N^2 - 1)]^\#}. \quad (27)$$

The increased power of N in the numerator due to the presence of more insertions is cancelled by the increased power due to more propagators in the denominator. For example, the diagram (c) with four insertions gets N^{12} coming from the vertex contractions and $N^{-\frac{21}{2}}$ coming from the propagators preserving the overall $N^{\frac{3}{2}}$ power. Based on this, we conclude that the leading terms in the one-loop contribution have N^3 behavior that comes from the classical contribution.

4.2. Comments on entropy of a D(-1)/D3 system

As widely known, the entropy of the extremal D3 brane configuration has N^2 behavior. Let us ponder the differences between a D0/D4 and a D(-1)/D3 systems.¹⁰ Although the Myers' term in the D0 brane context has been long-known, the direct string theoretic realization in the context of general Dp branes is very recent; it is the Eq. (31) in [39]¹¹ that is based on the earlier works [40–43]. We quote it for convenience; in the present notation, it is given by

$$\int d^{p+1}x \epsilon^{\mu_0 \dots \mu_p} \text{tr}(F_{\mu_0 \mu_1} \phi^k) \partial_k C_{\mu_2 \dots \mu_p}^{(p-1)} \quad (28)$$

up to a multiplicative constant. The μ -index denotes the worldvolume directions and the k -index, the transverse directions. The Myers' term in the D0 context arises upon reducing this term to time dimension. There is a peculiarity in the equation above; when the dielectric effect is realized in string theory context, it could also be realized through the closed string coupling to lower-dimensional objects than the dimension of the brane that one has started with. We will have more on this in the conclusion. Another implication of the result above is associated with how the effect of D4 branes should be incorporated in the D0-based description; one should start with D4 branes, and reduce them to get the D0 dielectric effect. This is, therefore, where an effect of the presence of D4 branes can be seen in the D0-based description: the presence of the D4 branes – which would not have any effect in the computation of the partition function otherwise – affects the system through the Myers terms given above.

For the Myers' terms of a D0/D4 system in the D0-based description, one should take $p = 4$ and reduce this to time dimension. For a D(-1)/D3 system in the matrix theory description, $p = 3$ should be chosen, followed by reduction to zero dimension. The factor $\text{tr}(F_{\mu_0 \mu_1} \phi^k)$ becomes $\text{tr}([\phi_{\mu_0}, \phi_{\mu_1}] \phi^k)$ which is totally antisymmetric in (μ_0, μ_1, k) . Unlike the $p = 4$ case, μ_0 or μ_1 must take the time direction in the case of $p = 3$. The whole term would be removed by taking a temporal gauge. This indicates that, for D(-1)/D3, it might not be the dielectric effect that is responsible for the N^2 behavior.¹²

¹⁰ A study of a D(-1)/D3 system can be found, e.g., in [38].

¹¹ Additional Myers' terms have also been found in [44].

¹² However, it could be higher α' -order dielectric effect that is responsible for the N^2 behavior.

5. Conclusion

In this work, we have reproduced the leading N^3 entropy behavior from D0 quantum mechanics with the Myers' terms. The effect of D4 brane is incorporated through the hypermultiplet in the fundamental representation. The leading N^3 behavior came from the classical contribution with the rest of the contributions yielding subleading expressions.

In the D0-based description, the presence of the D4 branes gets to introduce additional branches via w . However, the supergravity N^3 behavior corresponds to the branch where $w_0 = 0$. Therefore, the presence of D4 branes has no effect on the classical part of the free energy. It does not contribute to the one-loop part of the partition function either (as seen in the previous section) by choosing the localization term solely in terms of the gauge multiplet. This does not mean that the w -field has no effect on the free energy in general. It just means that the w -field is irrelevant for the particular class of the supergravity solutions under consideration.

Although natural, there are several assumptions that were made to derive the results in this paper. Firstly, we used a D0-based description in the spirit of [18]; it is not entirely clear whether the D0-based description would be capable of capturing the full physics of a D0/D4 system although it would certainly capture some aspects of it. This matter would be worth looking into. Secondly, we have noticed in Section 4 that the Myers' terms can be realized in a peculiar manner from the coupling between open and closed strings. It comes from the closed string coupling to a lower dimensional branes as well that "lie inside" of the branes that one started with. The lower dimensional branes can be viewed as soliton solutions of the branes that one started with. (This seems consistent with the view that the closed string is taken as a composite state of the open string theory.) A better understanding of this phenomenon would be desired. The finite temperature effect is another aspect that requires attention. Finally, it would be interesting to investigate whether the N^2 behavior of extremal D3 branes could be related to a higher order dielectric effect.¹³ We hope to report on some of these issues in the future.

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¹³ Note [45] in this respect, where the N^2 and N^3 entropy growth of effective potentials was observed within effective subdeterminant models of N coincident D3 and M2 branes.

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