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Procedia Engineering 137 (2016) 151 – 160

**Procedia
Engineering**www.elsevier.com/locate/procedia

GITSS2015

Optimization of Urban Single-Line Metro Timetable for Total Passenger Travel Time under Dynamic Passenger Demand

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Abstract

This paper studies the optimization of an urban single-line metro timetable for total passenger travel time adapted to dynamic passenger demand, which arises in an urban metro service and is a common problem in major cities. After analyzing the components of the total passenger travel time, a model is presented with the aim of minimizing total passenger travel time. An *S*-pattern function is proposed to represent the cumulative demand function for each pair of origin and destination in an urban single-line metro. Furthermore, a spatial branch and bound algorithm that is applicable to the model is presented. The advantages of designing a timetable that optimizes the total passenger travel time adapted to dynamic passenger demand are depicted through extensive computational experiments on several cases derived from a real urban single-line metro. An extensive computational comparison of a regular timetable, a timetable optimizing average waiting time, and a timetable optimizing total passenger travel time timetable are performed.

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Peer-review under responsibility of the Department of Transportation Engineering, Beijing Institute of Technology

Keywords: train timetable; total passenger travel time; dynamic demand; regular timetable; spatial branch and bound algorithm.

1. Introduction

Urban metro system planning is a sub-aspect of the complex public transportation system planning and is usually decomposed into several stages, including network design, line design, timetabling, rolling stock, and staffing [8,10]. These problems have been traditionally optimized from the operator's perspective. This paper provides an approach that considers the operator's restrictions and minimizes the passengers' total travel time,

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which is measured as the sum of passenger average waiting time (AWT) at stations and average riding time (ART) on trains. This study aims to solve the urban single-line metro timetable problem, which consists of determining departure time at each station. Furthermore, the optimal speed between every segment of each train service to and from each station along an urban single-line metro over a planning horizon, which is determined by a dynamic passenger origin-destination demand, is also addressed.

The urban metro timetable problem can be classified into two forms: periodic and non-periodic timetables. [17] proposed a mathematical model for the periodic scheduling problem, which considered the extension to the periodic phenomena of ordinary scheduling with precedence constraints. The advantage of periodic timetables is that they can be easily memorized by passengers and are able to deal with large-scale railway networks [15]. The alternative, which involves constructing a non-periodic timetable, is appropriate when the demand cannot be assumed constant over time. Various integer linear programming models have been proposed for the non-periodic timetable problem. Carey [6] presented a model, as well as algorithms and strategies to dispatch trains with different speeds and stopping patterns for a double track rail line dedicated to trains in one direction; they further extended it to include general and complex rail networks, with choices of lines and station platforms. Caprara [5] proposed a model to determine a periodic timetable for a set of trains that are unable to violate track capacities and able to satisfy some operational constraints. They proposed a graph theory-based model for modelling the problem using a directed multi-graph in which nodes correspond to departures/arrivals at certain stations and given time instants. Vansteenwegen and Oudheusden [19] dealt with improving passenger service on a small portion of the Belgian railway network by taking waiting times and delays into account. They established a two-phase model. First, ideal buffer times are calculated based on delay distributions of arriving trains and the weights of various waiting times to safeguard connections when the arriving train is late; and second, standard linear programming is used to construct an improved timetable with well-scheduled connections and ideal buffer times whenever possible. Cacchiani and Toth [4] surveyed existing studies that primarily dealt with the train timetable problem in its nominal and robust versions, which satisfied track capacity constraints. They aimed to optimize an objective function with various meanings that correspond to requests of the railway company. Most of the papers mentioned optimized an objective function relevant to the service operator. From the infrastructure operator's point of view, one common objective is to minimize deviation from a timetable plan proposed by the operator, which is frequently used in the periodic case. From the users' perspective, the objective of minimizing waiting time has been considered under dynamic passenger demand [2]. However, these contributions have not been explicitly considered the total travel time under dynamic passenger demand.

The following contributions focused on the dynamic structure of demand behaviour. Hänseler [11] investigated ways of computing dynamic OD matrices for train stations. Yano and Newman [20] proposed a dynamic programming algorithm for the timetable of trains used to transport containers that arrive dynamically at the origin. Cordone and Redaelli [7] took into account the reciprocal influence between timetable quality and transport demand amount captured by the railway. They proposed a mixed-integer nonlinear model with a non-convex continuous relaxation. Niu and Zhou [16] focused on optimizing a passenger train timetable in a heavily congested urban rail corridor under a dynamic demand scenario. The researchers were interested in system behaviour under congestion once demand exceeds capacity.

This paper focuses on constructing an urban single-line metro timetable that is adapted to dynamic passenger demand. The proposed model pursues two objectives: minimizing passenger AWT at stations and ART on trains, respectively, to represent the total passenger travel time (TTT). One of the main features of this model is the S-pattern function that is mostly used in urban metro systems. This pattern is proposed to describe the cumulative demand for each pair of origin and destination, as well as each spatial branch and bound algorithm to optimize the model. The solution is an urban single-line metro timetable adapted to dynamic passenger demand over a finite planning horizon.

The remainder of this paper is organized as follows. Section 2 formally states the problem and the objectives, and Section 3 presents the proposed mathematical model. Spatial branch and bound algorithm are further described in Section 4, and results of extensive computational experiments and conclusions are presented in Sections 5 and 6 respectively.

2. Problem statement and analysis

This study focuses on constructing an urban single-line metro timetable adapted to a dynamic passenger

demand using the objective AWT similar to [1,2,16], and ART. Urban single-line metro timetable is normally represented in time-space diagrams, as shown in Figure 1[2]. The x-axis represents the planning horizon, and the y-axis represents the stations in one line, more concretely, the distance from a certain station to the first one. Figure 1(a) illustrates a regular timetable, in which the headway between consecutive trains and the speeds of different trains between consecutive stations are constant. Figure 1(b) illustrates a non-regular timetable, in which headways and speeds are not necessarily constant and train frequency is normally high around peak hours.

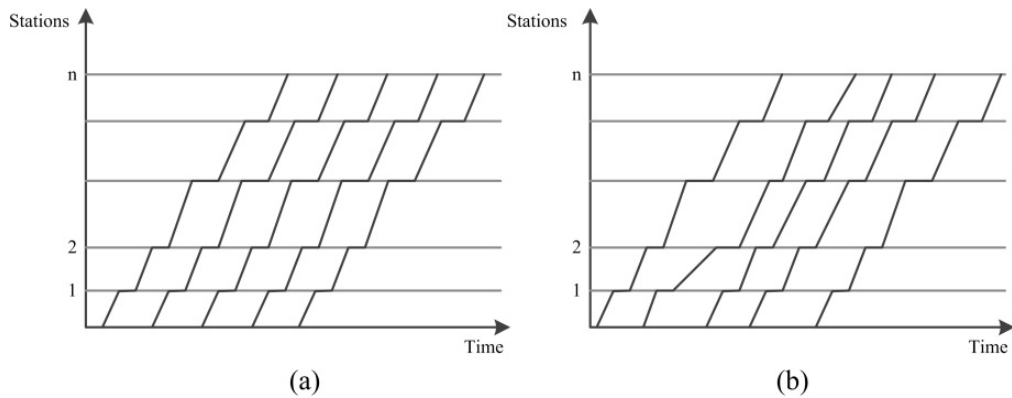


Figure 1 Time-space diagrams of train timetables for a one line metro. (a) Regular and (b) Non-regular timetables. Source: Barrena et al., 2014b.

Let $S = \{1, \dots, n\}$ be the ordered set of stations considered in one line as depicted by the y-axis in Figure 1. Planning horizon is discretized into time intervals of length δ , which can be set discretarily by the necessary precision. Thus, time instant $t \in T = \{0, 1, \dots, p\}$ corresponds to δt time units elapsed since the beginning of the planning horizon as showed by the x-axis in Figure 1. Let $d_{ij}(t)$ be the cumulative demand function from stations i to j before time t , $i, j \in S$, $i < j$. Let $l^{i-1,i}$ be the distance between stations $i-1$ and i , h_{\min} be the minimum headway which refers to the minimum time required between the departure of two consecutive trains at each station, w^i be the dwelling time at the station i , s_{\max} and s_{\min} be the inverse of the maximum and minimum travelling train speeds. Speed inverse is used to avoid problem non-linearity in the constraints. $t_k^{i,i-1}$ is the riding time between adjacent stations, and t_k^{ij} is the riding time between non-adjacent stations i and j . The metro operator has a set $M = \{1, \dots, m\}$ of available trains. The decision variable is x_k^i , which refers to the departure time of train k at station i . All the variables and functions used are listed in Table 1.

The model aims to determine train departure times at stations and speeds on rail segments, respectively, to minimize the sum of passenger AWT at stations and ART between each station. In the following, the demand pattern and reasons for choosing the sum of passenger AWT at stations and ART between each station are presented as objects.

2.1. Demand function

To determine an optimal timetable under dynamic passenger demand, the cumulative demand function for each pair of origin and destination is considered using cumulating function $d_{ij}(t)$. Several authors dealt with variable demand and described it through functions defined from different perspectives. Hurdle [14] starts from a stepwise cumulative demand function and assumes that the arrival pattern of passengers can be reasonably approximated through a differentiable function. Although not required, the algebraic expression of this function can be drawn. Barrena et al. [2] approximated the cumulative demand by using a smooth function as the sum of sigmoidal functions as well.

Given the problem of dynamic passenger demand, the cumulative demand function for each pair of origin and

destination generally exhibits a smooth growth during non-peak hours. However, it will achieve dramatic growth during peak hours. To describe the phenomenon, an *S*-pattern function is proposed as Equation (1) in Figure 2 (a), where K_{ij} , a_{ij} , and b_{ij} are parameters for each pair of origin and destination. Parameters can be achieved through fitting with actual data.

Table 1. Variables and functions associated with urban single-line metro timetable decisions

Variable/function	Description
i	Index of stations
j	Index of stations
k	Index of trains
m	Number of available trains
M	Set of available trains, $M = \{1, \dots, m\}$
n	Number of stations
S	Set of stations, $S = \{1, \dots, n\}$
δ	Unit of time interval
p	Number of time intervals
T	Set of time intervals, $T = \{0, 1, \dots, p\}$
$l^{i-1,i}$	Distance between station $i-1$ and station i , $i \in S$
$s_k^{i-1,i}$	Inverse of traveling speed of train k between station $i-1$ and station i , $i \in S$, $k \in M$
s_{\max}, s_{\min}	Inverse of the maximum and minimum traveling speed of a train
$t_k^{i,i-1}$	Riding time between adjacent stations
$t_k^{i,j}$	Riding time between station i and station j
h_{\min}	Minimum headway
w^j	Dwelling time at station i , $i \in S$
x_k^i	Departure time of train k at station i , $i \in S$, $k \in M$
$d_{ij}(t)$	Cumulative demand function from station i to station j before time t , $i, j \in S$, $i < j$

$$d_{ij}(t) = K_{ij} / (1 + a_{ij} e^{-b_{ij}t}) \tag{1}$$

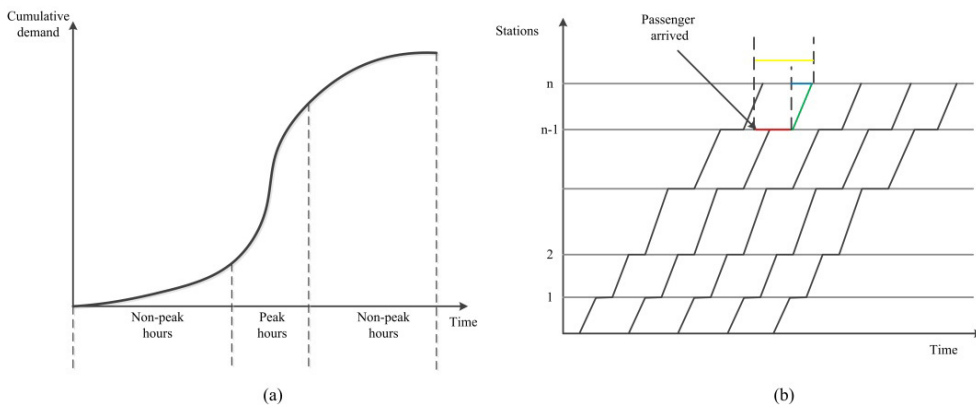


Figure 2 (a) S-pattern cumulative function and (b) Explanation for passenger waiting and riding times.

2.2. Illustration of passenger waiting and riding times

A service is defined as a trip from an origin to a final destination, where a train refers to the service it operates. The train operator can determine the departure time and the speed between stations. The trip can be divided into three stages for each passenger: (1) entering the station and waiting for the train; (2) riding on the train; (3) disembarking from the train and leaving the station.

Apart from the time of the last stage, which consists of disembarking from the train and leaving the station, which are decided by the passengers, most of the time spent during the first stage consisted of waiting for the train, which is decided by the departure time of the next train. Riding the train, which is another large proportion of the time, is decided by the speed of the train between stations. As shown in Figure 2 (b), the arrow marks the moment when passengers arrive at the station and begin to wait for the next train. Until the train departure, the waiting time is marked by the red line. Traveling time between station $n-1$ and station n is marked by the blue line in Figure 2 (b), which is the time between the train departure from station $n-1$ and arrival at station n . Riding time is decided by the speed of train, represented by the green line. Both the waiting and riding times for and on the train, respectively, are decided by the operator, which means that the TTT is decided by the operator, which is represented by the yellow line in Figure 2 (b). The model to minimize the passenger TTT in urban single-line metro is proposed in Section 3.

3. Integer programming model

3.1. Objective function

$$AWT = \left(\frac{1}{2} \sum_{i \in S} \sum_{k \in M \cup (m+1)} \left(\sum_{j \in S, j > i} d_{ij}(x_k^i) - \sum_{j \in S, j > i} d_{ij}(x_{k-1}^i) \right) (x_k^i - x_{k-1}^i) \right) \quad (2)$$

$$ART = \left(\sum_{i \in S} \sum_{j \in S, j > i} \sum_{k \in M} (d_{ij}(x_k^i) - d_{ij}(x_{k-1}^i)) t_k^{ij} \right) \quad (3)$$

$$\min(AWT + ART) \quad (4)$$

As demonstrated in Section 2.2, the sum of AWT and ART should serve as the objective function. The total demand is constant and should not be considered. The objective function (2) is referred to as passenger AWT and function (3) as passenger ART, which aims to minimize the sum of passenger AWT and ART as a function (4).

3.2. Constraints

$$x_0^i = 0, i \in S \quad (5)$$

$$x_{m+1}^i = p, i \in S \quad (6)$$

$$x_k^i + h_{\min} \leq x_{k+1}^i, i \in S, k \in M \setminus \{m\} \quad (7)$$

$$t_k^{i,i-1} = l^{i,i-1} s_k^{i,i-1}, i \in S, k \in M \quad (8)$$

$$s_{\min} \leq s_k^{i-1,i} \leq s_{\max}, i \in S, k \in M \quad (9)$$

$$t_k^{ij} = t_k^{j-1,j}, i+1 = j, k \in M \quad (10)$$

$$t_k^{ij} = \sum_{r=i+1}^j t_k^{r-1,r} + \sum_{q=i+1}^{j-1} w^q, i+1 < j, k \in M \quad (11)$$

$$x_k^i = x_k^{i-1} + t_k^{i-1,i} + w^i, i \in S, k \in M \quad (12)$$

$$\begin{aligned} x_k^i &\in N^+, i \in S, k \in M \\ x_k^i &\in [1, \dots, p], i \in S, k \in M \end{aligned} \quad (13)$$

Constraint (5) defines the first dummy train departure time whose role is to ensure that the objective function takes all waiting times and demands into account. Constraint (6) defines the last dummy train departure time whose role is the same as that of constraint (5). Constraint (7) ensures the departure of two trains from the same station beyond the minimum headway h_{\min} . Constraint (8) defines the relationship of riding time and train speed k between contiguous stations $i-1$ and i . Constraint (9) ensures that the train speed is under the control of the operator. Constraint (10) and (11) defines the passenger's riding time between stations i and j . This time is the sum of the train's run and dwell times between stations i and j , which denote the time period from the train starts running at station i to passengers getting off the train at station j , respectively. This riding time is an important part in the computation of the objective function. Constraint (12) defines the departure time between contiguous stations. The departure time for train k at station i is decided by the departure time for train k at station $i-1$ and the dwell time at station i , of which it is the sum. Given that we discretized the planning horizon into time intervals of length δ , which can be set discretionarily by the necessary precision, constraint (13) ensures x_k^i should be an integer and can only be a value between 1 and p .

4. Spatial branch and bound algorithm

The spatial branch and bound algorithm is an algorithm for solving non-convex NLPs and mixed-integer nonlinear programs; several variants like Dallwig et al. [8], Belottia et al. [3], Smith and Pantelides [18] exist. The spatial branch and bound algorithm used here is of the general form proposed by Horst and Tuy [13].

Most spatial branch and bound algorithms for global optimization conform to a general framework in the following form:

Step 1: Initialization

A list of regions is initialized to a single region that comprises the entire set of variable ranges. The convergence tolerance is set as $\varepsilon > 0$, the best objective function value determined from the current step as $U := \infty$, and the corresponding solution point is $x^* := (\infty, \dots, \infty)$.

Step 2: Choice of Region

If the list of regions is empty, the algorithm is terminated with solution x^* and objective function value U . Otherwise, a region R is selected from the list. R is deleted from the list. Feasibility-based bounds tightening is performed on R .

Step 3: Lower Bound

Convex relaxation of the original problem is generated in the selected region R and solved to obtain an underestimation l of the objective function with corresponding solution \bar{x} . If $l > U$ or the relaxed problem is infeasible, then Step 2 is performed again.

Step 4: Upper Bound

An attempt is made to solve the original problem in the selected region to obtain a (locally optimal) solution \tilde{x} with objective function value u . If it fails, then $u := U$ and $\tilde{x} = X^0$ is set.

Step 5: Pruning

If $u < U$, then $x^* = \tilde{x}$ and $U := u$ are set. All regions with lower bounds bigger than U in the list are deleted as they cannot possibly contain the global minimum.

Step 6: Check Region

If $u - l \leq \varepsilon$, then u is accepted as the global minimum for this region and Step 2 is performed again. Otherwise, the next step is performed because the region global minimum may not have been located.

Step 7: Branching

A branching rule is applied to the current region to split it into sub-regions. The sub-regions are added to the list of regions, and they are assigned an (initial) lower bound of l . Step 2 is then performed again.

5. Computational experiments

A series of implementation specifications is provided through cases used, and results of extensive computational experiments are presented in Section 5.2. Barrena et al. [2] proposed an optimizing timetable model that uses AWT as an objective. This AWT objective is adopted to run computational experiments using our dataset with our model to compare each model's advantages and disadvantages. The regular timetable's AWT and ART are computed for comparison purposes as well.

5.1. Set of cases

The set of cases is generated according to the following parameters:

- Number of stations n : 3, 4, 5;
- Horizon p : 120, 240, 480 min;
- Discretization constant δ : 1, 2 min;
- Numbers of trains: 3, 4, 5;
- Fixed dwelling time: 2 min;
- Maximum inverse speed of the trains s_{\max} : 3 min/km;
- Minimum inverse speed of the trains s_{\min} : 1 min/km;
- Minimum headway h_{\min} : 2min.

The distance parameters between each station and dwell time at each station, as well as the other data necessary in the problem are obtained from the Changping line of the Beijing metro. Passenger demand of the urban single-line metro used in the computational experiments indicates a pattern similar to that in Figure 2 between each pair of origin and destination.

These cases will be referred to as TT-n-p-d-m as Barrena et al.[2], e.g., TT-3-120-2-5, corresponding to a train timetable case with three stations: a planning horizon of 120 min, a discretization constant of 2 min, and a maximum of 5 trains.

5.2. Computational results

The summary of computational results for each improvement between the regular and the optimized AWT+ART and AWT timetables on all cases are presented in Table 2, 8, and 6, respectively.

As Table 2, 3, and 4 shows, for cases across three stations, the proposed model for optimizing TTT reduced AWT, ART, and AWT+ART by 78.19%, 36.19%, and 68.05%, respectively, compared with the regular timetable. Furthermore, it can reduce 4.50% ART and 1.57% AWT+ART although it increases 1.81% AWT compared with the optimized AWT timetable. For four stations scenario, the proposed model for optimizing the TTT can reduce AWT, ART, and AWT+ART by 72.89%, 38.11%, and 62.86%, respectively, compared with the regular timetable. Furthermore, it can reduce 12.70% ART and 4.04% AWT+ART compared with the optimized AWT timetable although it increases 8.61% AWT. For five stations scenario, the proposed model can reduce AWT, ART, and AWT+ART by 62.07%, 43.74%, and 56.30%, respectively, compared with the regular timetable. Although it increases 16.43% AWT, it can reduce 27.13% ART and 8.58% AWT+ART compared with the optimized AWT timetable.

Table 2. Improvement between the regular, optimized AWT, and optimized AWT+ART timetables on cases across three stations (Unit: %)

Case	AWT+ART VS Regular Improvement			AWT+ART VS AWT Improvement		
	AWT	ART	AWT+ART	AWT	ART	AWT+ART
TT-3-120-1-3	64.79	38.11	55.80	-1.24	1.52	0.08
TT-3-120-2-3	64.71	38.63	55.93	-1.12	3.35	1.03
TT-3-240-1-3	80.68	38.90	72.11	-2.06	14.57	6.15
TT-3-240-2-3	80.84	37.62	71.98	0.01	13.33	6.57
TT-3-480-1-3	89.87	38.72	83.99	-0.91	10.94	4.67
TT-3-480-2-3	89.69	39.90	83.97	-2.73	3.74	0.16
TT-3-120-1-4	64.20	35.69	52.76	-1.70	1.19	-0.10
TT-3-120-2-4	64.29	35.86	52.88	0.00	0.00	0.00
TT-3-240-1-4	80.57	35.69	69.13	-2.76	1.70	-0.34
TT-3-240-2-4	80.70	35.49	69.17	-1.06	2.73	0.99
TT-3-480-1-4	89.96	35.40	81.92	-0.96	2.74	1.03
TT-3-480-2-4	89.89	35.44	81.87	-1.60	2.78	0.74
TT-3-120-1-5	62.34	34.47	49.70	4.52	0.06	1.93
TT-3-120-2-5	64.07	33.88	50.38	-2.32	2.14	0.42
TT-3-240-1-5	81.88	33.29	67.37	0.15	8.86	5.66
TT-3-240-2-5	80.03	33.68	66.19	-5.22	2.88	-0.32
TT-3-480-1-5	89.61	35.34	79.99	-5.90	3.80	-0.11
TT-3-480-2-5	89.30	35.33	79.73	-7.76	4.75	-0.31
Average	78.19	36.19	68.05	-1.81	4.50	1.57

Table 3. Improvement between the regular, optimized AWT, and optimized AWT+ART timetables on cases across four stations (Unit: %)

Case	AWT+ART VS Regular Improvement			AWT+ART VS AWT Improvement		
	AWT	ART	AWT+ART	AWT	ART	AWT+ART
TT-4-120-1-3	56.53	42.99	51.07	-8.07	20.26	7.40
TT-4-120-2-3	59.50	38.06	50.85	-1.41	4.83	1.86
TT-4-240-1-3	75.34	44.34	67.21	-9.54	17.71	4.54
TT-4-240-2-3	75.16	44.40	67.08	-10.32	11.30	0.44
TT-4-480-1-3	87.66	40.54	80.38	-1.72	16.42	7.66
TT-4-480-2-3	87.45	40.49	80.19	-3.19	23.87	11.42
TT-4-120-1-4	56.90	37.58	47.89	-8.02	8.69	2.00
TT-4-120-2-4	55.97	38.06	47.62	-9.61	9.08	1.55
TT-4-240-1-4	76.15	36.73	63.62	-6.17	6.11	0.99
TT-4-240-2-4	75.80	37.05	63.48	-7.38	13.95	5.47
TT-4-480-1-4	86.94	38.74	77.57	-8.70	15.76	5.82
TT-4-480-2-4	87.27	37.05	77.50	-4.92	4.79	0.60
TT-4-120-1-5	53.88	32.04	42.65	-11.04	7.30	0.91
TT-4-120-2-5	57.40	34.17	45.46	-6.49	3.93	0.22
TT-4-240-1-5	71.34	38.96	59.54	-27.44	18.47	2.68
TT-4-240-2-5	77.22	31.37	60.52	-0.97	11.61	7.38
TT-4-480-1-5	85.33	36.33	74.05	-16.20	14.76	3.58
TT-4-480-2-5	86.12	37.01	74.81	-13.81	19.67	8.22
Average	72.89	38.11	62.86	-8.61	12.70	4.04

Table 4 Improvement between the regular, optimized AWT, and optimized AWT+ART timetables on cases across five stations (Unit: %)

Case	AWT+ART VS Regular Improvement			AWT+ART VS AWT Improvement		
	AWT	ART	AWT+ART	AWT	ART	AWT+ART
TT-5-120-1-3	37.14	51.96	43.78	-10.22	42.83	18.68
TT-5-120-2-3	35.80	54.08	43.99	-11.66	46.17	19.93
TT-5-240-1-3	57.39	56.73	57.18	-14.45	45.95	15.20
TT-5-240-2-3	57.39	56.73	57.18	-14.56	48.33	17.04
TT-5-480-1-3	73.35	60.38	70.87	-17.40	50.57	13.54
TT-5-480-2-3	73.65	58.83	70.81	-16.09	48.08	12.95
TT-5-120-1-4	48.77	38.54	43.61	-15.46	24.21	10.34
TT-5-120-2-4	53.01	34.56	43.70	-4.63	12.58	6.20
TT-5-240-1-4	68.64	40.52	58.34	-19.19	32.09	14.54
TT-5-240-2-4	70.46	38.19	58.65	-10.11	26.47	13.45
TT-5-480-1-4	84.26	35.91	72.82	-8.53	11.97	3.94
TT-5-480-2-4	83.61	38.12	72.85	-10.43	11.36	2.49
TT-5-120-1-5	44.54	34.20	38.90	-8.90	11.37	4.00
TT-5-120-2-5	45.75	35.38	40.09	-18.32	7.05	-1.94
TT-5-240-1-5	64.00	36.40	52.65	-29.38	16.78	0.97
TT-5-240-2-5	62.30	38.38	52.46	-19.40	11.37	-0.75
TT-5-480-1-5	76.28	40.60	66.45	-45.70	30.29	4.88
TT-5-480-2-5	81.00	37.90	69.12	-21.24	10.90	-1.04
Average	62.07	43.74	56.30	-16.43	27.13	8.58

6. Conclusions

A model on an urban single-line metro timetable problem for optimizing total passenger travel time TTT under dynamic passenger demand was proposed. Given that the model is an integer nonlinear program, spatial branch and bound algorithm are applied to the model. Extensive computational experiments on the cases show that the model improved AWT by around 70%, ART by around 40%, and TTT by around 60% compared with the regular timetable. Compared with the optimized AWT timetable, our model improved ART by around 15% and TTT by around 5%. The model that aims to optimize total passenger travel time TTT that was measured by the sum of passenger AWT and ART is reasonable and equitable for all passengers. Thus, this proposal can be adopted by operators to increase the performance of an urban metro system.

Acknowledgement

This research was partially supported by the Hang Lung Center for Real Estate, Tsinghua University.

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