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APPLICATION OF MAGNETIC FIELDS TO SYNOVIAL JOINTS

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Abstract—In order to assess the possibility of applications of the applied magnetic fields on physiological systems, a simple model for knee joints subjected to the applied magnetic field is presented in this paper. The model consists of two parallel porous cartilageous surfaces separated by a thin film of non-Newtonian lubricant representing the synovial fluid, which is assumed to behave like a paramagnetic fluid system. The upper surface normally approaches the lower one which is maintained at rest. A transverse magnetic field is applied to the system. The approximate solutions of the governing equations occur in three regions: the two porous matrices and a thin fluid film are obtained by introducing perturbation method. It has been observed that the suitably designed applied magnetic fields may possibly help in normal articulation particularly in diseased states. It helps in sustaining greater loads and in decreasing the coefficient of friction. Other results depending on the parameters involved in the analysis have also been brought out and discussed.

INTRODUCTION

The load bearing synovial joint is a self-acting dynamically loaded bearing which employs two poroviscoelastic bearing materials (articular cartilage) and a highly non-Newtonian lubricant (synovial fluid). Dowson [1] showed that a squeeze film action is capable of providing considerable protection to the cartilage surfaces once a fluid film is generated. Furthermore, Fein [2] discussed the possibility of increasing concentration of hyaluronic acid molecules in synovial fluid due to the squeezing sideways and the filtration of the base fluid into the porous cartilage during articulation.

Dowson *et al.* [3] also suggested that best squeeze film models may be given by parallel surfaces. McCutchen [4] and Maroudas [5] have discussed the non-Newtonian nature of the synovial fluid. Thus, the fluid under consideration must include the salient properties of the synovial fluid [6]. The viscoelastic fluids exhibit non-Newtonian flow properties observed in the flow of suspended deformable particles in viscous fluid as considered by Oldroyd [7] and Mow and Lai [8]. Tanner [9] showed that the movement of the suspending medium of synovial fluid into and out of the cartilage contributes considerably to its damped elasticity or viscoelasticity. The parameters of the constitutive equations for viscoelastic fluids representing the synovial fluid have also been estimated for normal, old and diseased systems [10].

Anatomy and physiology of the synovial joints cover the structure and functions of the human joints. Mechanics of flow, heat and mass transfer, and reaction kinetics play a unique role in bio-lubrication of synovial joints. Electric and magnetic fields both cause the flow of current in the tissue (cartilage) as well as through the lubricant [11]. The electric field within the body, though much weaker than the external fields, represents the flow of current in the conducting body tissues and alters physiological properties of the system. Magnetic fields pass through the

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body as if it were transparent, but induce currents which flow in circuitous paths. Biological effects due to the imposition of external fields have been observed in a large number of systems over a wide range of field strength [12]. The magnetic forces originate within the microstructures (suspended particles), which attempt to slip relative to the suspending medium and thereby transmit drag to the fluid, causing dispersion to move as a whole.

Recently, a magneto-hydrodynamic theory of lubrication has attracted considerable attention. Hughes and Elco [13,14] discussed the problem of thrust bearings under the action of an externally applied transverse magnetic fields. Later Hughes [15] considered the problem of slider bearings with uniform transverse magnetic fields. Most of the materials used extensively as rubbing surface materials are paramagnetic, ferromagnetic or diamagnetic. The lubricating performance of the lubricant is enhanced by applying a magnetic field to rubbing surfaces separated by a lubricant behaving like a paramagnetic fluid system [16]. In a recent article, Zahn and Rosenweig [17] described dynamics of magnetic fluids through porous media. They gave a modified form of Darcy's law, suitable for magnetic fluids. As a result, a body force $[\mu_0(M.\nabla)H]$ arises in the magnetic fluid due to the application of an external magnetic field H, where μ_0 and M are free space permeability and magnetization, respectively.

The externally applied magnetic field has, therefore, considerable effects on biological systems. It is believed that the applied magnetic field stimulates the functions of the various physiological systems and also regenerates the tissues in the body [18].

Applications of magnetic fields in artificial human joints have been recently developed through the models incorporating bone in-growth into the porous implants. Such models came out with remarkable success [19,20]. An applied magnetic field, if suitably designed, accelerates bone in-growth into the implants and simultaneously increases the load carrying capacity with the reduction of friction. It has been established that human joints, in general, are greatly affected by the application of external magnetic fields. Such models incorporate an upper poroelastic cartilageous surface approaching the lower one with a velocity $(-\frac{dh'}{dt})$ separated by the lubricant (synovial fluid) in the intra-articular gap, and the whole system is subjected to a transverse magnetic field.

Thus, extending the concept of two-dimensional squeeze film lubrication of the two approaching porous surfaces [21–23] subjected to the applied magnetic field, an attempt has been made in this paper to study the possible effects of applied magnetic fields. Particularly, in the case of arthritic synovial fluids which sustain less loads, the possibility of increasing load carrying capacity has been investigated due to the application of applied magnetic fields following Verma [24].

FORMULATION OF THE PROBLEM

Although, in general, load bearing synovial joints behave more or less the same way during articulation but, in order to draw some specific conclusion, we propose to study the effects of applied magnetic fields on knee joints. Figure 1 represents the simplified geometrical counter part of a knee joint for the purpose of analytical treatment. The joint consists of bone, cartilage and the synovial fluid. The joint surfaces are porous and separated by a fluid-film of thickness h'. A transverse magnetic field is applied externally. The synovial fluid is represented by viscoelastic fluid. The proposed model consists of two regions: (a) Fluid film region and (b) Porous matrix representing the cartilage.

The governing equations are described below for each region:

(a) Fluid Film Region

The synovial fluids in between the two approaching porous cartilageous surfaces has been represented by the Oldroyd model [7]. Introducing the well known lubrication assumptions [25], the governing equations of motion in the fluid film region may be written in the following form:

$$-\frac{\partial p'}{\partial x'} + \mu\left(\frac{\partial^2 u'}{\partial y'^2} - 3(\lambda_1 - \lambda_2)\eta\left(\frac{\partial u'}{\partial y'}\right)^2\frac{\partial^2 u'}{\partial y'^2}\right) + \mu_0(M.\nabla)H = 0, \quad (1)$$

and

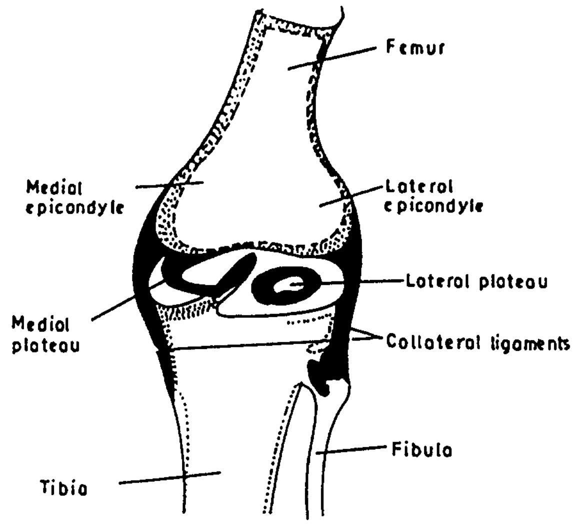

Figure 1(a). Human knee joint.

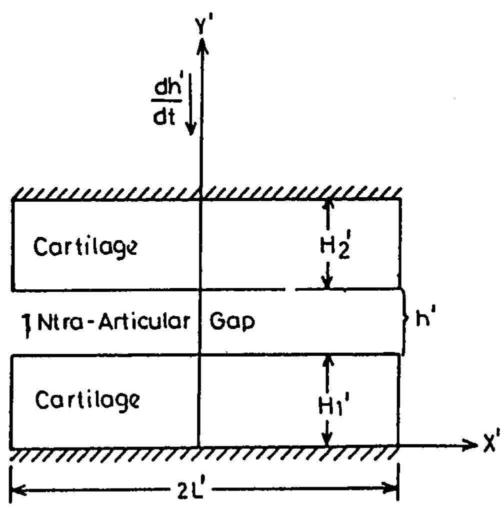

Figure 1(b). Geometrical counter-part of the knee joint.

$$-\frac{\partial p'}{\partial y'} + \mu_0(M.\nabla)H = 0. \tag{2}$$

The equation of continuity is given by

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0, \tag{3}$$

where $(u', v', 0)$ are the components of the velocity vector $\bar{v}$. $\overrightarrow{M}$ is the magnetization vector and $\overrightarrow{H}$ is the induced magnetic field. p' is the pressure in the film, μ is the coefficient of the viscocity, μ_0 is the free space permeability, λ_1 and λ_2 are the viscoelastic parameters.

The well known Maxwell's equations are reduced to the following forms:

$$\nabla.\overrightarrow{B} = 0, \tag{4}$$

$$\overrightarrow{B} = \mu_0(\overrightarrow{H} + \overrightarrow{M}), \tag{5}$$

$$\overrightarrow{M} = \bar{\mu}\,\overrightarrow{H}, \tag{6}$$

$$\mu_r = 1 + \bar{\mu}, \tag{7}$$

and

$$\nabla \times \overrightarrow{H} = 0, \tag{8}$$

where $\bar{\mu}$ is the constant magnetic permeability and μ_r is the relative magnetic permeability. $\overrightarrow{B}$ is the magnetic induction vector.

Assuming that the applied magnetic field is of the form:

$$\begin{aligned} &H = H(x') \quad \text{then} \\ &M = \bar{\mu}\, H \end{aligned} \tag{9}$$

Hence, we write

$$\begin{aligned} &M\,\frac{\partial H}{\partial x'} = \frac{1}{2}\,\bar{\mu}\,\frac{\partial H^2}{\partial x'} \\ &\frac{1}{2}\,\bar{\mu}\,\frac{\partial H^2}{\partial y'} = 0 \end{aligned} \tag{10}$$

Using Equation(10), the Equations (1) and (2) transform to

$$-\frac{\partial p'}{\partial x'} + \mu\left(\frac{\partial^2 u'}{\partial y'^2} - 3(\lambda_1 - \lambda_2)\,\eta\left(\frac{\partial u'}{\partial y'}\right)^2 \frac{\partial^2 u'}{\partial y'^2}\right) + \frac{1}{2}\,\mu_0\,\bar{\mu}\,\frac{\partial H^2}{\partial x'} = 0 \tag{11}$$

$$-\frac{\partial p'}{\partial y'} + \frac{1}{2}\,\bar{\mu}\,\mu_0\frac{\partial H^2}{\partial y'} = 0 \quad \text{or} \quad \frac{\partial p'}{\partial y'} = 0. \tag{12}$$

Defining modified pressure P' as

$$P' = p' - \frac{1}{2}\,\mu_0\,\bar{\mu}\,H^2, \tag{13}$$

Equations (11) and (12) transform into the following equations:

$$-\frac{\partial P'}{\partial x'} + \mu\left[\left(\frac{\partial^2 u'}{\partial y'^2}\right) - 3\eta\,(\lambda_1 - \lambda_2)\left(\frac{\partial u'}{\partial y'}\right)^2 \left(\frac{\partial^2 u'}{\partial y'^2}\right)\right] = 0 \tag{14}$$

and

$$\frac{\partial P'}{\partial y'} = 0. \tag{15}$$

(b) Porous Matrix

Introducing Darcy's law, the relative velocity components $\bar{u}'$ and $\bar{v}'$ may be defined as

$$\begin{aligned} \bar{u}' &= -\frac{\varnothing'}{\mu}\,\frac{\partial \bar{p}'}{\partial x'}, \\ \bar{v}' &= -\frac{\varnothing'}{\mu}\,\frac{\partial \bar{p}'}{\partial y'}, \end{aligned} \tag{16}$$

which satisfy the Laplace equation in two dimensions:

$$\frac{\partial^2 \bar{p}'}{\partial x'^2} + \frac{\partial^2 \bar{p}'}{\partial y'^2} = 0. \tag{17}$$

Therefore, the governing equations in the two porous regions are given below:

(a) In lower matrix

$$\frac{\partial^2 \bar{p}_1'}{\partial x'^2} + \frac{\partial^2 \bar{p}_1'}{\partial y'^2} = 0. \tag{18}$$

(b) In upper matrix

$$\frac{\partial^2 \bar{p}_2'}{\partial x'^2} + \frac{\partial^2 \bar{p}_2'}{\partial y'^2} = 0. \tag{19}$$

BOUNDARY CONDITIONS

To solve Equations (1)–(3), the appropriate mathematical forms of the boundary and matching conditions are given below:

(1) For Fluid Film Region

$$\begin{aligned}
u' &= -L_1' \ \frac{\partial u'}{\partial y'} && \text{at } y' = H_1', \\
u' &= -L_2' \ \frac{\partial u'}{\partial y'} && \text{at } y' = h' + H_1', \\
v' &= -v_1' && \text{at } y' = H_1', \\
v' &= \bar{v}_2' - \frac{dh'}{dt} && \text{at } y' = H_1' + h', \\
P' &= 0 && \text{at } x' = \pm L', \\
\frac{\partial p'}{\partial x'} &= 0 && \text{at } x' = 0.
\end{aligned} \tag{20}$$

(2) For Lower Porous Matrix

$$\begin{aligned}
\frac{\partial P_1'}{\partial y'} &= 0 && \text{at } y' = 0, \\
P_1' &= 0 && \text{at } x' = \pm L', \\
\frac{\partial \bar{P}_1'}{\partial x'} &= 0 && \text{at } x' = 0.
\end{aligned} \tag{21}$$

(3) For Upper Porous Matrix

$$\begin{aligned}
\frac{\partial \bar{P}_2'}{\partial y'} &= 0 && \text{at } y = h' + H_1' + H_2' \\
\bar{P}_2 &= 0 && \text{at } x' = \pm L', \\
\frac{\partial \bar{P}_2'}{\partial x'} &= 0 && \text{at } x'. = 0
\end{aligned} \tag{22}$$

(4) Matching Conditions

$$\begin{aligned}
P'(x') &= \bar{P}_1'(x', H_1'), \\
P'(x') &= \bar{P}_2'(x', h' + H_1'),
\end{aligned} \tag{23}$$

where L_1' and L_2' are parameters associated with slip velocity at the porous boundary, $\bar{v}_1'$ and $\bar{v}_2'$ are the components of the relative fluid velocity of the porous matrix.

SOLUTION OF THE PROBLEM

Introducing the following dimensionless variables and parameters (Collins [26]):

$$\begin{aligned}
&x = \frac{x'}{h_0},\ y = \frac{y'}{h_0},\ u = \frac{u'}{U_0},\ v = \frac{v'}{U_0},\ L_1 = \frac{L_1'}{h_0}, L_2 = \frac{L_2'}{h_0},\\
&\bar{P}_1 = \frac{\bar{P}_1'}{\rho U_0{}^2},\ \bar{P}_2 = \frac{\bar{P}_2'}{\rho U_0^2},\ N_1 = Re\frac{\varnothing_1}{h_0^2},\ N_2 = Re\frac{\varnothing_2}{h_0^2},\ L = \frac{L'}{h_0},\\
&P = \frac{P'}{\rho U_0^2},\ H_1 = \frac{H_1'}{h_0},\ \varepsilon = (\lambda_1 - \lambda_2)\eta\frac{U_0^2}{h_0^2},\ Re = \frac{\rho U_0 h_0}{\mu},\\
&H_2 = \frac{H_2'}{h_0},\ h = \frac{h'}{h_0},
\end{aligned} \tag{24}$$

where U_0 is the characteristic velocity and h_0 is the initial film thickness of porous pad, ρ is the mass density of the fluid, ε is the viscoelastic parameter, Re is the Reynold's number, $\varnothing_i (i = 1, 2)$ is the permeability of the porous cartilage.

The momentum equations within the intra-articular gap may be written in the dimensionless form:

$$\frac{\partial P}{\partial x} Re = \frac{\partial^2 u}{\partial y^2} - 3\varepsilon \left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^2 u}{\partial y^2}, \tag{25a}$$

and

$$\frac{\partial P}{\partial y} = 0. \tag{25b}$$

The equation of continuity may be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{26}$$

In the porous matrices, we obtain the second order Laplace differential equations for pressure distribution in non-dimensional form:

$$\nabla^2 \bar{P}_i = 0, \qquad i = 1, 2. \tag{27}$$

To obtain solutions for the fluid film region, perturbation technique is introduced, which is based on the following assumptions:

(i) The solutions are valid for small values of ε.
(ii) In the limiting case of $\varepsilon \to 0$, the corresponding solutions for viscous lubricants are derivable from the approximate solutions so obtained. The variables are assumed in a sequence of the functions in terms of the small parameters ε:

$$u = u_0 + \sum_{k=1}^{\infty} \varepsilon^k u_k, \tag{28}$$

where u_0 is the limiting solution for viscous fluids as $\varepsilon \to 0$.

The transformed non-dimensional boundary and matching conditions are given below:

$$
\begin{aligned}
&u = -L_1 \frac{\partial u}{\partial y} && \text{at} \quad y = H_1, \\
&u = -L_2 \frac{\partial u}{\partial y} && \text{at} \quad y = h + H_1, \\
&v = \bar{v}_2 - 1 && \text{at} \quad y = h + H_1, \\
&v = -\bar{v}_1 && \text{at} \quad y = H_1, \\
&P = 0 && \text{at} \quad x = \pm L, \\
&\frac{\partial \bar{P}_1}{\partial y} = 0 && \text{at} \quad y = 0, \\
&P = \bar{P}_1(x, H_1), && \\
&\bar{P}_1 = 0 && \text{at} \quad x = \pm L, \\
&\frac{\partial \bar{P}_2}{\partial y} = 0 && \text{at} \quad y = h + H_1 + H_2, \\
&P = \bar{P}_2(x, h + H_1), && \\
&\bar{P}_2 = 0 && \text{at} \quad x = \pm L, \\
&\frac{\partial \bar{P}_2}{\partial x} = 0 && \text{at} \quad x = 0, \text{ and} \\
&\frac{\partial P_1}{\partial x} = 0 && \text{at} \quad x = 0, \frac{\partial P}{\partial x} = 0 \text{ at } x = 0.
\end{aligned}
\tag{29}
$$

The zeroth and first order solutions of Equations (25) and (26) with the help of boundary conditions (29) are given below:

$$
u_0 = Re \frac{\partial P_0}{\partial x} \left(\frac{y^2}{2} + K_1 y - K_2 \right), \tag{30}
$$

$$
u_1 = Re \frac{\partial P_1}{\partial x} \left(\frac{y^2}{2} + K_1 y - K_2 \right) + \left(Re \frac{\partial P_0}{\partial x} \right)^3 \left(K_3 y - K_4 + \frac{(y + K_1)}{4} \right)^4, \tag{31}
$$

where

$$
\begin{aligned}
u &= u_0 + \varepsilon u_1, \\
K_1 &= \frac{(h/2)(h + 2H_1) + L_2(h + H_1) - L_1 H_1}{L_1 - h - L_2}, \\
K_2 &= \frac{H_1^2}{2} + L_1 H_1 + K_1(H_1 + L_1), \\
K_3 &= \frac{(h + K_1 + H_1)^4 - (H_1 + K_1)^4 + L_2(h + H_1 + K_1)^3 - 4L_1(H_1 + K_1)^3}{4(L_1 - h - L_2)}, \\
K_4 &= K_3(H_1 + L_1) + \frac{(H_1 + K_1)^4}{4} + L_1(H_1 + K_1)^3,
\end{aligned}
$$

we obtain the hydrostatic pressure in the porous region by solving the Laplace equation (27) and using the boundary condition described in (29):

$$
\begin{aligned}
\bar{P}_1 &= \sum_{n=0}^{\infty} 2B_n \cos(\lambda_n x) \cos h(\lambda_n y), \\
\bar{P}_2 &= \sum_{n=0}^{\infty} A_n \cos(\lambda_n x) R_1(y),
\end{aligned}
\tag{32}
$$

where

$$R_1(y) = \exp(-\lambda_n(2(h+H_1+H_2)-y)) + \exp(-\lambda_n\, y)$$
$$\lambda_n = \frac{(2n+1)\pi}{2L}, \qquad n = 0,1,2\dots.$$

The constants A_n and B_n are obtained by using the Fourier cosine series expansions of the function involved:

$$A_n = \frac{2\,B_n \cos(\lambda_n\, H_1)}{R_1(h+H_1)}, \tag{33}$$
$$B_n = \frac{(-1)^n}{A\,L\,Re\,K_5\,\lambda_n^3}, \tag{34}$$

where

$$R_1(h+H_1) = \exp(-\lambda_n(h+H_1+2H_2)) + \exp(-\lambda_n(h+H_1)),$$
$$R_2(h+H_1) = \exp(-\lambda_n(h+H_1+2H_2)) - \exp(-\lambda_n(h+H_1)),$$
$$A = \frac{1}{Re\,K_5\,\lambda_n}\left[(-2N_1 \sin h(\lambda_n\, H_1) + N_2 \cos h(\lambda_n\, H_1)\,\frac{R_2(h+H_1)}{R_1(h+H_1)} - \cos(\lambda_n\, H_1)\right],$$

and

$$K_5 = \frac{1}{6}\,h(h^2 + 3H_1^2 + 3h\,H_1) + \frac{K_1 h}{2}(h+2H_1) - K_2 h.$$

The normal components of the relative fluid velocity at the porous matrix are given below:

$$\begin{aligned}\bar{v}_1 &= -N_1\,\frac{\partial \bar{p}_1}{\partial y}\bigg|_{y=H_1},\\ \bar{v}_2 &= -N_2\,\frac{\partial \bar{p}_2}{\partial y}\bigg|_{y=h+H_1}.\end{aligned} \tag{35}$$

Hence,

$$\bar{v}_1 = -N_1 \sum_{n=0}^{\infty} 2B_n\,\lambda_n \cos(\lambda_n\, x)\,\sin(\lambda_n\, H_1),$$
$$\bar{v}_2 = -N_2 \sum_{n=0}^{\infty} A_n\,\lambda_n \cos(\lambda_n\, x)\,R_2(h+H_1),$$

and

$$R_2(h+H_1) = \exp(-\lambda_n(h+H_1+2H_2)) - \exp(-\lambda_n(h+H_1)).$$

PRESSURE DISTRIBUTION

The expression for pressure distribution in non-dimensional form is given by:

$$\begin{aligned}p =& \frac{1}{2}\,\bar{\mu}\,\mu_0\left(1-\frac{x^2}{L^2}\right)H_0^2 + \frac{1}{2\,Re\,K_5}\,(x^2-L^2) - \frac{K_6\epsilon}{4\,Re\,K_5^4}(x^4-L^4)\\ &+ \frac{1}{Re\,K_5}\sum_{n=0}^{\infty}\frac{M_n}{\lambda_n}\cos(\lambda_n\, x)\\ &+ \frac{6\,K_6\,\varepsilon}{Re\,K_5^4}\sum_{n=0}^{\infty}\frac{\cos(\lambda_n\, x)}{\lambda_n^3}\,M_n - \frac{K_6\,\varepsilon}{Re\,K_5^4}\left(\sum_{n=0}^{\infty} 3x^2\cos(\lambda_n\, x)\,.\frac{M_n}{\lambda_n}\right.\\ &\left.-\sum_{n=0}^{\infty}\frac{6x\sin(\lambda_n\, x).M_n}{\lambda_n^2}\right) - \frac{6\,K_6\,L\,\varepsilon}{Re\,K_5^4}\sum_{n=0}^{\infty}(-1)^n\frac{M_n}{\lambda_n^2},\end{aligned} \tag{36}$$

where

$$K_6 = \frac{K_3 h}{2}(h + 2H_1) - K_4 h + \frac{(h + H_1 + K_1)^5 - (H_1 + K_1)^5}{20},$$
$$M_n = -2\, N_1\, B_n \sin h(\lambda_n\, H_1) - N_2\, A_n\, R_1(h + H_1),$$

and

$$H^2 = \left(1 - \frac{x^2}{L^2}\right) H_0^2.$$

LOAD CARRYING CAPACITY

The expression for the load carrying capacity is given by

$$\begin{aligned} W = 2 \Bigg(& \frac{1}{3}\bar{\mu}\,\mu_0\, H_0^2\, L + \frac{K_6\, \varepsilon\, L^5}{5\, Re\, K_5^4} - \frac{L^3}{3\, Re\, K_5} \\ & + \sum_{n=0}^{\infty} \frac{M_n}{\lambda_n^2}(-1)^n \left(\frac{-3\, L^2\, K_6\, \varepsilon}{Re\, K_5^4} + \frac{1}{Re\, K_5} \right. \\ & \left. \left. - \frac{6\, K_6\, L\, \varepsilon}{Re\, K_5^4} + \frac{6\, K_6\, \varepsilon}{Re\, K_5^4\, \lambda_n^2} \right) \right). \end{aligned} \tag{37}$$

RESULTS AND DISCUSSION

In this paper, the possiblity of increased efficiency of joint articulation, particularly in diseased states, by the application of applied magnetic fields has been explored. A simple model of idealized knee joints subjected to an external magnetic field has been presented above.

In order to assess the effects of the parameters involved in the analysis, the following values of the magnetic parameters have been introduced for the synovial fluid for computational purposes:

$$\mu_0 = 4\pi \times 10^{-7} \frac{H}{m}, \quad H_0 = 10^7 \frac{A}{m^2},$$

and $\mu_r = 1 - 9 \times 10^{-6}$ as mentioned by Jordon and Balmain [27]. Such higher values are acceptable for requirements of any viable positive effects.

Figure 2 describes the local pressure distribution in the fluid film region for different values of the external magnetic field (H_0). The applied magnetic field increases the pressure distribution, which, in turn, increases the load carrying capacity as may be observed in Figure 5. One may conclude from this that applied external fields are useful, particularly in diseased states when the load carrying capacity decreases due to the structural changes in synovial fluid and hyaluronic acid concentration. The bio-lubrication ability of the lubricant also changes.

Similarly, Figure 3 depicts local pressure variation within the intra-articular gap with viscoelastic parameter. The viscoelastic parameter has the tendency to increase the pressure distribution. The increasing values of the viscoelastic parameter may be related to the increaseing values of the hyaluronic acid concentration and the viscosity of the lubricant and, therefore, the rise in pressure with the viscoelastic parameter is in order as observed by Tandon *et al.* [23,28].

Figure 4 describes the pressure distribution. As the gap decreases, the pressure at a point increases tremendously owing to the closure in the intra-articular gap. It has been observed by Tandon *et al.* [21–23,28] that as the gap closes, the base fluid squeezes out sideways and the concentration of the hyaluronic acid molecules in the intra-articular gap increases. This, in turn, increases the viscosity of the synovial fluid and, therefore, the rise in pressure is obvious, as depicted in Figure 4. This naturally enhances the load carrying capacity. This enrichment of the concentration of the hyaluronic acid molecules within the intra-articular gap, as it closes, has been observed by several authors both experimentally and theoretically [29,30].

Figures 5 and 6 describe the variation of load carrying capacity with the gap (h) for different values of the applied magnetic field (H_0) and the viscoelastic parameter (ε), respectively. One

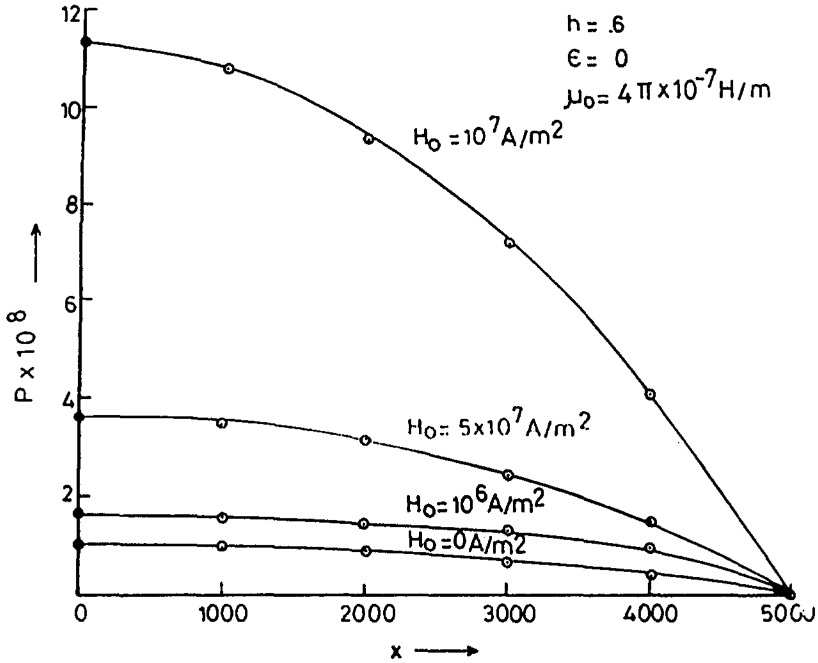

Figure 2. Variation of non-dimensional pressure distribution with axial distance for different H_0.

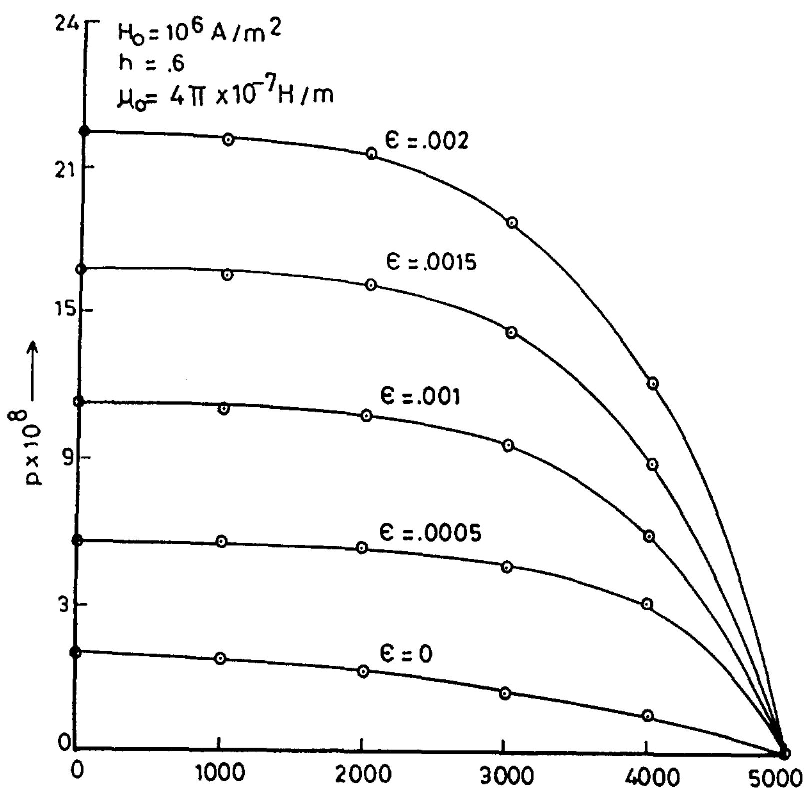

Figure 3. Variation of non-dimensional pressure distribution with axial distance for different ϵ.

may infer from this that, particularly in diseased states, the applied magnetic field helps in sustaining greater loads. Similarly, if the viscoelastic parameter increases the load carrying capacity also increases. The increasing values of the viscoelastic parameter describe the increase in the concentration of the suspended hyaluronic acid molecules which, in turn, increase the

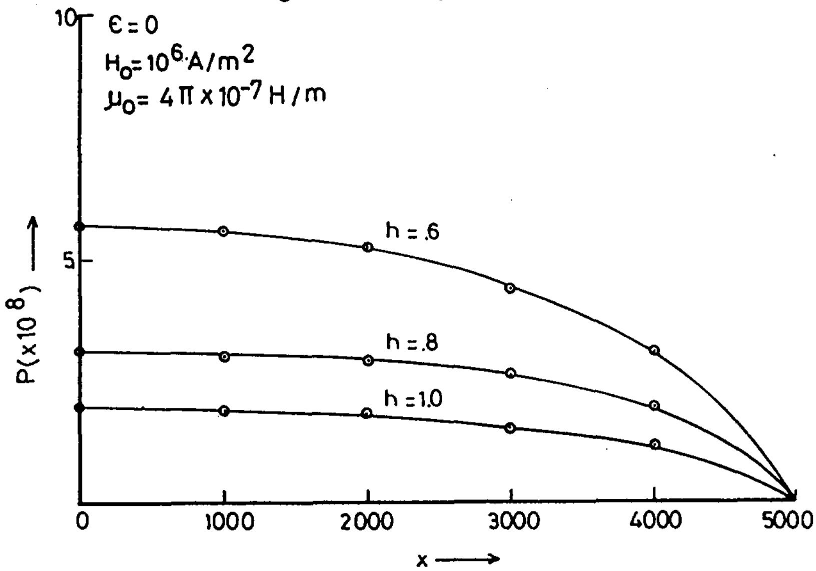

Figure 4. Variation of non-dimensional pressure distribution with axial distance for different h.

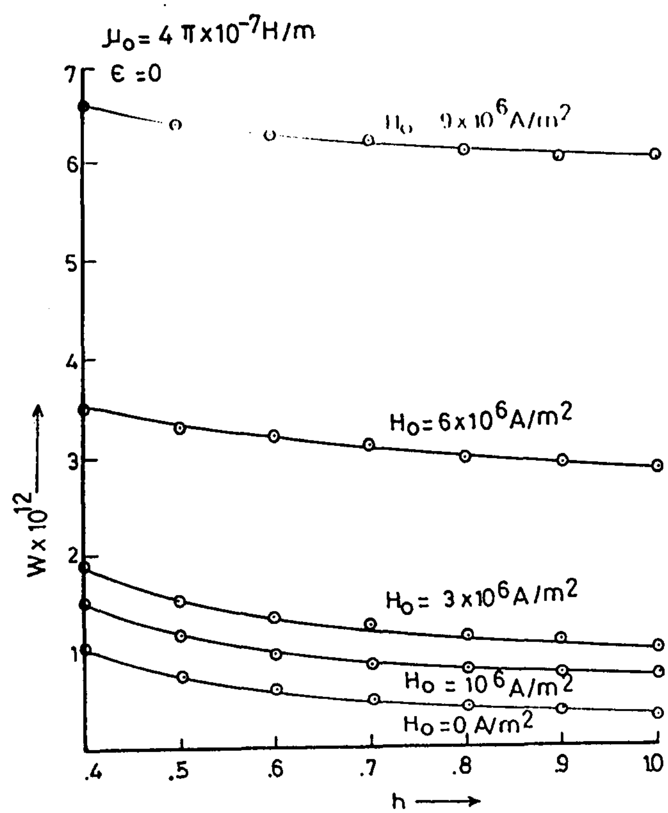

Figure 5. Variation of non-dimensional load capacity with h for different H_0.

overall viscosity of the lubricant. This helps again in sustaining greater loads. Breakdown of the hyaluronic acid component reduces the viscosity. This decrease in viscosity decreases the load carrying capacity of the joint. A comparison of these figures clearly demonstrates that in diseased states when the viscosity of the synovial fluid is lowered, the applied magnetic field can help in normal articulation by increasing the pressure in the intra-articular gap and the corresponding load carrying capacity of the joint as a whole.

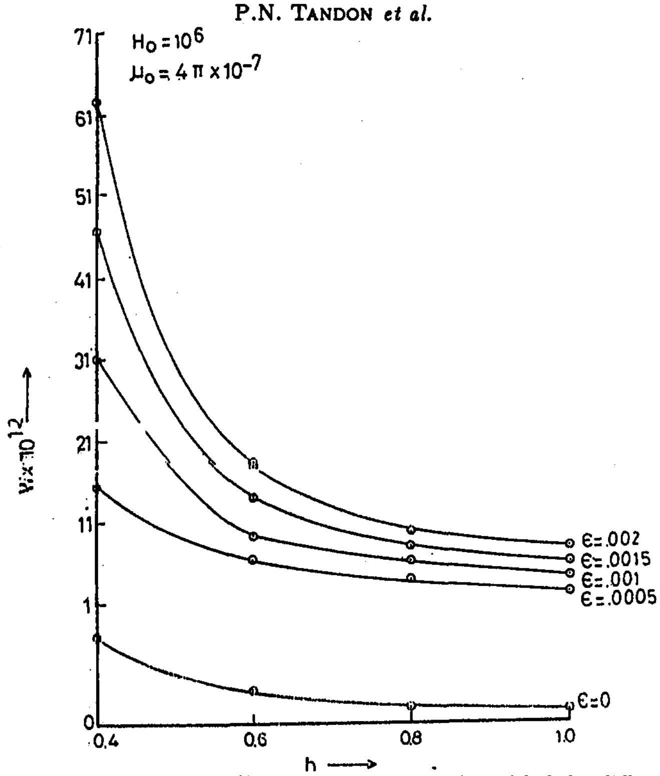

Figure 6. Variation of non-dimensional load capacity with h for different ε.

CONCLUDING REMARKS

The paper incorporates a more realistic model of synovial joints incorporating the poroelastic nature of cartilage and representing the synovial fluid by a more realistic viscoelastic fluid. The parameters of the viscoelastic fluid representing the synovial fluids have been taken from experimentally measured values for normal, diseased and old synovial fluids so as to draw some conclusions for clinical purposes. It may be observed that for the diseased joints, identified through the physiological characteristic of the synovial joints, there is less pressure distribution within the intra-articular gap and less load carrying capacities as compared to the corresponding values for normal joints. Under the assumption that the synovial fluid behaves like a paramagnetic fluid system, it has been observed that the suitably designed applied magnetic fields improve these performance characteristics. Thus, the applied magnetic fields may be used for better articulation, particularly in diseased states, although there is considerable scope to further the development in this direction both theoretically and experimentally, particularly in isolating the paramagnetic properties of the synovial fluid in diseased states.

REFERENCES

1. D. Dowson, Modes of lubrication in human joints, Lubrication and wear in living and artificial human joints, In *Proc. Inst. Mech. Engrs.*, **181** (3J), 45–54 (1966).
2. R.S. Fein, Are synovial joints squeeze film lubricated? Lubrication and wear in living and artificial human joints, In *Proc. Inst. Mech. Engrs.*, **181** (3J), 125–128 (1966–67).
3. D. Dowson, A. Unsworth and V. Wright, Analysis of boosted lubrication in human joints, *J. Mech. Eng. Sci.* **12** (5), 364–369 (1970).
4. C.W. McCutchen, Sponge-hydrostatic and weeping bearing, *Nature* **184** (7), 1284–1285 (1959).
5. A. Maroudas, Hyaluronic acid films, Lubrication and wear in living and artificial human joints, *Proc. Inst. Mech. Engrs.* **181** (3J), 122–124 (1967).
6. V.C. Mow, Effect of viscoelastic lubricant on squeeze film lubrication between impinging spheres, *J. Lub. Tech. Trans., ASME* **90** (1), 113–116 (1968).
7. J.G. Oldroyd, Non-Newtonian effects in some idealized elastic viscous liquids, In *Proc. Roy. Soc.* **4** (3), 278–297 (1958).
8. V.C. Mow, W.M. Lai and L. Redler, Some surface characteristics of the articular cartilage, *J. Biomech.* **7** (5), 449–456 (1974).

9. R.I. Tanner, Some illustrative problems in flow of viscoelastic non-Newtonian lubricants, *ASLE Trans.* **8** (2), 179–188 (1965).
10. W.M. Lai, S.C. Kuei and V.C. Mow, Rheological equations for synovial joints, *J. Biomech. Engg.* **100 k**, 169–186 (1978).
11. M. Zahn and K.E. Shenton, Magnetic fluids bibliography, *Magnetics IEEE Trans.* **16**, 387–409 (1980).
12. C.T. Brighton, Z.B. Fridenberg, E.I. Mitchell and R.E. Booth, Treatment of non-union with constant direct current, *Clin. Orthop.* **124**, 106–123 (1977).
13. R.A. Elco and W.F. Hughes, MHD pressurisation of liquid metal bearing, *Wear* **5**, 198–212 (1962).
14. W.F. Hughes and R.A. Elco, MHD lubrication flows between parallel rotating discs, *J. Fluid Mech.* **13** (1), 21–32 (1962).
15. W.F. Hughes, The magnetohydrodynamic inclined slider bearing with a transverse magnetic field, *Wear* **6**, 315–324 (1963).
16. Y. Yamamoto and S. Gonda, Effect of a magnetic field on boundary lubrication, *Tribology* **20** (6), 342–346 (1987).
17. M. Zahn and K.E. Rosenweig, Stability of a magnetic fluid penetration through a porous medium with uniform magnetic field oblique to the interface, *IEEE Trans. Magnetics* **16**, 275–280 (1980).
18. M.L. Bansal, *Magneto Therapy*, Jain Publishers, New Delhi, (1976).
19. J. Bagwell, J. Klawitter, B. Sauer and A. Weinstein, A study of bone growth into porous polyethylene, Presented at the *Sixth Annual Biomaterials Symposium*, pp. 20–24, Clemson University, Clemson, SC, USA, (1974).
20. R.M. Pilliar, H.V. Cameron and I. MacNab, Porous surface layered prosthetic devises, *Biomedical Engg.* **4** (1), 12–16 (1975).
21. P.N. Tandon, J.K. Misra, R.S. Gupta and Satyanand, Role of ultafiltration in lubrication of human joints, *Intn. J. Mech. Sci.* **27** (1/2), 29–37 (1985).
22. P.N. Tandon and R. Agarwal, Role of synovial fluid in the functioning of knee joints, In *Proc. of 2nd Int. Conf. on Physiological Fluid Dynamics*, pp. 245–249, IIT, Madras, (1987).
23. P.N. Tandon, P. Nirmala, T.S. Pal and R. Agarwal, Rheological study of lubricant gelling in synovial joints during articulation, *Int. J. Appl. Math. Modelling* **12** (1), 72–77 (1988).
24. P.D.S. Verma, Magnetic fluid-based squeeze film, *Int. J. Engg.* **24** (3), 395–401 (1986).
25. A. Cameron, *Basic Lubrication Theory*, Ellis Horwood Ltd., Chichester, (1976).
26. R. Collins, A model of lubricant gelling in synovial joints, *J. Appl. Math. and Phys. (ZAMP)* **33**, 93–123 (1982).
27. E.C. Jordan and K.G. Balmain, *Electromagnetic Waves and Rotating Systems*, Prentice Hall of India Pvt. Ltd., New Delhi, (1976).
28. P.N. Tandon and P. Nirmala, Role of ultrafiltration in lubrication and gel formation in synovial joints, In *Proc. of Natl. W/S on Computer Applications to Continuum Mechanics*, pp. 75–82, (1986).
29. D. Dowson, M. Longfield, P. Walker and V. Wright, An investigation of the friction and lubrication in human joints, *Proc. Inst. Mech Engrs.* **182** (3N), 68–70 (1968).
30. V.C. Mow and W.M. Lai, Recent development in synovial joint biomechanics, *SIAM Review* **22** (3), 275–317 (1980).