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$U(1)$ effective confinement theory from $SU(2)$ restricted gauge theory via the Julia–Toulouse approach

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ABSTRACT

We derive a $U(1)$ effective theory of color confinement by applying the so-called Julia–Toulouse approach for defects condensation to the $SU(2)$ restricted gauge theory defined by means of the Cho decomposition of the non-abelian connection. Cho's geometric construction naturally displays the topological degrees of freedom of the theory and can be used to put the Yang–Mills action into an abelianized form under certain conditions. On the other hand, the use of the Julia–Toulouse prescription to deal with the monopole condensation leads to an effective action describing the phase whose dynamics is dominated by the magnetic condensate. The effective theory we found describes the interaction between external electric currents displaying a short-range Yukawa interaction plus a linear confinement term that governs the long distance physics.

1. Introduction

It is known that due to the Meissner effect the usual superconductors should confine magnetic monopoles. This point led to the conjecture that the QCD vacuum could be a condensate of chromomagnetic monopoles, a dual superconductor as originally proposed in [1]. Such a chromomagnetic condensate should be responsible for the dual Meissner effect that is expected to lead to the confinement of color charges immersed in this medium. In dual superconductor models of color confinement, magnetic monopoles usually appear as topological defects in points of the space where the abelian projection becomes singular. For a review, see for example [13].

In this Letter we follow a different path to reveal the magnetic monopole condensate in the pure $SU(2)$ gauge theory. First, instead of just writing down an effective dual abelian Higgs model compatible with the residual gauge symmetry obtained from the abelian projection of the $SU(2)$ gauge theory, we use the so-called Cho decomposition [2] of the $SU(2)$ connection, which has the feature of explicitly exposing the abelian component of the non-abelian connection revealing the topological structures of the theory without resorting to any singular gauge fixing procedure like the abelian projection. We work with the subsector of the complete gauge theory called restricted gauge theory which contains the full $SU(2)$ gauge degrees and is expected to be responsible for the color confinement, as claimed in [2].

Next we fix the so-called magnetic gauge where we show that the action acquires the form of the Maxwell theory minimally coupled to external chromoelectric charges and non-minimally coupled to chromomagnetic monopoles. At this point we are in position to apply the Julia–Toulouse Approach (JTA) for defects condensation [6] as generalized by some of us in [10,11]. This is a prescription used to obtain an effective theory for a phase with condensed defects starting from the theory defined in the phase where the defects are diluted — and exploit the consequences of the monopole condensation. As the result, the effective theory describing the interaction between the chromoelectric charges immersed in the chromomagnetic condensate features two parts:

- the first one is a Yukawa-like term that dominates the short-range physics in the magnetic condensate — a typical feature of the confinement scenarios due to monopole condensation;
- the second one describes, in the static case, a linear potential in the interquarks separation, thus being responsible for the chromoelectric confinement that dominates the physics at large distances.

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2. Setting the problem

We begin with a brief review of the Cho decomposition of the $SU(2)$ connection. The starting point is the introduction of a unitary color triplet, $\hat{n}:\mathbb{R}^{1,3}_{(\text{spacetime})}\to S^2\subset\mathbb{R}^3_{(\text{color})}$, $x\mapsto \hat{n}(x)|\hat{n}^2(x)=1$, and the definition of the so-called restricted connection, $\hat{A}_\mu$, which leaves $\hat{n}$ invariant under parallel transport on the principal bundle [2],

$$\hat{D}_\mu\hat{n}:=\partial_\mu\hat{n}+g\hat{A}_\mu\times\hat{n}\equiv 0 \quad\Rightarrow\quad \hat{A}_\mu=A_\mu\hat{n}-\frac{1}{g}\hat{n}\times\partial_\mu\hat{n}. \tag{1}$$

As we are going to see in a moment:

- the unitary triplet $\hat{n}$ selects the abelian direction in the internal color space for each spacetime point;
- A_μ transforms like a $U(1)$ connection;
- the restricted connection $\hat{A}_\mu$ is already an $SU(2)$ connection.

Due to the fact that the space of connections is an affine space, a general $SU(2)$ connection, $\vec{A}_\mu$, can be obtained from the restricted connection, $\hat{A}_\mu$, by adding a field $\vec{X}_\mu$ that is orthogonal to $\hat{n}$ [2]. Thus, the general form of the Cho decomposition of the $SU(2)$ connection is given by

$$\vec{A}_\mu=A_\mu\hat{n}-\frac{1}{g}\hat{n}\times\partial_\mu\hat{n}+\vec{X}_\mu,$$
$$\hat{n}^2=1 \quad\text{and}\quad \hat{n}\cdot\vec{X}_\mu=0. \tag{2}$$

From the infinitesimal $SU(2)$ gauge transformation defined by

$$\delta\vec{A}_\mu=\frac{1}{g}D_\mu\vec{\omega}:=\frac{1}{g}(\partial_\mu\vec{\omega}+g\vec{A}_\mu\times\vec{\omega}),$$
$$\delta\hat{n}=-\vec{\omega}\times\hat{n}, \tag{3}$$

it follows that

$$\delta A_\mu=\frac{1}{g}\hat{n}\cdot\partial_\mu\vec{\omega},$$
$$\delta\hat{A}_\mu=\frac{1}{g}\hat{D}_\mu\vec{\omega},$$
$$\delta\vec{X}_\mu=-\vec{\omega}\times\vec{X}_\mu. \tag{4}$$

We see from (4) that A_μ transforms like a $U(1)$ connection, being the abelian component of the $SU(2)$ connection explicitly revealed by the Cho decomposition without any gauge fixing procedure (like the abelian projection). Thus, we say that the unitary triplet field $\hat{n}$ selects the abelian direction in the color space for each spacetime point. Furthermore, we also see from (3) and (4) that the restricted connection, $\hat{A}_\mu$, transforms like the general $SU(2)$ connection, $\vec{A}_\mu$, since the restricted covariant derivative is expressed (in the adjoint representation), like the general covariant derivative, in terms of the $SU(2)$ structure constants ϵ_{abc}. Hence, as anticipated, the restricted connection is already an $SU(2)$ connection carrying all the gauge degrees (but not all the dynamical degrees) of the non-abelian gauge theory, being $\vec{X}_\mu$ a source term called the valence potential which carries the remaining dynamical degrees of the theory.

We shall concentrate our attention from now on into the restricted connection, since it already gives us an $SU(2)$ theory, whose properties we are interested in analyze in this Letter. In fact, as claimed in [2], the restricted gauge theory governs the subdynamics of the complete gauge theory that characterizes the vacuum of the theory and would be responsible for the color confinement, that is what we are looking for.

The restricted curvature tensor is given by

$$\hat{F}_{\mu\nu}:=\partial_\mu\hat{A}_\nu-\partial_\nu\hat{A}_\mu+g\hat{A}_\mu\times\hat{A}_\nu=(F_{\mu\nu}+H_{\mu\nu})\hat{n}, \tag{5}$$

where

$$F_{\mu\nu}:=\partial_\mu A_\nu-\partial_\nu A_\mu,$$
$$H_{\mu\nu}:=-\frac{1}{g}\hat{n}\cdot(\partial_\mu\hat{n}\times\partial_\nu\hat{n}). \tag{6}$$

Parametrizing $\hat{n}$ over S^2 by the polar angle, θ, and azimuthal angle, φ, $\hat{n}=(\sin(\theta)\cos(\varphi),\sin(\theta)\sin(\varphi),\cos(\theta))$, we rewrite the last equation as

$$H_{\mu\nu}=-\frac{1}{g}\sin(\theta)(\partial_\mu\theta\partial_\nu\varphi-\partial_\mu\varphi\partial_\nu\theta). \tag{7}$$

At this point we go to the so-called magnetic gauge defined by fixing the local color vector field $\hat{n}$ in the $\hat{z}$-direction in the internal color space [2]. In this gauge, the curvature tensor is written as $\hat{F}_{\mu\nu}=(F_{\mu\nu}+H_{\mu\nu})\hat{z}$. Defining the so-called magnetic potential by the expression

$$\tilde{C}_\mu:=\frac{1}{g}\big(\cos(\theta)\partial_\mu\varphi+\partial_\mu\gamma\big), \tag{8}$$

we see that we can rewrite $H_{\mu\nu}$ in the abelianized form,

$$H_{\mu\nu}=\partial_\mu\tilde{C}_\nu-\partial_\nu\tilde{C}_\mu. \tag{9}$$

The angle γ is the third of the Euler angles used to define a general $SO(3)$ transformation that rotates $\hat{n}$ into $\hat{z}$ in $\mathbb{R}^3_{(\text{color})}$.

It is easy to see now that the restricted connection transforms like $\hat{A}_\mu\mapsto(A_\mu+\tilde{C}_\mu)\hat{z}$ under the gauge transformation that leads us to the magnetic gauge.

The magnetic potential, $\tilde{C}_\mu$, describes the potential of a monopole, being singular over its associated Dirac string [12], as we can easily see following the example discussed in [14]: if we consider $\gamma=-\varphi$, we have from (8) that

$$\tilde{C}_\mu=\frac{1}{g}\big(\cos(\theta)-1\big)\partial_\mu\varphi. \tag{10}$$

Since we have for the gradient in spherical coordinates that $\partial_0:=\partial_t\equiv\partial/\partial t$, $\partial_1:=\partial_r\equiv\partial/\partial r$, $\partial_2:=\partial_\theta\equiv(1/r)\partial/\partial\theta$ and $\partial_3:=\partial_\varphi\equiv(1/r\sin(\theta))\partial/\partial\varphi$, we see from (10) that

$$\tilde{C}_\mu=\frac{1}{g}\frac{(\cos(\theta)-1)}{r\sin(\theta)}\delta_{\mu\varphi}, \tag{11}$$

which is the monopole potential singular over the Dirac string arbitrarily placed (by the choice made for γ) in the negative $\hat{z}$-axis ($\theta=\pi$).

This singularity is a gauge artifact and must not show up in the final expressions for the physical observables. Hence, in order to define a regular finite action, we must subtract the unphysical singularity that arises in the expression for $H_{\mu\nu}$ due to the flux tube inside the Dirac string, introducing a δ-distribution, $\Lambda^M_{\mu\nu}$, that localizes the world surface spanned by the magnetic Dirac string and exactly cancels out the singularity in $H_{\mu\nu}$, as discussed for example, in Chapter 8 of [7] and Chapter 2 of [13]. This reasoning leads us to write the Lagrangian density for the restricted theory as:

$$\mathcal{L}=-\frac{1}{4}\hat{F}^2_{\mu\nu}$$
$$=-\frac{1}{4}F^2_{\mu\nu}-\frac{1}{2}F_{\mu\nu}\big(H^{\mu\nu}-\Lambda^{\mu\nu}_M\big)-\frac{1}{4}\big(H_{\mu\nu}-\Lambda^M_{\mu\nu}\big)^2. \tag{12}$$

Now we follow the reasoning presented in the model reviewed in Section 5.2 of [13] and minimally couple external chromoelectric currents, j_μ (described by electric Dirac strings, $\tilde{\Lambda}^E_{\mu\nu}$, through the relation $j_\mu = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\partial^\nu \tilde{\Lambda}_E^{\alpha\beta}$), to the restricted potential expressed in the magnetic gauge, $(A_\mu + \tilde{C}_\mu)$, obtaining the following Lagrangian density:

$$\bar{\mathcal{L}} = -\frac{1}{4}F^2_{\mu\nu} - \frac{1}{2}F_{\mu\nu}\left(H^{\mu\nu} - \Lambda_M^{\mu\nu}\right) - \frac{1}{4}\left(H_{\mu\nu} - \Lambda^M_{\mu\nu}\right)^2 - (A_\mu + \tilde{C}_\mu)j^\mu. \tag{13}$$

We shall refer generically to the magnetic (electric) Dirac string and to its world surface as a "magnetic (electric) Dirac brane".

3. The Julia–Toulouse approach for the monopole condensation and the $U(1)$ effective theory of confinement

Notice that absorbing the singular monopole field $\tilde{C}_\mu$ into the regular abelian gluon field A_μ by redefining $(A_\mu + \tilde{C}_\mu) \mapsto A_\mu$, we can rewrite (13) in the following form:

$$\bar{\mathcal{L}} = -\frac{1}{4}\left(F^{\mu\nu} - \Lambda_M^{\mu\nu}\right)^2 - j_\mu A^\mu, \tag{14}$$

where now the field A_μ is singular over the magnetic Dirac branes. Eq. (14) describes the Maxwell theory with the vector potential A_μ minimally coupled to electric currents and non-minimally coupled to monopoles. We have exploited the monopole condensation phenomenon in this action and in its dual counterpart in great details in [11]. However, notice that here the monopoles were not included in the theory by hand, instead they were naturally revealed in the YM theory by the Cho decomposition of the non-abelian connection. Furthermore, the quantum theory associated to (14) must be invariant under deformations of the unphysical Dirac strings. This is accomplished provided we impose the non-abelian version of the Dirac quantization condition [12,13], $g\tilde{g} = 4\pi n$, $n \in \mathbb{Z}$, where $\frac{g}{2}$ (which is present in the electric string term, $\tilde{\Lambda}^E_{\mu\nu} := \frac{g}{2}\tilde{\delta}_{\mu\nu}(x; S_E)$, being S_E the world surface of the electric Dirac string) is the $SU(2)$ chromoelectric charge of the quarks, being g the QCD coupling constant, and $\tilde{g} = \frac{4\pi n}{g}$ (which is present in the magnetic string term, $\Lambda^M_{\mu\nu} := \tilde{g}\tilde{\delta}_{\mu\nu}(x; S_M)$, being S_M the world surface of the magnetic Dirac string) is the chromomagnetic charge of the monopoles in the n-th homotopy class of the mapping $\Pi_2(SU(2)/U(1) \simeq S^2) = \mathbb{Z}$ defined by the unitary triplet $\hat{n}$ [2].

We are now in position to apply the JTA to obtain an effective theory describing the phase where the monopoles are condensed. In (14), the field A_μ is regular only over $\mathbb{R}^{1,3}_{(\text{spacetime})}\backslash\mathcal{M}$, where $\mathcal{M}$ is the geometric place of the magnetic Dirac branes. As the monopoles proliferate, the magnetic potential can only be defined over an increasingly smaller region in the space until we reach the critical case where the monopoles proliferate occupying the whole space. In this case, A_μ cannot be defined anywhere. Equivalently, the Dirac branes of the condensing monopoles occupy the whole space and should be elevated to the field category describing the long wavelength fluctuations of the condensate. The Julia–Toulouse procedure consists in the observation that the regular physical combination $(F_{\mu\nu} - \Lambda^M_{\mu\nu})$ should be taken as the fundamental field $Y_{\mu\nu}$ describing the magnetic monopole condensate [6]. This becomes the magnetic equivalent to the Stuckelberg procedure where the condensate field "eats up" the gauge field to become massive. Notice that in doing so we have effectively promoted the kinetic term for the 1-form gauge field describing the normal or diluted phase to a mass term for the 2-form Kalb–Ramond field describing the monopole condensate in the condensed phase — this mass generation accompanied by the rank-jump of the field describing the defects condensate is the main signature of the JTA [8,10,11]. Next, we must give dynamics to the 2-form describing the magnetic condensate supplementing the action with a kinetic term for it which, usually, results from a Lorentz and gauge symmetry preserving derivative expansion [9] and the outcome of such approach is the following effective theory for the magnetic condensed phase [6,11]:

$$\bar{\mathcal{L}}_c = \frac{1}{12}(\partial_\mu Y_{\alpha\beta} + \partial_\alpha Y_{\beta\mu} + \partial_\beta Y_{\mu\alpha})^2 + \frac{m_Y}{4}Y_{\mu\nu}\epsilon^{\mu\nu\alpha\beta}\tilde{\Lambda}^E_{\alpha\beta} - \frac{m_Y^2}{4}Y^2_{\mu\nu}. \tag{15}$$

In Minkowski spacetime the dual Kalb–Ramond field, $\tilde{Y}_{\mu\nu} := \frac{1}{2!}\epsilon_{\mu\nu\alpha\beta}Y^{\alpha\beta}$, implies the relations:

$$\begin{cases} \frac{1}{12}(\partial_\mu Y_{\alpha\beta} + \partial_\alpha Y_{\beta\mu} + \partial_\beta Y_{\mu\alpha})^2 = -\frac{1}{2}\left(\partial_\mu \tilde{Y}^{\mu\nu}\right)^2, \\ Y^2_{\mu\nu} = -\tilde{Y}^2_{\mu\nu}, \\ \frac{1}{4}Y_{\mu\nu}\epsilon^{\mu\nu\alpha\beta}\tilde{\Lambda}^E_{\alpha\beta} = \frac{1}{2}\tilde{Y}_{\mu\nu}\tilde{\Lambda}_E^{\mu\nu}, \\ Y_{\mu\nu} = -\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\tilde{Y}^{\alpha\beta}, \end{cases}$$

such that in terms of $\tilde{Y}_{\mu\nu}$ the effective action describing the condensed phase is written as:

$$\bar{S}_{eff}[\tilde{Y}_{\mu\nu}] = \int d^4x\left[-\frac{1}{2}\left(\partial_\mu \tilde{Y}^{\mu\nu}\right)^2 + \frac{m_Y^2}{4}\tilde{Y}^2_{\mu\nu} + \frac{m_Y}{2}\tilde{Y}_{\mu\nu}\tilde{\Lambda}_E^{\mu\nu}\right]. \tag{16}$$

Eq. (16) is the result of the application of the JTA, as formulated in the relativistic field theory context by Quevedo–Trugenberger, to the Maxwell theory. This equation, however, features an undesirable point: it is not invariant under deformations of the unphysical electric Dirac strings. If we deform $S_E \mapsto S'_E$, $\partial S_E = \partial S'_E$, where ∂ is the border operator, through $\tilde{\delta}_{\mu\nu}(x; S_E) \mapsto \tilde{\delta}_{\mu\nu}(x; S'_E) = \tilde{\delta}_{\mu\nu}(x; S_E) + \partial_\mu\tilde{\delta}_\nu(x; V) - \partial_\nu\tilde{\delta}_\mu(x; V)$, $\partial V = S_E \cup S'_E$, the theory is modified. In the sequel we shall approach this point carefully by using an extension of the JTA we have presented in [10,11]. The procedure is as follows.

The dual of the Maxwell action is given by:

$$*\bar{S} = \int d^4x\left[-\frac{1}{4}\left(\tilde{F}_{\mu\nu} - \tilde{\Lambda}^E_{\mu\nu}\right)^2 - \tilde{A}^\mu\tilde{j}_\mu\right], \tag{17}$$

where the couplings are inverted relatively to the ones present in (14): here the dual vector potential $\tilde{A}_\mu$ couples minimally to the monopoles and non-minimally to the electric charges.

We suppose that for the electric charges there are only a few fixed (external) worldlines L_E while for the monopoles we suppose that there is a fluctuating ensemble of closed worldlines L_M that can eventually proliferate. The magnetic current is written in terms of the magnetic Dirac brane as $\tilde{j}^\sigma = \frac{1}{2}\epsilon^{\sigma\rho\mu\nu}\partial_\rho\Lambda^M_{\mu\nu} = \tilde{g}\delta^\sigma(x; L_M)$, $L_M = \partial S_M$. In order to allow the monopoles to proliferate we must give dynamics to their magnetic Dirac branes since the proliferation of them is directly related to the proliferation of the monopoles and their worldlines. Thus we supplement the dual action (17) with a kinetic term for the magnetic Dirac branes of the form $-\frac{\epsilon_c}{2}\tilde{j}^2_\mu$, which preserves the local symmetries of the system. This is an activation term for the magnetic loops. Hence, the partition function associated to the extended dual action reads:

$$Z^c := \int \mathcal{D}\tilde{A}_\mu\delta\left[\partial_\mu\tilde{A}^\mu\right]e^{i\int d^4x[-\frac{1}{4}(\tilde{F}_{\mu\nu} - \tilde{\Lambda}^E_{\mu\nu})^2]}Z^c[\tilde{A}_\mu], \tag{18}$$

where the Lorentz gauge has been adopted for the dual gauge field $\tilde{A}_\mu$ and the partition function for the brane sector $Z^c[\tilde{A}_\mu]$ is given by,

$$Z^c[\tilde{A}_\mu] := \sum_{\{L_M\}} \delta\big[\partial_\mu \tilde{j}^\mu\big] \exp\left\{ i \int d^4x \left[-\frac{\epsilon_c}{2} \tilde{j}_\mu^2 + \tilde{j}_\mu \tilde{A}^\mu \right] \right\}, \tag{19}$$

where the functional δ-distribution enforces the closeness of the monopole worldlines.

Next, use is made of the Generalized Poisson's Identity (GPI) (see Appendix A of [11] for a detailed discussion on the subject) in $d = 4$:

$$\sum_{\{L_M\}} \delta\big[\eta_\mu(x) - \delta_\mu(x; L_M)\big] = \sum_{\{\tilde{V}\}} e^{2\pi i \int d^4x \tilde{\delta}_\mu(x;\tilde{V})\eta^\mu(x)}, \tag{20}$$

where L_M is a 1-brane and $\tilde{V}$ is the 3-brane of complementary dimension. The GPI works as an analogue of the Fourier transform: when the lines L_M in the left-hand side of (20) proliferate, the volumes $\tilde{V}$ in the right-hand side become diluted and vice versa. We shall say that the branes L_M and $\tilde{V}$ (or the associated currents $\delta_\mu(x; L_M)$ and $\tilde{\delta}_\mu(x; \tilde{V})$) are Poisson-dual to each other. Using (20) we can rewrite (19) as:

$$\begin{aligned} Z^c[\tilde{A}_\mu] &= \int \mathcal{D}\eta_\mu \sum_{\{L_M\}} \delta\left[\tilde{g}\left(\frac{\eta_\mu}{\tilde{g}} - \delta_\mu(x; L_M) \right) \right] \\ &\quad \times \delta\left[\tilde{g}\left(\partial_\mu \frac{\eta_\mu}{\tilde{g}} \right) \right] \exp\left\{ i \int d^4x \left[-\frac{\epsilon_c}{2} \eta_\mu^2 + \eta_\mu \tilde{A}^\mu \right] \right\} \\ &= \int \mathcal{D}\eta_\mu \sum_{\{\tilde{V}\}} e^{2\pi i \int d^4x \tilde{\delta}_\mu(x;\tilde{V}) \frac{\eta^\mu}{\tilde{g}}} \\ &\quad \times \int \mathcal{D}\tilde{\theta} e^{i \int d^4x \tilde{\theta} \partial_\mu \frac{\eta^\mu}{\tilde{g}}} \exp\left\{ i \int d^4x \left[-\frac{\epsilon_c}{2} \eta_\mu^2 + \eta_\mu \tilde{A}^\mu \right] \right\} \\ &= \sum_{\{\tilde{V}\}} \int \mathcal{D}\tilde{\theta} \int \mathcal{D}\eta_\mu \exp\left\{ i \int d^4x \left[-\frac{\epsilon_c}{2} \eta_\mu^2 \right. \right. \\ &\quad \left. \left. - \eta^\mu \frac{1}{\tilde{g}} \big(\partial_\mu \tilde{\theta} - \tilde{\theta}_\mu^V - \tilde{g} \tilde{A}_\mu \big) \right] \right\}, \end{aligned} \tag{21}$$

where we defined the Poisson-dual current $\tilde{\theta}_\mu^V := 2\pi \tilde{\delta}_\mu(x; \tilde{V})$.

Integrating the auxiliary field η_μ in the partial partition function (21) and substituting the result back in the complete partition function (18) we obtain, as the effective action for the condensed phase in the dual picture, the London limit of the $U(1)$ Dual Abelian Higgs Model (DAHM):

$$\begin{aligned} *\bar{S}^L_{DAHM} = \int d^4x \Big[&-\frac{1}{4} \big(\tilde{F}_{\mu\nu} - \tilde{\Lambda}^E_{\mu\nu} \big)^2 + \frac{m_{\tilde{A}}^2}{2\tilde{g}^2} \big(\partial_\mu \tilde{\theta} - \tilde{\theta}_\mu^V \\ &- \tilde{g} \tilde{A}_\mu \big)^2 \Big], \end{aligned} \tag{22}$$

where we defined $m_{\tilde{A}}^2 := \frac{1}{\epsilon_c}$. The Poisson-dual current, $\tilde{\theta}_\mu^V$, appears in (22) as a vortex-like defect for the scalar field $\tilde{\theta}$ describing the magnetic condensate in the dual picture, being a parameter that controls the monopole condensation [11].

Next we are going to dualize this result and one could be concerned with the fact that (22) constitutes a nonrenormalizable theory, thus requiring a cutoff in order to be well defined as an effective quantum theory. However, one can always think of its UV completion, in this case the complete DAHM, which is renormalizable, and then take its dual, taking the London limit afterwards [13]. At least in the case considered here, the result is exactly the same one obtains by directly dualizing the London limit (22) of the DAHM, thus justifying the procedure we shall adopt in the sequel.

The dual action to (22) is given by [11]:

$$\bar{S}^V_{eff}[\tilde{Y}_{\mu\nu}] = \int d^4x \left[-\frac{1}{2} \big(\partial_\mu \tilde{Y}^{\mu\nu} \big)^2 + \frac{m_Y^2}{4} \tilde{Y}_{\mu\nu}^2 + \frac{m_Y}{2} \tilde{Y}_{\mu\nu} \tilde{L}_E^{\mu\nu} \right], \tag{23}$$

where we have identified the phenomenological parameters $m_{\tilde{A}} \equiv m_Y$ and defined the electric brane invariant:

$$\tilde{L}_E^{\mu\nu} := \frac{g}{2} \big(\tilde{\delta}^{\mu\nu}(x; S_E) + \partial^\mu \tilde{\delta}^\nu(x; \tilde{V}) - \partial^\nu \tilde{\delta}^\mu(x; \tilde{V}) \big), \tag{24}$$

where we have also used the non-abelian version of the Dirac quantization condition to write $\frac{2\pi}{\tilde{g}} = \frac{g}{2}$. $\tilde{L}_E^{\mu\nu}$ is an electric brane invariant provided we have $\tilde{\delta}^\mu(x; \tilde{V}) \mapsto \tilde{\delta}^\mu(x; \tilde{V}') = \tilde{\delta}^\mu(x; \tilde{V}) - \tilde{\delta}^\mu(x; V)$, $\partial V = S_E \cup S'_E$ under the deformation $S_E \mapsto S'_E$, $\partial S_E = \partial S'_E$ of the electric Dirac branes.

In fact, the complete form of the electric brane transformation, $S_E \mapsto S'_E$, $\partial S_E = \partial S'_E$, $\partial V = S_E \cup S'_E$, is given by (see Eqs. (17) and (22)):

$$\begin{cases} \tilde{\delta}_{\mu\nu}(x; S_E) \mapsto \tilde{\delta}_{\mu\nu}\big(x; S'_E\big) \\ \quad = \tilde{\delta}_{\mu\nu}(x; S_E) + \partial_\mu \tilde{\delta}_\nu(x; V) - \partial_\nu \tilde{\delta}_\mu(x; V), \\ \tilde{\delta}_\mu(x; \tilde{V}) \mapsto \tilde{\delta}_\mu\big(x; \tilde{V}'\big) = \tilde{\delta}_\mu(x; \tilde{V}) - \tilde{\delta}_\mu(x; V), \\ \tilde{A}_\mu \mapsto \tilde{A}'_\mu = \tilde{A}_\mu + \frac{g}{2} \tilde{\delta}_\mu(x; V). \end{cases} \tag{25}$$

Eq. (23) is the generalization of Eq. (16) compatible with the local electric brane symmetry corresponding to the freedom of deforming the unphysical electric Dirac strings through the spacetime without modifying the physics. This result was first obtained by some of us in [11] and it is in consonance with the impossibility of spontaneously breaking local symmetries, a fact widely known as Elitzur's theorem [5]. Since we have electric brane symmetry in the diluted phase [7,10,11] it must be preserved also in the condensed phase. Indeed, just like the local gauge symmetry implies the current conservation, the local brane symmetry implies the charge quantization. If one of these local symmetries could really be broken, there would be no current conservation or charge quantization in the broken phase, a fact that is not observed in Nature. The explanation is again the fact that a local symmetry is really never broken [5]. However, notice that the brane symmetry is hidden in the electric brane invariant $\tilde{L}_E^{\mu\nu}$ and it is this hidden realization of the brane symmetry that is called the "spontaneous breaking of the brane symmetry" [7,10,11].

To see whether this effective theory gives us chromoelectric confinement or not we must integrate the field of the monopole condensate, $Y_{\mu\nu}$, in order to obtain the effective action describing the interaction between the electric currents in the condensed phase.

Integrating the Kalb–Ramond field in the partition function we obtain the following effective action describing the interaction between prescribed electric currents immersed in the monopole condensate (see Section 3.8.1 of [13]):

$$\bar{S}^V_{eff} = \int d^4x \left[-\frac{1}{4} \big(\tilde{L}^E_{\mu\nu} \big)^2 - \frac{1}{2} \partial_\mu \tilde{L}_E^{\mu\nu} \frac{1}{\partial^2 + m_Y^2} \partial^\alpha \tilde{L}^E_{\alpha\nu} \right]. \tag{26}$$

Noticing that we can rewrite the electric current j_μ in terms of the electric brane invariant $\tilde{L}^E_{\mu\nu}$ as $j_\mu = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} \partial^\nu \tilde{L}_E^{\alpha\beta}$, it can be shown that:

$$\begin{aligned} &\partial_\mu \tilde{L}_E^{\mu\nu} \frac{1}{\partial^2 + m_Y^2} \partial^\alpha \tilde{L}^E_{\alpha\nu} \\ &\quad = j_\mu \frac{1}{\partial^2 + m_Y^2} j^\mu - \frac{1}{2} \big(\tilde{L}^E_{\mu\nu} \big)^2 + \frac{m_Y^2}{2} \tilde{L}^E_{\mu\nu} \frac{1}{\partial^2 + m_Y^2} \tilde{L}_E^{\mu\nu}. \end{aligned} \tag{27}$$

Substituting (27) in (26) it is a simple algebraic task to show that:

$$\bar{S}^{V}_{eff} = \int d^4x \left[-\frac{m_Y^2}{4} \tilde{L}^{E}_{\mu\nu} \frac{1}{\partial^2 + m_Y^2} \tilde{L}^{\mu\nu}_{E} - \frac{1}{2} j_\mu \frac{1}{\partial^2 + m_Y^2} j^\mu \right]. \quad (28)$$

The first term in (28) is responsible for the charge confinement: it "spontaneously breaks the electric brane symmetry" such that the electric brane invariant $\tilde{L}^{E}_{\mu\nu}$ acquires energy and constitutes the electric flux tube connecting two charges of opposite sign immersed in the monopole condensate. The flux tube has a thickness equal to the penetration depth of the electric field in the dual superconductor constituted by the magnetic condensate. The shape of the electric flux tube that corresponds to the stable configuration that minimizes the energy of the system is that of a straight tube (minimal space). Substituting in the first term of (28) such a solution for the brane invariant, $\tilde{L}^{E}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\frac{1}{n\cdot\partial}(n^\alpha j^\beta - n^\beta j^\alpha)$, where $n^\mu := (0, \vec{R} := \vec{R}_1 - \vec{R}_2)$ is a straight line connecting the electric charges $+\frac{g}{2}$ in $\vec{R}_1$ and $-\frac{g}{2}$ in $\vec{R}_2$, and taking the static limit we obtain a linear confining potential between the electric charges [13].

The second term in (28) describes the Yukawa-like short-range interaction between the electric currents in the condensed phase.

Taking the limit $m_Y \to 0$ leads us back to the diluted phase, eliminating the monopole condensate and destroying the confinement. Indeed, we can see that in this limit the interaction between the electric currents in (28) becomes of the long-range (Coulomb) type and the confining term goes to zero.

It is very important to make a final remark regarding the JTA. In [11] we made the observation that under a complete monopole condensation (*id est*, when we consider that the monopoles proliferate until occupy the whole space) we have $\tilde{\theta}^{V}_{\mu} \to 0$ and we recover from (23) the Quevedo–Trugenberger result (16). However, as we discussed here, (16) is incompatible with the local electric brane symmetry and since a local symmetry cannot be broken [5], (16) must be substituted by (23). The fact is that it is impossible to have a complete monopole condensation when we include external electric charges in the system since the electric fields generated by them, although expulsed of almost all the space by the dual Meissner effect, cannot simply vanish: they are confined into straight flux tubes connecting electric charges of opposite sign immersed in the monopole condensate. These vortices with opposite electric charges in their borders do not vanish (only vortices disconnected from the electric charges can vanish) and thus there is no complete monopole condensation when there are external electric charges immersed in the dual superconductor. Furthermore, the physics described by (23) features not only the electric charge confinement but also the charge quantization since the brane symmetry is maintained in the condensed phase. The scenario is quite different when we consider (16), where although the electric charge confinement is present, the charge quantization is lost due to the explicitly breaking of the brane symmetry. To have the right physics with electric charge confinement and charge quantization in the condensed phase we must be very careful and give a proper treatment of brane symmetry as we did in this section.

Kleinert was the first one to point out that the brane symmetry is a kind of local symmetry different from the gauge symmetry [7]. We generalized the JTA [6] as done in [10,11], making it compatible with Elitzur's theorem and the local brane symmetry.

4. Conclusion

In this Letter we used the Julia–Toulouse condensation mechanism [6], as generalized by some of us in [10,11], to study the confinement problem for an $SU(2)$ gauge theory.

We took as the starting point to the novel reasoning presented in this Letter to approach the monopole condensation, in the non-abelian case, the expression for the restricted $SU(2)$ gauge theory defined by means of the Cho decomposition of the non-abelian connection. We showed that, in the magnetic gauge, the action can be put in the form of the Maxwell theory minimally coupled to external chromoelectric charges and non-minimally coupled to chromomagnetic monopoles. This was the crucial point that allowed us to apply the generalized JTA for defects condensation and obtain an effective theory compatible with Elitzur's theorem and local electric brane symmetry for the phase where the monopoles are condensed.

In order to obtain the physics describing the interaction between external chromoelectric charges immersed in the magnetic condensate we integrated out in the partition function the field of the monopole condensate. The effective action found displays a Yukawa short-range interaction between the electric currents in the condensed phase and a term responsible for the confinement physics at large distances, giving a linear potential in the interquarks separation when we consider the static case. Furthermore, since our generalized approach to the JTA preserves the local electric brane symmetry, the charge quantization that is present in the diluted phase is maintained in the condensed phase.

The result here achieved also confirms that the restricted gauge theory proposed by Cho is indeed the subsector of the complete gauge theory responsible for the confinement physics.

It is also important to say that the decomposition (2) was also approached in a different way by Faddeev and Niemi [3]. The difference between Cho's approach and Faddeev–Niemi's approach regards the specific form of the valence potential, $\vec{X}_\mu$. In Cho's construction the field $\hat{n}$ is regarded as a topological variable and its 2 degrees of freedom are not counted as transverse modes for the gluons — in doing so, the valence potential in Cho's interpretation of the decomposition (2) carries 4 transverse modes, being the other 2 transverses modes carried by the abelian component, A_μ [2]. On the other hand, Faddeev and Niemi interpret the 2 degrees of freedom of $\hat{n}$ as being 2 of the 6 transverse modes of the gluons and in doing so, their valence potential has only 2 transverse modes, the other 2 transverse modes being carried by A_μ [3]. The total number of degrees of freedom present in the connection described by (2) after gauge fixing in Cho's approach is 8 (6 transverse modes + 2 topological modes) while in Faddeev–Niemi's approach is 6 (6 transverse modes). Thus, in Faddeev–Niemi's approach the number of physical degrees of freedom of the $SU(2)$ connection (in $3+1$ there are 6 of them) is preserved by the decomposition, while in Cho's approach it is not (in the last of the references in [2], Cho discusses that his interpretation of the decomposition (2) indeed modifies the quantum theory). Regarding our result, as we did not specify the form of our valence potential, since we discharged it in our discussion, we expect that it remains unchanged in either approach (Cho or Faddeev–Niemi), since both of them agree about the form of the restricted connection that was the essential element we used in this Letter.

More recently, Faddeev and Niemi proposed a novel decomposition of the $SU(2)$ connection in terms of spin-charge separated variables [4] constructed directly in terms of the components of the non-abelian connection. The lowest order effective Lagrangian density expressed in terms of the spin-charge separated variables is given by Eq. (60) of [4]. To make contact with our results we notice that we can recover the functional form of the restricted gauge theory from the complete gauge theory by setting $\rho = 0$ in Eq. (60) of [4].

It is further claimed in [4] that the condition $\rho = \text{const}$ is related with the non-perturbative contribution of the $\langle A^2 \rangle$ condensate [15]. It would be interesting to have a better understanding of the interplay between this condensate and the monopole condensate studied in the present Letter.

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