L’Eredit arabo-islamica nelle scienze e nelle arti del calcolo dell’Europa medievale (The Arabic-Islamic heritage in the sciences and arts of calculation in Medieval Europe).

For anyone wanting to learn about the development of medieval arithmetic and algebra there have always been ample sources available at two extremes: brief accounts found mostly in encyclopedias and popular books, and focused works published mainly in academic journals and collected volumes. Less common are works such as the one under review, that address the overall development of the mathematics in reasonable depth utilizing current, and in this case original, research.

Ambrosetti is interested specifically in the diffusion of techniques of calculation that medieval Europeans obtained from Arabic sources. These include the base ten number system with the algorithms for performing operations (called in Latin algorism), and algebra. Both are the subjects of famous books by the 9th century Arabic scholar Muḥammad ibn Mūsā al-Khwārizmī that were later translated and reworked in European languages as they served the computational needs of the developing continent. Even if al-Khwārizmī's influence quickly subsided in the Islamic world due to a proliferation of later textbooks, the sway of the Baghdad scholar continued to be directly felt in Europe down to the 16th century.

Ambrosetti’s book is organized chronologically, and where appropriate the treatment is further broken down geographically. Chapter 1 covers the works of late antiquity and the early medieval period that formed a backdrop to later medieval mathematics. Here she discusses Martianus Capella, Boethius, Cassiodorus, and Isidore of Seville. Like in succeeding chapters, she describes in detail the contents of the more important books she writes about, with ample references to the secondary literature. For example, 13 pages are devoted to Boethius. After some historical remarks she proceeds with a detailed description of the contents of De Institutione Arithmetica, and she finishes with her own chronogeographical analysis of the surviving manuscripts for insights into the diffusion of the work.

After briefly reviewing the Greek and Indian sources of Arabic mathematics in Chapter 2, our author moves on to al-Khwārizmī in Chapter 3. Here some 30 pages are devoted to al-Khwārizmī’s Algebra alone. She covers not only the rules given in the first part of the book, including the solutions to the six simplified equations and their proofs, but also the many worked-out problems that follow. This attention is justified, given the wide reach of this book in both Latin and Italian translation. Al-Khwārizmī’s book on Hindu calculation is likewise covered, as well as a few books by other Arabic authors that became relevant in some way to European mathematics.

After reviewing mathematical activity around the Mediterranean down to the 12th century translation movement in Chapter 4, Ambrosetti devotes Chapter 5 to the Latin trans-
lations of al-Khwārizmi’s *Algebra*, and all of Chapter 6 to a particular family of manuscripts of Gerard of Cremona’s translation. In his 1986 critical edition of Gerard’s 12th century translation Barnabas Hughes identified what he calls the “Modus family” of manuscripts. The four manuscripts in this family date from the 15th and 16th centuries, and are characterized by “the frequent use of *cosa* for *res* and the addition of two lengthy paragraphs which begin ‘Modus dividendi’” [Hughes, 1986, p. 229]. One of these manuscripts is part of a codex that belonged to Regiomontanus, and was presumably copied by him. Ambrosetti provides us with a critical edition of this family on pp. 154–195. The value of such a late witness to al-Khwārizmi’s already 600 year old book lies of course not with what it can tell us about Arabic algebra or Gerard’s program, but for how it relates to late medieval mathematics.

Ambrosetti used all four manuscripts for the critical edition. The first part of the text, on the rules of algebra, is very close to Gerard’s original. Here Ambrosetti reproduces Hughes’ edition, noting variations in the Modus family in the critical apparatus. The second part, containing worked-out problems and the extra paragraphs, differ enough to warrant their own edition. Ambrosetti gives us the rest of Hughes’ edition, followed by the remainder of the text as it appears in the Modus family.

The most notable error I found is that the diagram on p. 171 is a repeat of the diagram on the previous page. For the correct diagram see [Hughes, 1986, p. 246]. In reading Chapter V, pp. 168–172, I found only a few minor errors: there is an extraneous “;” on p. 170.71; on p. 171.4 “est ab” should be “est a b”; and on p. 171.86 “eb” should be “eh”. I should also note that Ambrosetti changes Hughes’ duobus to duabus on p. 168.4 (I cannot check which reading is in the MSS, but “duabis” is grammatically correct). Throughout the edition Hughes’ references to the folios of the 13th century Paris MS are frequently garbled, like at p. 170.66, where “(P 113rb)” appears as “(P I i 3rb)”. These seem to be due to a computer assisted scan of Hughes’ edition.

Chapter 7 is devoted to the Latin translation of al-Khwārizmi’s book on Hindu calculation, as usual with a description of the surviving manuscripts, their contents, and references to the literature. Chapter 8 is all about Fibonacci, his books, and his sources. At only 18 pages, this part of the book does not get the detailed attention one might expect, especially because just about everything Fibonacci wrote about derives from Arabic sources, and he in turn influenced the writers of later medieval abacus books as well as Luca Pacioli. Chapter 9, “Gli algorismi”, covers the Latin and some vernacular texts that teach base ten calculation, including those by John of Halifax and Jordanus de Nemore. Jordanus of course is also noted for his *De Numeris Datis*, a kind of “Euclidization” of medieval algebra. We are also treated to graphical analyses of *algorism* manuscripts by country, by century/country, and by language/century.

In Chapter 10 Ambrosetti starts off with a description of the *libri d’abbaco* (abbacus books) of late medieval Italy. These are the textbooks written in connection with the abacus schools in which (mainly) future merchants were taught calculation and sometimes algebra. From there she surveys similar books produced all across Europe, from England to the Byzantine Empire. As a sign of just how far the author searched for sources, she notes that she had a difficult time collecting information on arithmetic and algebra in Slavic countries, due to language and the scarcity of online catalogs (p. 278).

The final Chapter 11 describes the waning of the influence of Arabic algebra in the course of the emergence of humanism in the 15th and 16th centuries. Even if it is not directly related to the theme of the book, it is interesting to read about David King’s recent theory of a link between Piero della Francesca’s enigmatic painting “Flagellation of Christ” and
an astrolabe constructed by Regiomontanus and presented to the Greek cardinal Bessarion. Piero, who also wrote a *Trattato d’Abaco* himself, likely met with Regiomontanus and the cardinal while the latter were traveling in Italy in the early 1460s. Back to the mathematics, Ambrosetti describes the works of Luca Pacioli, whose printed 1494 *Summa de Aritmetica, Geometria, Proportioni e Proportionalità* amassed the practical mathematical knowledge of his century, and provided the foundation for the rapid developments in algebra of Scipione del Ferro, Nicolo Tartaglia, Girolamo Cardano, and others in the following century.

Next comes the bibliography, followed by four appendices: (1) a 9 page list of manuscripts of Boethius’ *Arithmetic*, (2) a 34 page list of manuscripts on *algorism* in Latin and vernacular languages, (3) 4 pages listing algebra manuscripts in vernacular languages, and (4) a list of the 96 worked-out problems from the chapter on algebra in Fibonacci’s *Liber Abaci*. In this last list there are numerous errors. I will mention only the first three I found: in problem (32) the “−4” should be removed, in (34) a “4” should be placed before the square root, and in (36) the “x^2” should be “4x^2”. Also, a couple of problems are omitted.

Ambrosetti has given us a book rich in detail and with a unified overall approach. Her close attention to a vast amount of recent literature along with her own investigations make this a valuable book from several perspectives. In addition to being a good general history of arithmetic and algebra in premodern Europe, it serves as a handbook on medieval calculation with ample up-to-date references (as of 2008) where one can get quick information on mathematicians like Nicolas Chuquet, Regiomontanus, or even Roland of Lisbon. It is useful also for Ambrosetti’s analyses of the distribution of manuscripts, which should be of interest even to non-specialists curious about medieval mathematics. And of course the critical edition of the Modus family, presented alongside Hughes’ edition of Gerard’s original, is a welcome addition to the scholarship. All of these qualities overshadow the small errors I found, and together with the low price make this a very attractive book.

References


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Available online 12 April 2012

http://dx.doi.org/10.1016/j.hm.2012.03.008

1 Ambrosetti was evidently unaware that Barnabas Hughes includes a similar list in one of his recent articles [Hughes, 2004].