An Inventory-Distribution System with LTL Deliveries – Mixed Integer Approach

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Abstract

The paper concerns a class of supply chain management problems called Joint Transportation-and-Inventory Problems (JTIPs). These problems are characterized by the presence of both transportation and inventory considerations, either as policy-variables or constraints. The research presented in this paper aims at determining an optimal joint inventory/transportation policy. Mathematical modeling is applied to find the optimal solution to JTIPs thus defined. The set partitioning formulation for solving the vehicle routing problem is incorporated into a discrete time, finite-horizon inventory planning problem. Two models will be discussed: time-discretized integer programming and the new approach with predefined quantities of delivery. As is shown in this paper, the approach allows us to use a column generation technique.

Keywords: inventory-distribution system; supply chain management; mixed integer programming; column generation; inventory routing

1. Introduction

The increasing popularity of management concepts based on the cooperation of businesses along the supply chain encourages businesses to use modern methods for their operational planning. The main feature expected from contemporary approaches is to enable the simultaneous analysis of the flow of goods along the chain at multiple links of the chain. Nowadays, an appropriate structure of a distribution system and selection of distribution and inventory policies affects not only the cost of product supply execution, but also inventory cost and customer service quality. In their efforts to improve the selected characteristics of the entire or a particular part of the supply chain, the businesses have to solve problems which prove much more complex than in the case of traditional approaches, where a single subject is analyzed only.

An example of one such traditional method to distribution planning is the vehicle routing problem (VRP). In this problem, a vendor, knowing the quantities of orders placed by territorially dispersed customers, plans delivery routes for his vehicle fleet. While placing an order, a customer specifies the required delivery date. Each delivery

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plan concerns a single period and the planning process is repeated in consecutive periods. In the planning of delivery operation, this classic problem has with growing frequency been generalized to admit the possibility of the vendor determining delivery dates. Naturally, the vendor must not design delivery routes freely. The parties often agree upon specified levels of cooperation parameters, such as minimum or maximum quantity of delivery, minimum or maximum stock, or the maximum number of deliveries which the vendor has to comply with. When considered with respect to the delivery routing problem, this modification facilitates the execution of the same quantities of delivery with a shorter total route length. Unfortunately, the vendor takes over the duties of analyzing stock at their customers and planning routes under complex conditions.

In business practice, depending on the time of transfer of goods ownership from the vendor to the customer, such solutions are classified as consignment inventory (where it is not the physical delivery that means the change of ownership, but only the customer’s sale of goods or the use thereof in a production process) and the vendor managed inventory (VMI, where it is at the time of delivery that the customer assumes the ownership of products supplied). A third situation is also possible, where there is no change of ownership, because both the vendor and the customer represent the same organization.

The direct stimulus for the commencement of the following research presented herein has been an actual decision-making situation, where a company’s management board had to define criteria for future deliveries. In the environment of active competition, a decision was adopted to extend the distribution system to include another layer. Unlike under the previous approach, where goods were delivered from a production plant directly to the customer and at the customer’s expense, a new solution assumes that a customer collects goods from one of the 30 new warehouses. However, to estimate the parameters of such a distribution system, one has to have earlier developed an appropriate delivery policy.

2. Joint Transportation-and-Inventory Problems

Planning deliveries which exclusively respond to customer-placed orders (the vehicle routing problem) is a relatively complex problem. If a vehicle can deliver goods to multiple customers without the need to replenish the stock of carried goods, it as a NP-hard problem. In the supply planning for the strategy described above, where it is the vendor who sets the delivery date, one further dimension is added to the space of admissible solutions: delivery date. Moreover, additional conditions need to be taken into consideration, such conditions reflecting the specific task: the vendor assumes the liability for deficits, if any, or for exceeding the maximum allowed quantity of goods.

In operations research, the problems of simultaneous determination of delivery routes and dates are known as inventory routing problems (IRPs). Apart from the two components referred to above (determination of delivery routes and dates), these problems also include numerous further assumptions, which makes the entire IRP problem group so distinct. Among the criteria used to classify IRP problems, the most commonly used include: length of the planning horizon, inclusion/exclusion of storage cost, a customer’s demand (variable or constant in time, expressed deterministically or stochastically), and type of solution searched for. The class of IRP problems in which both the transportation cost and the inventory cost are taken into consideration is known as joint transportation-and-inventory problems (JTIPs).

A JTIP problem concerns a repeated distribution of a uniform product from a vendor to $n$ customers over a given planning horizon $T$. For each customer $i$, the daily product consumption $u_i$ and maximum warehouse capacity $c_i$ (for this product) are given. During the period 0, the state of the warehouse is described as $I_i$. The vendor operates a homogenous fleet of $m$ vehicles with a capacity of $Q$ per vehicle, used to deliver the supplies.

The objective is to minimize the total distribution and inventory cost over the planned horizon $T$, under the assumption that none of the customers reports stockout. A solution to a JTIP problem is a detailed way of distributing goods (delivery quantities and dates, as well as the delivery routes for each vehicle in every period). Such a solution is known as a distribution and stock replenishment policy.

The paper by Beltrami and Bodin (1974) may be deemed a pioneering work regarding IRP problems. This work, presented in the 1970’s, focused on modeling and simple solution techniques. In the following papers by Fisher et al. (1982) and Bell et al. (1983), mixed integer programming was used first to obtain a solution for the IRP instance.
Subsequently, the first approach to solve a large IRP instance was made by Golden et. al. (1984) and by Dror (1985), they investigated the large distribution system of liquid propane to residential and industrial customers. In the former, the basic components of the IPR problem are discussed and the simulation approach with vehicle routing algorithms is proposed. The latter, contains the comparison of several different computational schemes and some computation results are presented.

The IRP research directions can be divided into three main streams. In the first stream, the vehicle routing formulation is extended to take into consideration time horizon and inventory issues. This field was initiated in research by Fisher et al. (1982) and Bell et al. (1983), where the set partitioning formulation is incorporated into a discrete time inventory planning problem. This approach is described as the application of time-discretized integer programming models to determine the set of customers to be visited, as well as the amount of product to deliver. Actually, such problem formulation enables a route covering timetable to be generated. Recent studies in this area include research by Bertazzi et al. (2007) and Campbell and Savelsberg (2004). The first paper covers the analysis of interrelations between transportation costs and inventory costs for a simple task with five customers. The second branch of research analyzes the use of similarly defined mixed integer programming models to determine suggested delivery quantities over a long time horizon. Subsequently, the values thus determined are used in the second phase to compute the exact quantities of delivery over a short time horizon. The most promising formulation in this field was proposed by Archetti et al. (2007) and Archetti et al. (2009). In the first paper, the exact procedure is presented while the second describes the hybrid heuristic where the MIP formulations and tabu search are combined into an iterative algorithm. The average error of hybrid heuristics for small instances (up to 50 customers and up to 6 periods) is lower than 0.1% whereas the maximum error is lower than 2%. In comparison, the heuristic proposed by Bertazzi et al. (2002) guarantees an average error below 3% and a maximum error lower than 15%.

Savelsberg’s results may also be included in the second stream of research, where the planning horizon is shortened through the computation of suggested replenishment quantities over a long time horizon and, based on the results obtained, the subsequent determination a supply timetable and routes for the next few days. The first papers in this branch were: Dror et al. (1985) and Dror and Ball (1987). This approach was extended upon and improved in Trudeau and Dror (1992). The last paper in this stream Bard et al. (1998a, 1998b), works with a rolling horizon of IRP where a short term planning problem is defined for a two-week period. According to the rolling horizon approach only decisions for the first week are implemented.

The third research stream considers the division of a customer base into delivery groups based on their respective demands and other method-specific parameters. Then, each delivery is performed to all customers in a given group with routes determined with use of the classical VRP or TSP algorithms. Such approach is used, among others, in papers Anily, S., Federgruen (1990, 1991 and 1993). Gallego and Simchi-Levi(1990) and Bramel and Simchi-Levi(1995). The most recent use of these ideas was used in a paper by Chan et al. (1998) and Gaur and Fisher (2004).

The last field of research attempts to solve the stochastic version of the IRP problem. This field started with papers by Dror and Trudeau (1986) and Dror Laporte and Trudeau (1989), where authors consider the VRP with stochastic demand. However, due to the fact that in this paper stochastic formulation is not considered, a more detailed review of the paper has been omitted. A complete review of methods and algorithms for solving stochastic IRP problems may be found in Schwarz et al. (2004) or Cordeau et al. (2007).

The research presented herein below represents an attempt to use time-discretized integer programming models to solve a real IRP problem. Given the relatively small size of the problem (30 locations and a 20-day planning horizon), integer programming has been used to determine a distribution and stock replenishment policy. The solution has been obtained with a model based upon Bertazzi et al. (2007). Further, in order to reduce the number of variables in the model and accelerate the optimizing process, a modification of the original model has been suggested. A new formulation assumes predefined quantities of delivery. Moreover, the model having been simplified, a column generation technique is applicable. Despite research into relevant literature, the author has not been able to identify existing studies using this technique to solve IRP problems.

3. Formulation proposals
The formulation of the inventory and transportation model presented below is a set partitioning formulation for solving the vehicle routing problem with a discreet, cyclic supply planning model. The set partitioning formulation has been selected for the possibility of precise determination of carried-out routes. Such approach is very often put into practice where the decision-maker takes into account in the first place, the specificity of the problem and the area where transportation is carried out and determines a set of routes possible to carry out.

In the formulation presented by the formulas (4)-(11) the indexes \(i, j \) and \(k\) are, respectively, customers, routes and periods. The parameter \(a_{i,j}\) contains routes possible to carry out. The element has a value of 1, when a customer \(i\) is serviced in a route \(j\) and of 0 otherwise. The parameters \(b_i\) and \(c_j\) stand respectively for the costs of inventory of a goods unit with a customer \(i\) and for the costs of carrying out of a route \(j\). Then the parameter \(d_i\) is the customer demands, lastly \(g_i\) is the minimum inventory level to be kept at customer \(i\). Additionally, the capacity of a vehicle used in transportation is denoted by an \(L\) symbol. The parameter \(M\) is also used in the formulation, signifying positive value greater than the sum of all recipient demands in the analyzed horizon. This parameter is used in the model for technical reasons.

Three groups of decision variables were used in the formulation. The group of binary \(x_{j,k}\) variables should be enumerated as the first one. They have the value of 1 when a route \(j\) is carried out in a period \(k\) and of 0 otherwise. Further, two groups of variables related to supplies were used \(-\ y_{i,k}\ and \ z_{i,j,k}\). The first is the stock in customer \(i\) in a period \(k\), while the second is the delivery quantity to a customer \(i\) in a route \(j\) and in a period \(k\).

\[
\text{Minimize} \quad \sum_{j=1,...,m; k=1,...,p} c_j x_{j,k} + \sum_{i=1,...,n; k=1,...,p} b_i y_{i,k} \\
\text{subject to} \quad \sum_{i=1,...,n} a_{i,j} x_{j,k} M \leq 0 \quad \forall \ i = 1,...,n, \ j = 1,...,m, \ k = 1,...,p , \quad (2)
\]

\[
\sum_{i=1,...,n} z_{i,j,k} L \leq 0 \quad \forall \ j = 1,...,m, \ k = 1,...,p , \quad (3)
\]

\[
y_{i,k} + \sum_{j=1,...,m} z_{i,j,k} - d_i - y_{i,k+1} = 0 \quad \forall \ i = 1,...,n, \ k = 1,...,p-1 , \quad (4)
\]

\[
y_{i,k} + \sum_{j=1,...,m} z_{i,j,k} - d_i - y_{i,1} = 0 \quad \forall \ i = 1,...,n, \ k = 1,...,p , \quad (5)
\]

\[
x_{j,k} \in \{0,1\} \quad \forall \ j = 1,...,m, \ k = 1,...,p , \quad (6)
\]

\[
y_{i,k} \geq g_i \quad \forall \ i = 1,...,n, \ k = 1,...,p , \quad (7)
\]

\[
z_{i,j,k} \geq 0 \quad \forall \ i = 1,...,n, \ j = 1,...,m, \ k = 1,...,p . \quad (8)
\]
The goal function expressed by the formula (1) ensures minimizations of joint costs in the entire planning horizon. The first sum components are the transportation costs, while the second one are the inventory costs. The constraint (2) provides the combination of the $z_{i,j,k}$ and $x_{j,k}$ variables. It guarantees that deliveries will be carried out only in periods when a route encompassing a given customer will be executed. The constraint (3) is responsible for maintaining the maximum vehicle capacity. The formulas (4) and (5) express the constraints guaranteeing the continuity of the inventory policy. Thus, the constraint (4) guarantees continuity in the periods from 1 to $p-1$. Regarding the constraint (5), as it is assumed that transportation is cyclic, combines states from the period $p$ with those from the period 1. The constraints (6), (7) and (7) are bounds for the decision variables $x$, $y$ and $z$.

The decision variable in the presented model guarantees the optimal delivery quantity. However, due to a large number of decision variables in this group (the customers, routes and periods product) a negative impact concerning the solution of this formulation occurs. In order to use this formulation for practical instances we are able to take into consideration only the small set of routes or we need to implement the column generation technique.

The observation of real transportation systems in which one of the crucial factors in evaluating transportation is the maximum use of vehicle capacity suggests an approach in which not only a customer’s presence to a route will be indicated in a route’s definition, but it will also contain the delivery’s quantity. According to this observation, a new formulation is proposed where the delivery quantities are given. The main difference between the previously presented model and proposed formulation here lies in the way of incorporating into the model, the delivery quantities. While in the former model the value of delivery quantities is represented as decision variables, in the latter, the delivery quantities are represented as parameters. The model expressed by the formulas (9)-(13) instead of the binary parameter $a_{i,j}$ uses the $e_{i,j}$ parameter, the value of which determines the delivery quantity to a customer $i$ in a route $j$. In this formulation only two groups of decision variables are used - $x_{j,k}$ and $y_{i,k}$. Their interpretation is analogical to the one from the previous formulation.

\[
\text{Minimize} \quad \sum_{j=1, \ldots, m, k=1, \ldots, p} c_j x_{j,k} + \sum_{i=1, \ldots, n, k=1, \ldots, p} b_i y_{i,k}
\]

subject to

\[
y_{i,k} + \sum_{j=1, \ldots, m} e_{i,j} x_{j,k} - d_i - y_{i,k+1} = 0 \quad \forall \; i = 1, \ldots, n, \; k = 1, \ldots, p-1, \quad (10)
\]

\[
y_{i,k} + \sum_{j=1, \ldots, m} e_{i,j} x_{j,k} - d_i - y_{i,k+1} \geq 0 \quad \forall \; i = 1, \ldots, n, \; k = 1, \ldots, p, \quad (11)
\]

\[
x_{j,k} \in \{0,1\}
\]

\[
y_{i,k} \geq g_i \quad \forall \; i = 1, \ldots, n, \; k = 1, \ldots, p. \quad (13)
\]

The constraint (10) ensures the continuity of the inventory policy. The constraint (11) is a modified version of the constraint (5) from the previous model. Its task is to provide cyclic process in the application of the specified inventory policy. In this case, however, due to the delivery quantities defined as parameters of the problem which could reduce the set of possible solutions in a too radical manner, the sign of equality was replaced by the sign of minority. In this situation, the described constraint should make admissible transportation policies in which, in the planning horizon, to each customer at least deliveries in compliance with the customer’s demand will be carried out.

When proceeding with predefined delivery quantities the fundamental problem is the one regarding generation of routes. Apart from checking the feasibility of a route and determining its costs before the start of the optimization process it is indispensable to determine the delivery quantities carried out in a route. The column generation technique presented in this paper consists of using two presented formulation by turns. The solution to the first
formulation suggests appropriate delivery quantities for the second, while the dual variables analysis of the second formulation is applied to the extension of the routes definition set in the first one.

The optimization procedure of solving the JITP problem proposed in this paper consists of using two presented formulation iteratively. The solution to the first formulation suggests appropriate delivery quantities for the second, while the dual variable analysis of the second formulation is applied to the extension of the routes definition set in the first one. The optimization process starts with solving the former formulation with a restricted set of routes. Only the routes to one customer are taken into consideration in the first step of the first iteration. In the second step, the calculation in the first step, quantities of deliveries, are incorporated into the latter formulation and the problem is solved. Finally, the analysis of optimal dual variables associated with constraints (10) and (11) are conducted and the routes with negative reduced cost are found and added to the first formulation.

In order to generate additional routes the dual formulation to the second formulation (9)-(13) is taken into consideration (14)-(19). The indexes and parameters are the same as in the primal formulation.

\[
\text{Maximize} \quad \sum_{i=1}^{n} \sum_{k=1}^{p} d_i \pi_{i,k}
\]

subject to

\[\pi_{i,k} + \pi_{i,k+1} \leq b_i \quad \forall \ i = 1,\ldots,n, \ k = 1,\ldots,p-1, \quad (15)\]
\[\pi_{i,k} + \pi_{i+1,k} \leq b_j \quad \forall \ i = 1,\ldots,n, \ k = 1,\ldots,p, \quad (16)\]
\[\sum_{i=1}^{n} \sum_{k=1}^{p} e_{i,j} \pi_{i,k} \leq c_j \quad \forall \ j = 1,\ldots,m, \quad (17)\]
\[\pi_{i,k} \in \mathbb{R} \quad \forall \ i = 1,\ldots,n, \ k = 1,\ldots,p-1, \quad (18)\]
\[\pi_{i,k} \geq 0 \quad \forall \ i = 1,\ldots,n, \ k = p. \quad (19)\]

Every route in the second primal formulation corresponds to one of the constraints (17) in the dual formulation. Let \(\pi_{i,k}\) be the corresponding optimal dual variables associated with constraints (10) and (11). If \(\pi_{i,k}\) in the current solution satisfies every constraint (17), then the current solution is the optimal one. Consequently, if we can find a route definition which violates one of the constraints (17) i.e. the route for which constraint (20) is met then the route just found is added to the first formulation, which is solved once again. The optimization process repeats until no violated constraints are found.

\[\sum_{i=1}^{n} \sum_{k=1}^{p} e_{i,j} \pi_{i,k} > c_j \quad (20)\]

The specific route generation procedure wasn’t taken into consideration. In the practical problem, by which this research is motivated, the company took into consideration only switching from routes to only one customer into routes with either one or two customers. According to this business assumption, we needed to consider only restricted set of routes. It results in simple route generation procedure where the whole set of the feasible routes was searched in order to determine the routes which meet the constraints (20).

4. Application Example

Formulations presented in the previous section were used to solve a real decision-making problem. As already mentioned, the direct stimulus for commencing research was a decision-making situation which required that a company’s distribution system be modified. However, the need to define an appropriate delivery policy proved the key problem in estimating parameters of a new system. The basic question to be answered was whether combining supplies to multiple regional warehouses may result in material savings. The targeted solution assumes that the
business in question would use external warehouses owned by businesses which would take over the operation of the external warehouses and delivery of goods to customers. However, the fee for the operators’ services depends on the quantities of inventories stored in the warehouses. This requires that both the transportation cost and the inventory cost be included in the model used.

The analyzed distribution system comprises of 30 regional warehouses and one central warehouse located next to the production plant. The detailed data is set forth in Table 1. Preliminary values of daily distribution and inventory cost were also set: at PLN 2/km and PLN 1.2/T, respectively. The capacity of vehicles used in the distribution process is 25T. The presented analysis refers to a planned distribution system and accordingly, a sensitivity analysis is envisaged for certain parameters (in particular, financial parameters).
Table 1. Distribution system data

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<th>Locations</th>
<th>Longitude</th>
<th>Latitude</th>
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<th>Safety stock [T]</th>
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In this situation, the main research objective is to define the possible formulation of and restrictions for mixed integer programming for IRPs, assuming the use of state-of-the-art optimizing tools. The use of a column generation algorithm has also been planned, for the first time ever in formulations of this type. Further, the preliminary formulation has been modified with a view to reducing the number of variables in the model and acceleration, if possible, of the solution finding procedure. The use of the second formulation is feasible thanks to the use of the column generation technique.

The first phase of the research included an attempt to determine the minimum monthly cost of distribution system operation, assuming routes to a single customer are covered only. The exact formulation (1)–(8) has been used to solve the instance. The minimum monthly cost of distribution system operation obtained in the solution amounts to PLN 178,296. Then a solution was obtained taking into consideration all routes to single customers and routes to
pairs of customers (regional warehouses). For such an instance, including a set of 465 routes, an attempt to find an optimal solution within one hour proved unsuccessful. The result obtained was 3.3% worse than the lower bound. The minimum cost of distribution system operation stood at PLN 177,215. The obtained solution contains all the direct routes and only one route included two locations.

In the second phase of the research the column generation technique was implemented. For the initial set of routes, the 30 routes to single customers were only taken into consideration. At the beginning of the optimization process the optimal solution was determined using the first formulation. In the second step we calculate the solution using the second model. The quantity of the delivery $e_{i,j}$ in this step was based on the solution obtained in the first formulation. Lastly, based on the analysis of values of dual variables corresponding to constraints (10), routes to pairs of customers were added to the initial set of routes. Then the next iteration was started by solving the first formulation with modified set of routes. After four iterations (in each of which the optimizing problem was solved and additional routes were generated) a solution was arrived at. For this solution, the monthly cost of a distribution system operation amounted to PLN 176,424. A total of nine new routes, each visiting two customers, were added to the original set of routes. Only two of them were used in optimal solution. The solving time stood at 15 minutes.

In order to test the presented approach on more than one test instance, the next 30 instances was generated. The company didn’t provide us with more real data because of business matters. The generated test instances have the following characteristics: time horizon 20, number of customers 30, demand at customer randomly generated as an integer number between the interval $[6, 25]$, and constant over time for a single customer. The costs parameters are constant with the real life test instance. As previous, only routes consisting of 1 or 2 customers are taken into consideration. The proposed formulations have been implemented in AIMMS and run on an Intel Dual Core 1.86 GHz with 3 GB RAM personal computer. The first formulation was used to solve the test instances. In all occurrences, within one hour, the optimal solution wasn’t obtained. The average gap between lower bound and the solution achieved in one hour is 4.2%. In the second approach, where the column generation technique was implemented the solution was obtained in less than 15 minutes. The cost of distribution system operation for the solution thus obtained was slightly lower than for the solutions found within the first formulation.

5. Conclusion

Two formulations of mixed integer programming used to solve the inventory routing problem are presented in this paper. The use of the first formulation, based on Bertazzi et al. (2007), has proved that the obtainment of an optimal solution globally is infeasible. However, with the solving time limited to one hour, the solutions obtained were by at most 3.3% worse than the lower bound. The second analyzed formulation assumed predefined quantities of delivery. This formulation was developed with a view to such a simplification of a model of the inventory routing problem which would make the use of the column generation technique feasible. Upon such a modification, with use of the proposed approach to solve a testing problem, a solution was obtained within 15 minutes. The cost of distribution system operation for the solution thus obtained was slightly lower than for the solutions found within the first formulation.

It should be emphasized, though, that the limitation of the maximum number of customers visited along a single route had a material effect on the results obtained in the tests presented here. In the case analyzed, the limitation directly resulted from the decision-making situation, giving rise to the research reported here. Undoubtedly, the limitation significantly simplified the solving process within the first formulation. The column generation technique, used with the second formulation, is designed to limit the number of routes considered in the model. However, it is far from certain whether this formulation would support equally short solving process in the case of a larger number of customers visited along a single route. A detailed analysis of the presented column generation technique, including its possibilities and limitations, will certainly be the subject of future research.

From the business perspective, the results obtained will support making a decision regarding whether combined deliveries should be taken into consideration in planning the operation of the distribution system, which is under development. To maintain confidentiality, the storage cost was averaged, which renders adopting an unequivocal decision based on the results presented difficult. On the other hand, the research has proved that combining
deliveries may give measurable savings but the solving process is complex, time-consuming, and requires the use of advanced tools.

References


