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Right-handed electrons in radiative muon decay

L.M. Sehgal

Institute of Theoretical Physics (E), RWTH Aachen, D-52056 Aachen, Germany

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Abstract

Electrons emitted in the radiative decay $\mu^- \to e^- \bar{\nu}_e \nu_\mu \gamma$ have a significant probability of being right-handed, even in the limit $m_e \to 0$. Such "wrong-helicity" electrons, arising from helicity-flip bremsstrahlung, contribute an amount $\frac{\alpha}{4\pi}\Gamma_0$ to the muon decay width ($\Gamma_0 \equiv G_F^2 m_\mu^5/(192\pi^3)$). We use the helicity-flip splitting function $D_{hf}(z)$ of Falk and Sehgal [Phys. Lett. B 325 (1994) 509] to obtain the spectrum of the right-handed electrons and the photons that accompany them. For a minimum photon energy $E_\gamma = 10$ MeV (20 MeV), approximately 4% (7%) of electrons in radiative μ-decay are right-handed.

It is usually thought that in V–A theory, electrons emitted in muon decay are purely left-handed, in the limit $m_e \to 0$. This statement, however, is not true for electrons in the radiative decay $\mu^- \to e^- \bar{\nu}_e \nu_\mu \gamma$, where the photon is the result of inner bremsstrahlung. We show in this Letter that radiative muon decay contains a well-defined constituency of right-handed electrons, contributing an amount $\frac{\alpha}{4\pi}\Gamma_0$ ($\Gamma_0 \equiv G_F^2 m_\mu^5/(192\pi^3)$) to the decay width. We calculate the spectrum of these "wrong-helicity" electrons, and of the photons that accompany them. These spectra are compared with the unpolarized spectra, summed over electron helicities. This comparison provides a quantitative measure of the right-handed fraction and its distribution in phase space.

The appearance of "wrong-helicity" electrons in the decay $\mu^- \to e^- \bar{\nu}_e \nu_\mu \gamma$, even in the limit $m_e \to 0$, is a consequence of helicity-flip bremsstrahlung in quantum electrodynamics, a feature first noted by Lee and Nauenberg [1]. It was found that in the radiative scattering of electrons by a Coulomb field, the probability of helicity-flip (i.e., $e_L \to e_R$ or $e_R \to e_L$) did not vanish in the limit $m_e \to 0$. This unexpected result, in apparent contradiction to the naive expectation of helicity conservation in the $m_e \to 0$ limit, arises from the fact, that the helicity-flip cross section for bremsstrahlung at small angles has the form

$$\frac{d\sigma}{d\theta^2} \sim \frac{\left(\frac{m_e}{E_e}\right)^2}{\left(\left(\frac{m_e}{E_e}\right)^2 + \theta^2\right)^2}, \tag{1}$$

which, when integrated over angles, gives a finite non-zero answer in the limit $m_e \to 0$.

In Ref. [2], Falk and Sehgal examined the helicity structure of the bremsstrahlung process in an equivalent particle approach, and showed that helicity-flip radiation $e_L^- \to e_R^- + \gamma(z)$, in the limit $m_e \to 0$, can be described by a simple and universal splitting (or frag-

E-mail address: sehgal@physik.rwth-aachen.de (L.M. Sehgal).

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mentation) function

$$D_{hf}(z)=\frac{\alpha}{2\pi}z, \tag{2}$$

where $z=E_\gamma/E_e$ is the ratio of the photon energy to the energy of the radiating electron. This function is analogous to the familiar Weizsäcker–Williams function describing helicity-conserving (non-flip) bremsstrahlung

$$D_{nf}(z)=\frac{\alpha}{\pi}\frac{1+(1-z)^2}{z}\log\left(\frac{E_e}{m_e}\right). \tag{3}$$

Several applications of the helicity-flip function $D_{hf}(z)$ were considered in [2], including the process $e_R^- + p\to\nu_L+\gamma+X$ ("fake right-handed currents") and $e_\lambda^- e_\lambda^+ \to f\bar{f}\gamma$ (wrong-helicity e^+e^- annihilation). It was shown that the splitting function approach reproduced the results of the usual bremsstrahlung calculation in which the limit $m_e\to 0$ was taken at the end [3,4]. Subsequently, the equivalent-particle technique has been successfully applied to other helicity-flip processes such as $\pi^-\to e_L^-\bar{\nu}\gamma$ and $Z^0\to e_L^- e_L^+\gamma$ [5].

Recently, in an analysis of radiative corrections to the electron spectrum in muon decay, $\mu^-\to e^-\bar{\nu}_e\nu_\mu$, Fischer et al. [6] have noted that the radiative correction to the helicity of the electron, calculated in an early paper by Fischer and Scheck [7], can be reproduced in a simple way using the helicity-flip function $D_{hf}(z)$. This has motivated us to examine the helicity-dependence of the radiative decay $\mu^-\to e^-\bar{\nu}_e\nu_\mu\gamma$, to determine the incidence and spectrum of wrong-helicity (right-handed) electrons in this channel.

The electron spectrum in ordinary (non-radiative) muon decay $\mu^-\to e^-\bar{\nu}_e\nu_\mu$ has, in Born appoximation, the well-known form

$$\left(\frac{d\Gamma}{dx\,d\cos\theta_e}\right)^{\text{non-rad}} = \Gamma_0\big[x^2(3-2x)+x^2(1-2x)\cos(\theta_e)\big], \tag{4}$$

where $x=2E_e/m_\mu$ and θ_e is the angle of the electron relative to the spin of the muon. We can obtain from this the spectrum of the radiative channel $\mu^-\to e^-\bar{\nu}_e\nu_\mu\gamma$, using the splitting functions D_{hf} or D_{nf}. In the specific case of right-handed electrons in the final state, the spectrum, in the limit $m_e\to 0$ (collinear bremsstrahlung), is given by

$$\left(\frac{d\Gamma}{dx_e\,d\cos\theta_e}\right)^{\text{rad}}_{e_R^-} = \int_0^1 dx\int_0^1 dz\left(\frac{d\Gamma}{dx\,d\cos\theta_e}\right)^{\text{non-rad}} D_{hf}(z) \times\delta\big(x_e-x(1-z)\big)\theta(xz-x_{\gamma 0}), \tag{5}$$

where the θ-function in the integrand has been inserted to allow for a minimum energy cut on the photon:

$$x_\gamma\equiv\frac{2E_\gamma}{m_\mu}\geqslant x_{\gamma 0}. \tag{6}$$

The result of the integration is

$$\left(\frac{d\Gamma}{dx_e\,d\cos\theta_e}\right)^{\text{rad}}_{e_R^-} = \Gamma_0\frac{\alpha}{2\pi}\big[A(x_e,x_{\gamma 0})+\cos(\theta_e)B(x_e,x_{\gamma 0})\big], \tag{7}$$

where

$$\begin{aligned} A(x_e,x_{\gamma 0}) &= -\frac{2}{3}\big[1-(x_e+x_{\gamma 0})^3\big] + \frac{1}{2}\big[1-(x_e+x_{\gamma 0})^2\big](2x_e+3) - 3x_e\big[1-(x_e+x_{\gamma 0})\big],\\ B(x_e,x_{\gamma 0}) &= -\frac{2}{3}\big[1-(x_e+x_{\gamma 0})^3\big] + \frac{1}{2}(1+2x_e)\big[1-(x_e+x_{\gamma 0})^2\big] - x_e\big[1-(x_e+x_{\gamma 0})\big]. \end{aligned} \tag{8}$$

Integrating over $\cos\theta_e$ and x_e $(0\leqslant x_e\leqslant 1-x_{\gamma 0})$, we obtain

$$\Gamma^{\text{rad}}_{e_R^-}(x_{\gamma 0})=\Gamma_0\frac{\alpha}{\pi}\left[\frac{1}{4}-x_{\gamma 0}^2+x_{\gamma 0}^3-\frac{1}{4}x_{\gamma 0}^4\right]. \tag{9}$$

If no cut is imposed on the photon energy (i.e., $x_{\gamma 0}=0$) the spectrum of right-handed electrons given in Eq. (7) reduces to

$$\left(\frac{d\Gamma}{dx_e\,d\cos\theta_e}\right)^{\text{rad}}_{e_R^-} = \Gamma_0\frac{\alpha}{2\pi}\frac{1}{6}(1-x_e)^2 \times\big[(5-2x_e)-\cos(\theta_e)(2x_e+1)\big], \tag{10}$$

which coincides with the result obtained by Fischer and Scheck [7].

The helicity-flip fragmentation function also gives a simple way of calculating the spectrum of photons accompanying right-handed electrons in $\mu^- \to e^- \bar{\nu}_e \nu_\mu \gamma$. In the collinear limit ($m_e \to 0$), we have

$$\left(\frac{d\Gamma}{dx_\gamma\, d\cos\theta_\gamma}\right)^{\text{rad}}_{e^-_R} = \int_0^1 dx \int_0^1 dz \left(\frac{d\Gamma}{dx\, d\cos\theta_e}\right)^{\text{non-rad}}_{\theta_e=\theta_\gamma} \times D_{hf}(z)\delta(x_\gamma - xz) = \Gamma_0 \frac{\alpha}{2\pi} x_\gamma (1-x_\gamma)\big[(2-x_\gamma) - x_\gamma \cos(\theta_\gamma)\big]. \quad (11)$$

Integrating over all photon energies and over $\cos\theta_\gamma$ we get $\Gamma^{\text{rad}}_{e^-_R} = \frac{\alpha}{4\pi}\Gamma_0$, which is the same as Eq. (9) for $x_{\gamma 0} = 0$.

The decay width into right-handed electrons, for a given minimum energy $x_{\gamma 0}$ (Eq. (9)), can be compared with the width summed over electron helicities. The helicity-summed photon spectrum in $\mu^- \to e^- \bar{\nu}_e \nu_\mu \gamma$ was calculated by Kinoshita and Sirlin [8] and Eckstein and Pratt [9], and the integrated width, for $x_\gamma > x_{\gamma 0}$ is [9]

$$\Gamma^{\text{rad}}_{e^-_L+e^-_R}(x_{\gamma 0}) = \Gamma_0 \frac{2\alpha}{\pi}\Bigg\{\left(-\frac{17}{12} + \log\left(\frac{m_\mu}{m_e}\right)\right)\log\left(\frac{1}{x_{\gamma 0}}\right) - \frac{1}{2}(1-x_{\gamma 0})\left[\frac{1}{6}(1-x_{\gamma 0})^3 + 1\right] \times \log\left[\frac{m_\mu^2}{m_e^2}(1-x_{\gamma 0})\right] + \frac{1-x_{\gamma 0}}{288}\big(601 - 159x_{\gamma 0} + 171{x_{\gamma 0}}^2 - 61{x_{\gamma 0}}^3\big) - \frac{\pi^2}{12} + \frac{1}{2}\sum_{n=1}^{\infty}\frac{(x_{\gamma 0})^n}{n^2}\Bigg\}. \quad (12)$$

This function is plotted in Fig. 1 and compared with the right-handed width $\Gamma^{\text{rad}}_{e^-_R}(x_{\gamma 0})$ calculated in Eq. (9). The right-handed fraction $\Gamma^{\text{rad}}_{e^-_R}/\Gamma^{\text{rad}}_{e^-_R+e^-_L}$ is shown in Fig. 2, as a function of $x_{\gamma 0}$. For a photon energy cut $E_\gamma > 10$ MeV (20 MeV), this fraction is approximately 4% (7%). (It may be noted here that the branching ratio of $\mu^- \to e^- \bar{\nu}_e \nu_\mu \gamma$, summed over electron spins, with a photon energy cut $E_\gamma > 10$ MeV, was measured in Ref. [10] to be (1.4 ± 0.4)%. The theoretical expression Eq. (12) yields for this quantity the value 1.3%.)

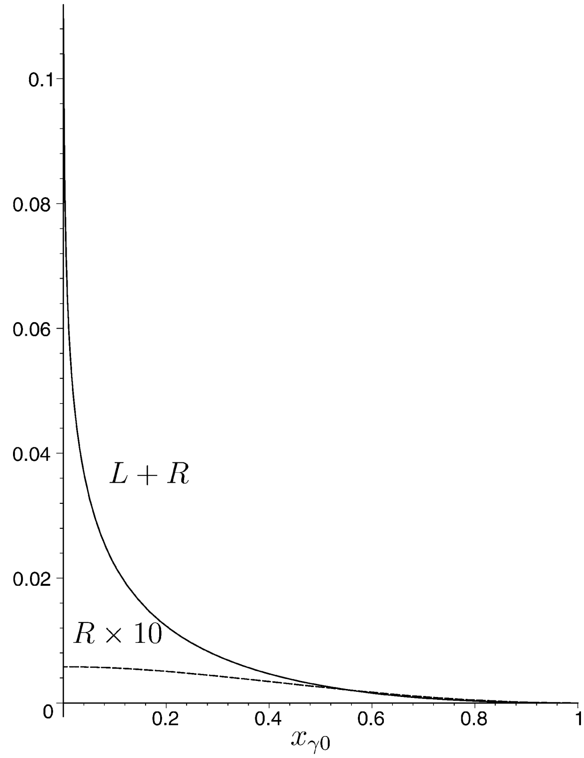

Fig. 1. Radiative decay width $\Gamma^{\text{rad}}_{e^-_L+e^-_R}(x_{\gamma 0})$ (full line), compared with the right-handed decay width $\Gamma^{\text{rad}}_{e^-_R}(x_{\gamma 0})$ (multiplied by factor 10, dashed line) as function of minimum photon energy $x_{\gamma 0}$. Decay widths in units of Γ_0.

A complete analysis of the channel $\mu^- \to e^- \bar{\nu}_e \nu_\mu \gamma$ involves a study of the decay intensity in all kinematical variables. A variable of particular interest is the angle $\theta_{e\gamma}$ between the electron and the photon. For helicity-flip radiation, the characteristic angular distribution is [2]

$$\frac{dD_{hf}(z,\theta^2)}{d\theta^2} \approx \frac{\alpha}{2\pi} z \left(\frac{m_e}{E_e}\right)^2 \frac{1}{\big[\theta^2 + \big(\frac{m_e}{E_e}\big)^2\big]^2}, \quad (13)$$

which is maximum at $\theta = 0$ (forward direction). By contrast, the helicity-conserving bremsstrahlung has

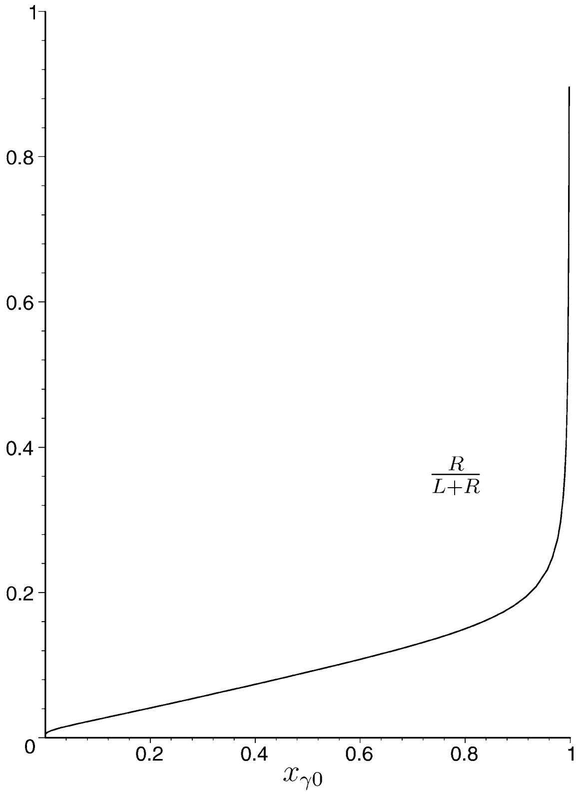

Fig. 2. Right-handed fraction $\Gamma^{\text{rad}}_{e^-_R}/\Gamma^{\text{rad}}_{e^-_R+e^-_L}$ as function of minimum photon energy $x_{\gamma 0}$.

the spectrum [2]

$$\frac{dD_{nf}(z,\theta^2)}{d\theta^2} \approx \frac{\alpha}{2\pi}\frac{1+(1-z)^2}{z}\frac{\theta^2}{\left[\theta^2+\left(\frac{m_e}{E_e}\right)^2\right]^2}, \quad (14)$$

which peaks at $\theta^2 \approx m_e^2/E_e^2$. This suggests that in the decay $\mu^- \to e^-\bar{\nu}_e\nu_\mu\gamma$, the distribution in the angle $\theta_{e\gamma}$ between the electron and photon could be a useful discriminant in separating the two electron helicities. A full analysis of the helicity-dependent decay spectrum in different kinematical variables will be reported elsewhere.

Summary:

(i) The decay $\mu^- \to e^-\bar{\nu}_e\nu_\mu\gamma$ contains in the final state a constituency of right-handed electrons, which contribute an amount $\frac{\alpha}{4\pi}\Gamma_0$ to the decay width, in the limit $m_e \to 0$.

(ii) The spectrum of the right-handed electrons is given by Eq. (7), and reduces to Eq. (10) if no cut on photon energy is imposed. The latter differs in a characteristic way from the spectrum of left-handed electrons, which (on account of the soft $1/x_\gamma$ nature of helicity-conserving bremsstrahlung) tends to follow the non-radiative pattern Eq. (4). Thus the energy spectra, integrated over angles are $(d\Gamma/dx_e)_R \sim (1-x_e)^2(5-2x_e)$, $(d\Gamma/dx_e)_L \sim x_e^2(3-2x_e)$, while the angular distribution, integrated over energies, is $(d\Gamma/d\cos\theta_e)_{L,R} \sim (1-\frac{1}{3}\cos\theta_e)$, the same for e^-_L and e^-_R.

(iii) The photon spectrum associated with right-handed electrons is $(d\Gamma/dx_\gamma)_R \sim x_\gamma(1-x_\gamma)(2-x_\gamma)$, and is hard compared to that accompanying left-handed electrons $(d\Gamma/dx_\gamma)_L \sim 1/x_\gamma$.

(iv) The right-handed fraction $\Gamma_{e^-_R}/(\Gamma_{e^-_R}+\Gamma_{e^-_L})$ has been calculated as a function of the photon energy cut $x_{\gamma 0}$, and amounts to 4% (7%) for $E_\gamma > 10$ MeV (20 MeV).

(v) The radiatively corrected decay width of the muon, usually written as

$$\Gamma_\mu = \Gamma_0\left[1+\frac{\alpha}{\pi}\left(\frac{25}{8}-\frac{\pi^2}{2}\right)\right] \quad (15)$$

can be regarded as a sum of two mutually exclusive helicity contributions [6]

$$\Gamma_\mu = \Gamma_\mu(e^-_L) + \Gamma_\mu(e^-_R),$$

where

$$\Gamma_\mu(e^-_L) = \Gamma_0\left[1+\frac{\alpha}{\pi}\left(\frac{23}{8}-\frac{\pi^2}{2}\right)\right],$$

$$\Gamma_\mu(e^-_R) = \Gamma_0\frac{\alpha}{\pi}\frac{1}{4}.$$

(vi) A full analysis of $\mu^- \to e^-\bar{\nu}_e\nu_\mu\gamma$, aimed at finding regions of phase space with enhanced concentration of right-handed electrons will be reported elsewhere.

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