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Cardiac CT Image Reconstruction Based on Compressed Sensing

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Abstract

In this paper, the authors review the compressed sensing concept and analyze the basics for CT image reconstruction. The application of compressed sensing is investigated in cardiac CT image reconstruction and a comprehensive sparse reconstruction approach is presented. This approach incorporates the total variation norm, l_1 norm and subtraction of different images norm minimization to reconstruct image from Sheep-Logan and Cardiac phantom. The reconstruction results show that our algorithm can achieve high quality cardiac image.

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Keywords: Compressed sensing; Norm; Cardiac image; Reconstruction

1. Introduction

In signal acquisition and processing, one always seeks efficient way to facilitate signal processing procedure. In general signal processing, the first step is to sample and convert the analog signal into digital signal. The Shannon/Nyquist sampling theory states that the sampling rate for signal should be more than twice the highest frequency of signal at least so as to avoid the signal aliasing. This theory has been widely utilized since that time. However, over the last few years, compressed sensing theory enables the complete recovery of signal by using far below Nyquist sampling rate if signal is sparse enough. Recently, more and more researches have been made in compressed sensing and signal recovery [1-3]. In CT image reconstruction, the small number of projection is captured to reduce the radiation dose by compressed sensing technique. Hence, it is an interested and significant research to apply compressed

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sensing theory in CT image reconstruction. In this paper, the authors apply a comprehensive compressed sensing method to reconstruct cardiac CT image. The reconstruction images are showed to verify our algorithm.

2. Signal compressed sensing

2.1. Signal Representation

The basic aim of compressed sensing reconstruction is to reconstruct an image from fewer than necessary number of measurements without information loss of the signal. The signal measurement is the process to decompose signal f as superposition of signal basic components, such as orthogonal basis or tight frame. There are many orthogonal basis functions, e.g., sinusoids, wavelet, curvelet and Gabor function. If basis function is chosen suitably, the signal representation will be much simpler than original signal. Although the signal is not usually sparse in time domain, it may be sparse in transform domain. For example, discrete cosine transform DCT is typical signal transform because most energy of signal is located at low frequencies. Therefore, signal is often sparse in DCT domain. Moreover, the wavelet transform of signal is more effective transform which would transform signal in different resolution level. There are few large wavelet coefficients and many small wavelet coefficients. Generally speaking, the wavelet has more sparse representation than DCT [4].

2.2. Compressed sensing

Sparse signal means many zero components in signal. Assume the number of signal samples is N and the signal is only nonzero on S samples. If signal is sufficient sparse, M random samples can be captured to recover the signal with $M \geq C.S \log N$ condition. The linear measurement can be expressed as $Ax=b$. Where A is an m -by- n matrix and b is the set of actual measurement. For linear systems, there are different types of linear systems $Ax=b$. If $m=n$ is satisfied and matrix A is invertible, there exists a unique solution is $x=A^{-1}b$. In practical case, $m \ll n$ is usually satisfied. The underdetermined system has an infinite many solutions [5]. We can solve the question by prior knowledge and constrain. It requires the utility of the signal sparse character. The challenge is that zero-norm is not a norm and combinatorial search is exponential time. This solution can be replaced by easier optimization l_1 -norm.

$$x^* = \operatorname{argmin} \|x\|_1 \text{ s.t. } Ax=b \quad (1)$$

As stated above, the compressed sensing indicate that a sparse signal $x \in R^N$ can be recovered from a small number of m linear measurements $b = Ax$ by solving a convex program ($m \ll N$). The problems can be classified into linear programs (LPs) and second-order cone programs (SOCPs). The LPs are solved using a generic path-following primal-dual method [6]. The SOCPs are solved with a generic log-barrier algorithm. Alternatively, greedy approaches such as orthogonal pursuits (OMP) can be used to find sparse solution in certain cases.

2.3. Sparse transform

If the signal x is not sparse in time domain, we need find a sparse transform ψ to sparsify the signal and use l_1 minimization to recover the signal.

$$x^* = \operatorname{argmin} \alpha \|\psi x\|_1 + \frac{1}{2} \|Ax-b\|_2^2 \text{ s.t. } Ax=b \quad (2)$$

$\psi(x)$ can be DFT, wavelet and total variation of x . α is the control parameter.

3. Signal compressed sensing

The CT image reconstruction based on compressed sensing is the reconstruction of sparse images from very few projections by means of solving a minimization problem. For CT imaging, this can be applied for achieving good reconstruction from a limited projections or views, thereby reducing the radiation dose.

Compressed sensing is a promising method for image reconstruction in CT imaging. Practically, the medical image often has smoothly varying pixel values, which can be forced during reconstruction by a total variation penalty term [7].

$$x^* = \underset{x}{\operatorname{argmin}} \alpha \|TV(x)\|_1 + \frac{1}{2} \|Ax - b\|_2^2 \quad \text{s.t. } Ax = b \quad (3)$$

3.1. Sheep-Logan Image Reconstruction

Sheep-Logan phantom for CT reconstruction is piecewise constant image. Obviously, this image is sparse in the gradient space. The calculation shows that the amount of zero components in original phantom image is only 50% and however the amount of zero components in gradient image is up to 96% [7]. Hence, total variation can be used to reconstruct image efficiently. In total variation regularization, the initial image is reconstructed by sparse limited projection. In first step, standard FBP algorithm is utilized for initial image reconstruction. In second step, the iterative algorithm is used to reconstruct the final image. The related parameters are defined to control iterative procedure. These parameters include iterative times, desired precision and iterative step.

The 256×256 Sheep-Logan image is shown in Fig 1(a). Only 30 views are used to capture projection. Each view has 256 projections. The reconstruction image is shown in Fig 1(b). Log-barrier method is used to solve the l_1 -norm minimum problem in which Newton iterative method followed by the backtracking line search is applied. The image is then reconstructed exactly using TV norm method in Fig 1(c). The algorithm is followed as [6]:

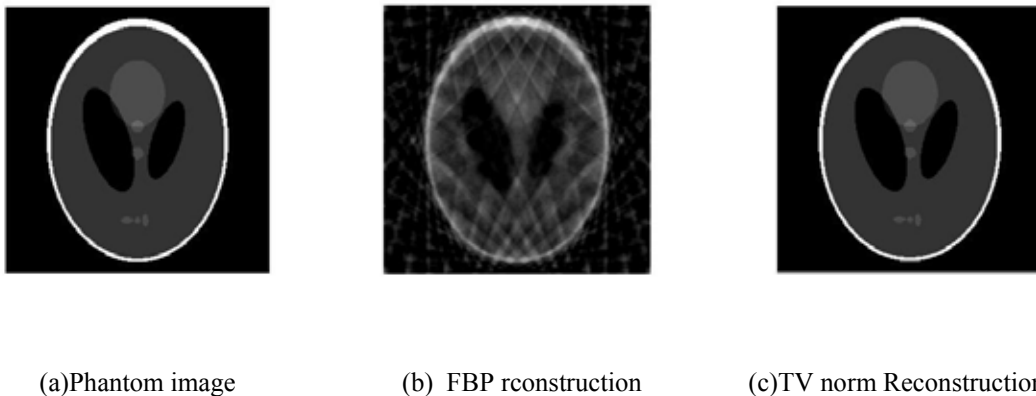


Fig.1 Sheep Logan image reconstruction

3.2. CT Cardiac Image reconstruction

The recent development for cardiac CT improved significantly the performance. However, the cardiac CT still faces the difficulty to obtain high temporal resolution. The main problem is that the heart always beats during projection acquisition. Thus, the general CT reconstruction algorithm can not work very well. In cardiac imaging, there are two major means for overcoming this problem. One is ECG gating technique which capture the projection data corresponding to a same cardiac phase and then reconstruct image by using multi-segment projection data. This approach can reconstruct heart with high spatial resolution at quiescent cardiac phases, e.g. at the end systole or late diastole. In this mode, a small part of the projection data is applied for image reconstruction. The second approach is to incorporate motion model into reconstruction algorithm [8]. Generally speaking, this method includes image space correction

and projection space correction. In image space correction, the registration algorithm is utilized. A series of image at different cardiac phases is needed to reconstruct and one image is selected as reference. Other images were performed registration with reference image to find the motion vector. The motion vector can be used for motion correction in reconstruction. Analytic and iterative algorithms are applied to reconstruct image. In limited views, iterative reconstruction algorithms are more efficient than FBP algorithm.

3.3. Compressed Sensing in Cardiac CT

The Compressed sensing algorithm can be applied to reconstruct dynamic cardiac CT image. In order to improve the temporal resolution, the projection data is acquired in multi-heart cycles. The projection data in several heart cycles is combined to reconstruct the final image. Actually, the spiral scan was utilized in each heart cycles. Moreover, the low pitch should be applied to ensure the projection data covering between the different cycles. In this case, there exists redundancy in the projection data. Therefore, appropriate processing may lead to the sparse representation.

In paper [9], a prior image x_p was reconstructed by using the standard filtered back-projection (FBP) reconstruction algorithm from the collection of the overlapped projection data sets. The subtraction of the target image at each time frame from this prior image can remove all of the static structures in the target image. Therefore, the target image can be sparsified by the subtraction operation $x-x_p$. After the subtraction, the sparsifying transforms ψ_1 are utilized to further sparsify the subtracted image $x-x_p$. The conventional compressed sensing objective function $\|\psi_2(x)\|_1$ has been incorporated into the algorithm with a relative weight of $(1-\alpha)$. The reconstruction algorithm is implemented by solving the following constrained minimization problem:

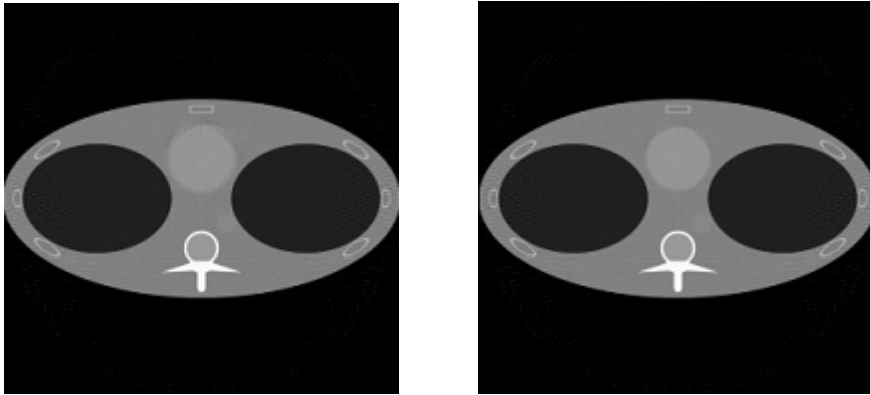
$$x^* = \operatorname{argmin} \alpha \|\psi_1(x-x_p)\|_1 + (1-\alpha) \|\psi_2(x)\|_1 \quad \text{s.t. } Ax=b \quad (4)$$

In noise and disturbance existence, the sole discrete gradient transform may yield defect. In order to eliminate the noise defect, TV-norm minimization is combined with l_1 norm minimization to reconstruct image. Moreover, sparse transform $\psi_1(x-x_p)$ can be simplified as $x-x_p$ for efficient computation in reconstruction. Practically, the projection data are acquired in several heart cycles to improve the temporal resolution. For simplicity, two heart cycles are applied to reconstruct image. Two pieces of projection data are captured respectively. We use all projection to reconstruct initial image and then utilize projection data from the first cycle to reconstruct image x_1 and use the projection data from second cycle to reconstruct image x_2 . These two images are blurred due to heart motion and limited projection. Because image x_1 are significantly correlated with image x_2 in time, we can subtract the summation of x_1 and x_2 image from x to sparsify the signal. Therefore, the better reconstruction image can be achieved by motion compensation between two cycles. We also introduce three adjusted parameter to improve the quality of image. This strategy can reduce the noise and keep sharp edge. Furthermore, the elastic image registration based on B-spline functions can be exploited to estimate the motion transform T between the neighboring heart cycles to acquire the motion vectors. The reconstruction is followed as:

$$x^* = \operatorname{argmin} \alpha \|(x-x_1-x_2)\|_1 + \beta \|\operatorname{TV}(x)\|_1 + \gamma \|x\|_1 \quad \text{s.t. } Ax=b \quad (5)$$

The above problem solving is implemented in two separate steps: In the first step, the standard FBP algorithm was used to reconstruct initial image. In the second step, the weighted summation of the total variation (TV), the l_1 norm of the discrete gradient image and the subtracted image $x-x_1-x_2$. The norm is minimized using the standard steepest descent method.

CT Cardiac phantom is used for reconstruction. The white circular in middle of image represents heart. We assume that the radius of heart moves in sinusoidal function. The reconstruction image only using TV norm minimization is illustrated in Figure 2.(a). The Figure 2.(b) is the reconstructed image using our comprehensive norm minimization. Obviously, the heart image in Figure2.(a) is blur in edge and the edge of reconstruction heart image in Figure 2.(b) is sharp. This shows that the improved approach can achieve better image in motion case.



(a) TV reconstruction

(b) Our method reconstruction

Fig.2 Cardiac phantom reconstruction

4. Conclusion

In this paper, we investigate basic principle for compressed sensing and present a comprehensive sparse recovery approach, which combine the TV norm, l_1 norm and subtraction of image in different time minimization to reconstruct image from Sheep-Logan and Cardiac phantom. The proposed algorithm is applied in CT image reconstruction. The simulated results show that our algorithm can improve the quality of image.

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