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Large direct *CP* violation in $B^0 \to \pi^+\pi^-$ and an enhanced branching ratio for $B^0 \to \pi^0\pi^0$

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Abstract

Recent measurements of $B^0 \to \pi\pi$ decays reveal two features that are in conflict with conventional calculations: the channel $B^0(\bar{B}^0) \to \pi^+\pi^-$ shows a large direct *CP*-violating asymmetry, and the channel $B^0(\bar{B}^0) \to \pi^0\pi^0$ has an unexpectedly high branching ratio. We show that both features can be understood in terms of strong-interaction mixing of $\pi\pi$ and $D\bar{D}$ channels in the isospin-zero state, an effect that is important because of the large experimentally observed ratio $\Gamma(B^0/\bar{B}^0 \to D^+D^-)/\Gamma(B^0/\bar{B}^0 \to \pi^+\pi^-) \approx 50$. Our dynamical model correlates the branching ratios and the *CP*-violating parameters $\mathcal{C}$ and $\mathcal{S}$, for the decays $B^0(\bar{B}^0) \to \pi^+\pi^-$, $B^0(\bar{B}^0) \to \pi^0\pi^0$, $B^0(\bar{B}^0) \to D^+D^-$ and $B^0(\bar{B}^0) \to D^0\bar{D}^0$.
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The Belle Collaboration has presented new data [1] which support their original evidence [2] for large direct *CP* violation in the decays $B^0(\bar{B}^0) \to \pi^+\pi^-$, the asymmetry parameter $\mathcal{C}$ $(= -\mathcal{A})$ being measured to be $\mathcal{C} = -0.58 \pm 0.15 \pm 0.07$. In a related development, both the BaBar [3] and Belle [4] Collaborations have reported a sizable branching ratio for the decay $B^0(\bar{B}^0) \to \pi^0\pi^0$, with an average value $\mathrm{Br}(B^0/\bar{B}^0 \to \pi^0\pi^0) = (1.9 \pm 0.6) \times 10^{-6}$. Both of these observations are unexpectedly large from the standpoint of conventional calculations [5–7] based on a short-distance, effective weak Hamiltonian and the assumption of factorization of products of currents in matrix elements for physical hadron states. In this paper, we carry out a calculation based upon the idea [8] of final-state interactions involving the mixing of $\pi\pi$ and $D\bar{D}$ channels. This dynamics provides a natural, correlated explanation of the new experimental facts, and leads to several further predictions.

To fix notation, we write the three $\bar{B} \to \pi\pi$ amplitudes as

$$A(\bar{B}^0 \to \pi^+\pi^-) = N(\lambda_u a_1 + \lambda_c a_p),$$
$$A(\bar{B}^0 \to \pi^0\pi^0) = N(\lambda_u a_2 - \lambda_c a_p)/\sqrt{2},$$
$$A(B^- \to \pi^-\pi^0) = N\lambda_u(a_1 + a_2)/\sqrt{2}. \tag{1}$$

Here a_1, a_2, a_p are, in general, complex numbers and N is a positive normalization factor. The parameters λ_u and λ_c are CKM factors, defined as $\lambda_u = V_{ub}V^*_{ud}$,

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$\lambda_c = V_{cb}V_{cd}^*$, with magnitudes $|\lambda_u| \cong 3.6 \times 10^{-3}$, $|\lambda_c| \cong 8.8 \times 10^{-3}$ and phases given by $\lambda_u = |\lambda_u|e^{-i\gamma}$, $\lambda_c = -|\lambda_c|$, with $\gamma \approx 60°$ [9]. The amplitudes in Eq. (1) are defined so that their absolute square gives the branching ratio, and they satisfy the isospin relation [10]

$$\frac{1}{\sqrt{2}}A(\bar{B}^0 \to \pi^+\pi^-) + A(\bar{B}^0 \to \pi^0\pi^0) = A(B^- \to \pi^-\pi^0). \tag{2}$$

From the results of the models discussed in [5–7], the parameters appearing in Eq. (1) have the following rough representation. The constants a_1, a_2, a_p are approximately real (to within a few degrees), with magnitudes $a_1 \approx 1.0$, $a_2 \approx 0.2$, $a_p \approx -0.1$. The normalization factor is $N \approx 0.75$; it is here fixed by the empirical branching ratio for $B^- \to \pi^-\pi^0$. The fact that the parameters a_1, a_2, a_p are nearly real implies immediately that there is very little direct *CP*-violating asymmetry between $\bar{B}^0 \to \pi^+\pi^-$ and $B^0 \to \pi^+\pi^-$, as well as in the channels $\pi^0\pi^0$ and $\pi^\pm\pi^0$. Furthermore, the absolute branching ratios following from the above parametrization are as follows (with experimental values given in parentheses):

$$\begin{aligned}
&\mathrm{Br}(B^\pm \to \pi^\pm\pi^0) = 5.3 \times 10^{-6}\\
&\quad[\text{exp. } (5.3 \pm 0.8) \times 10^{-6}],\\
&\mathrm{Br}(B^0/\bar{B}^0 \to \pi^+\pi^-) = 9.3 \times 10^{-6}\\
&\quad[\text{exp. } (4.6 \pm 0.4) \times 10^{-6}],\\
&\mathrm{Br}(B^0/\bar{B}^0 \to \pi^0\pi^0) = 0.2 \times 10^{-6}\\
&\quad[\text{exp. } (1.9 \pm 0.6) \times 10^{-6}].
\end{aligned} \tag{3}$$

The most striking feature is the strong enhancement of the $\pi^0\pi^0$ rate compared to this model expectation.

It was pointed out in Ref. [8] that the *CP*-violating asymmetries and branching ratios in the $B \to \pi\pi$ system would be strongly affected by final-state interactions involving the mixing of the $\pi\pi$ and $D\bar{D}$ channels in the isospin $I = 0$ state, as a consequence of the large ratio of partial decay widths $\Gamma(B^0 \to D^+D^-)/\Gamma(B^0 \to \pi^+\pi^-) \approx \frac{3}{14}|V_{cb}|^2/|V_{ub}|^2 \approx 26$ expected in the Bauer–Stech–Wirbel model [5]. A large ratio has now been confirmed by the Belle measurement [11] of the branching ratio $\mathrm{Br}(B^0/\bar{B}^0 \to D^+D^-) = 2.5 \times 10^{-4}$, which is about 50 times larger than $\mathrm{Br}(B^0/\bar{B}^0 \to \pi^+\pi^-)$. This fact gives new urgency to an investigation of $\pi\pi \leftrightarrow D\bar{D}$ mixing as a way of resolving the puzzling observations in $B \to \pi\pi$ decays.

The $\pi\pi$ system exists in the states $I = 0$ or $I = 2$, while the $D\bar{D}$ system has $I = 0$ or $I = 1$. Mixing can occur between the isospin-zero states

$$\begin{aligned}
&|\pi\pi\rangle_0 = \sqrt{\frac{2}{3}}|\pi^+\pi^-\rangle - \sqrt{\frac{1}{3}}|\pi^0\pi^0\rangle,\\
&|D\bar{D}\rangle_0 = \sqrt{\frac{1}{2}}\big[|D^+D^-\rangle + |D^0\bar{D}^0\rangle\big].
\end{aligned} \tag{4}$$

By contrast, the $I = 2$ $\pi\pi$ state and the $I = 1$ $D\bar{D}$ state, given by

$$\begin{aligned}
&|\pi\pi\rangle_2 = \sqrt{\frac{1}{3}}|\pi^+\pi^-\rangle + \sqrt{\frac{2}{3}}|\pi^0\pi^0\rangle,\\
&|D\bar{D}\rangle_1 = \sqrt{\frac{1}{2}}\big[|D^+D^-\rangle - |D^0\bar{D}^0\rangle\big]
\end{aligned} \tag{5}$$

are unaffected by mixing. The physical decay amplitudes of $\bar{B}^0$ to the above four states are

$$\begin{aligned}
&A^{(0)}_{\pi\pi} = \sqrt{\frac{2}{3}}\,A_{\pi^+\pi^-} - \sqrt{\frac{1}{3}}\,A_{\pi^0\pi^0},\\
&A^{(2)}_{\pi\pi} = \sqrt{\frac{1}{3}}\,A_{\pi^+\pi^-} + \sqrt{\frac{2}{3}}\,A_{\pi^0\pi^0},\\
&A^{(0)}_{D\bar{D}} = \sqrt{\frac{1}{2}}\,[A_{D^+D^-} + A_{D^0\bar{D}^0}],\\
&A^{(1)}_{D\bar{D}} = \sqrt{\frac{1}{2}}\,[A_{D^+D^-} - A_{D^0\bar{D}^0}].
\end{aligned} \tag{6}$$

These physical decay amplitudes are related to the “bare” amplitudes calculated in the absence of final-state interactions, i.e., with no mixing, which we denote by $\tilde{A}$:

$$\begin{aligned}
&\begin{pmatrix} A^{(0)}_{\pi\pi} \\ A^{(0)}_{D\bar{D}} \end{pmatrix} = S^{\frac{1}{2}} \begin{pmatrix} \tilde{A}^{(0)}_{\pi\pi} \\ \tilde{A}^{(0)}_{D\bar{D}} \end{pmatrix},\\
&A^{(2)}_{\pi\pi} = \tilde{A}^{(2)}_{\pi\pi}, \qquad A^{(1)}_{D\bar{D}} = \tilde{A}^{(1)}_{D\bar{D}}.
\end{aligned} \tag{7}$$

Here S denotes the strong-interaction S matrix connecting the isospin-zero states $|\pi\pi\rangle_0$ and $|D\bar{D}\rangle_0$

which can be written generally as[1]

$$S = \begin{pmatrix} \cos 2\theta e^{i2\delta_1} & i \sin 2\theta e^{i(\delta_1+\delta_2)} \\ i \sin 2\theta e^{i(\delta_1+\delta_2)} & \cos 2\theta e^{i2\delta_2} \end{pmatrix}, \tag{8}$$

where θ is a mixing angle, and δ_1 and δ_2 are the strong-interaction phase shifts for the elastic scattering of $\pi\pi$ and $D\bar{D}$ systems in the $I = 0$ state, at $\sqrt{s} = M_B$. For any choice of these three parameters, the matrix $S^{\frac{1}{2}}$ can be calculated numerically, and the set of four equations (7) solved to obtain the physical amplitudes $A_{\pi^+\pi^-}$, $A_{\pi^0\pi^0}$, $A_{D^+D^-}$ and $A_{D^0\bar{D}^0}$ in terms of the bare amplitudes. The bare amplitudes are identified with those calculated in the factorization model [5–7], which we list below

$$\tilde{A}_{\pi^+\pi^-} = N(\lambda_u a_1 + \lambda_c a_p),$$
$$\tilde{A}_{\pi^0\pi^0} = N(\lambda_u a_2 - \lambda_c a_p)/\sqrt{2},$$
$$\tilde{A}_{D^+D^-} = N'\lambda_c a_1, \qquad \tilde{A}_{D^0\bar{D}^0} = 0, \tag{9}$$

where the first two equations are as in Eq. (1), and the factor N' is determined from the empirical [11] branching ratio $\mathrm{Br}(B^0/\bar{B}^0 \to D^+D^-) = N'^2|\lambda_c|^2 a_1^2 = 2.5 \times 10^{-4}$ to be $N' = 1.79$.

In order to show, in a transparent way, how the mixing mechanism gives rise to large direct *CP* violation in $B^0 \to \pi^+\pi^-$, as well as an enhanced branching ratio for $B^0 \to \pi^0\pi^0$, we consider, for illustration, the case where the elastic phases δ_1 and δ_2 in the S matrix (Eq. (8)) are neglected, so that $S^{\frac{1}{2}}$ may be written as

$$S^{\frac{1}{2}} = \begin{pmatrix} \cos\theta & i \sin\theta \\ i \sin\theta & \cos\theta \end{pmatrix}. \tag{10}$$

The amplitudes $A_{\pi^+\pi^-}$ and $A_{\pi^0\pi^0}$ for $\bar{B}^0$ decay are then given by

$$A_{\pi^+\pi^-} = \frac{1+2\cos\theta}{3}\tilde{A}_{\pi^+\pi^-} + \sqrt{2}\,\frac{1-\cos\theta}{3}\tilde{A}_{\pi^0\pi^0} + i\sin\theta\frac{1}{\sqrt{3}}(\tilde{A}_{D^+D^-} + \tilde{A}_{D^0\bar{D}^0}),$$

$$A_{\pi^0\pi^0} = \sqrt{2}\,\frac{1-\cos\theta}{3}\tilde{A}_{\pi^+\pi^-} + \frac{2+\cos\theta}{3}\tilde{A}_{\pi^0\pi^0} - i\sin\theta\frac{1}{\sqrt{6}}(\tilde{A}_{D^+D^-} + \tilde{A}_{D^0\bar{D}^0}). \tag{11}$$

Clearly for $\theta = 0$, the physical amplitudes reduce to the bare amplitudes. Inserting the bare amplitudes from Eq. (9), we can rewrite $A_{\pi^+\pi^-}$ and $A_{\pi^0\pi^0}$ as linear combinations of λ_u and λ_c:

$$A_{\pi^+\pi^-} = N\left[\lambda_u\left\{\frac{1+2\cos\theta}{3}a_1 + \frac{1-\cos\theta}{3}a_2\right\} + \lambda_c(a_p\cos\theta + a_m)\right],$$
$$A_{\pi^0\pi^0} = \frac{N}{\sqrt{2}}\left[\lambda_u\left\{\frac{2(1-\cos\theta)}{3}a_1 + \frac{2+\cos\theta}{3}a_2\right\} - \lambda_c(a_p\cos\theta + a_m)\right], \tag{12}$$

where

$$a_m = i\frac{1}{\sqrt{3}}\sin\theta\frac{N'}{N}a_1. \tag{13}$$

Note that the isospin relation in Eq. (2) continues to be fulfilled. The important new feature of the amplitudes in Eq. (12) is the appearance of the *imaginary* term a_m in the coefficient of λ_c, in striking contrast to the real term a_p. The imaginary nature of this dynamical term is an inescapable consequence of S-matrix unitarity, which enforces the factor i in the off-diagonal matrix element in Eq. (10). The term a_m, given in Eq. (13), has a magnitude $|a_m| \approx 1.39\sin\theta$, and dominates the term $a_p\cos\theta$ even for a modest mixing angle ~ 0.1. We will now show that the mixing term a_m has profound consequences for direct *CP* violation in the decays $B^0 \to \pi^+\pi^-$, and for the branching ratio of the channel $B^0 \to \pi^0\pi^0$.

1. $\mathcal{C}$ and $\mathcal{S}$ parameters for $B^0 \to \pi^+\pi^-$ and $B^0 \to \pi^0\pi^0$

The $\mathcal{C}$ and $\mathcal{S}$ parameters derived from the time-dependent asymmetry between $\bar{B}^0$ and B^0 decays into $\pi^+\pi^-$ are defined as

$$\mathcal{C}_{+-} = \frac{1-|\lambda_{+-}|^2}{1+|\lambda_{+-}|^2},$$
$$\mathcal{S}_{+-} = \frac{2\,\mathrm{Im}\,\lambda_{+-}}{1+|\lambda_{+-}|^2}, \tag{14}$$

[1] The two-channel S-matrix has been discussed, in particular in [12,13]. The $S^{\frac{1}{2}}$ prescription is given in [6,12]. An alternative prescription, using $\frac{1}{2}[\mathbf{1} + S]$ in place of $S^{\frac{1}{2}}$, has been discussed by Kamal [14], and was used in Ref. [8].

where

$$\lambda_{+-} = \frac{q}{p}\frac{A(\bar{B}^0 \to \pi^+\pi^-)}{A(B^0 \to \pi^+\pi^-)} \tag{15}$$

with

$$\frac{q}{p} = e^{-i2\beta}, \quad 2\beta \approx 45^\circ. \tag{16}$$

$\mathcal{C}_{+-}$ is the parameter for direct *CP* violation (i.e., $|A(\bar{B}^0 \to \pi^+\pi^-)/A(B^0 \to \pi^+\pi^-)| \neq 1$). Using the amplitude $A_{\pi^+\pi^-}$ in Eq. (12), we obtain

$$\lambda_{+-} = e^{-i2\beta}\left[\lambda_u\left(\frac{1+2\cos\theta}{3}a_1 + \frac{1-\cos\theta}{3}a_2\right) + \lambda_c(a_p\cos\theta + a_m)\right] \times \left\{\lambda_u^*\left(\frac{1+2\cos\theta}{3}a_1 + \frac{1-\cos\theta}{3}a_2\right) + \lambda_c^*(a_p\cos\theta + a_m)\right\}^{-1}. \tag{17}$$

The asymmetry parameters $\mathcal{C}_{+-}$ and $\mathcal{S}_{+-}$ calculated from the above expression are plotted as functions of θ in Fig. 1. Good approximate agreement with data is obtained for $\theta \approx 0.2$ (see Table 1, where we also list $\mathcal{C}_{00}$ and $\mathcal{S}_{00}$). We note that the amplitudes in Eq. (12) have been derived from the matrix $S^{\frac{1}{2}}$ in Eq. (10), which was obtained from (8) by neglecting the phase shifts δ_1 and δ_2. We have also explored numerically *S* matrices with non-zero phases, and indicate in Figs. 1 and 2 two examples, obtained with the values $\delta_1 = \pm 10^\circ$, $\delta_1 + \delta_2 = -30^\circ$. Table 1 gives numerical values for a few choices of parameters. In all cases, there is a large direct *CP* violation.

Discussions of the direct *CP*-violating parameter $\mathcal{C}_{+-}$ are often based on an amplitude for $\bar{B}^0 \to \pi^+\pi^-$ written in the form

$$A_{\pi^+\pi^-} \sim \left[e^{-i\gamma} + \frac{P_{\pi\pi}}{T_{\pi\pi}}\right]. \tag{18}$$

The parametrization in Eq. (1), based on the models [5–7], gives $|P_{\pi\pi}/T_{\pi\pi}| \cong 0.24$, and $\arg(P_{\pi\pi}/T_{\pi\pi}) \cong 0$. The small phase of the "penguin-to-tree" ratio $P_{\pi\pi}/T_{\pi\pi}$ is a generic feature of these models, and is responsible for the prediction $\mathcal{C}_{+-} \approx 0$, which is now contradicted by data [1]. In our approach, the role of $P_{\pi\pi}/T_{\pi\pi}$ is played by the ratio

$$\text{“}P/T\text{”} = -\frac{|\lambda_c|(a_p\cos\theta + a_m)}{|\lambda_u|(\frac{1+2\cos\theta}{3}a_1 + \frac{1-\cos\theta}{3}a_2)}. \tag{19}$$

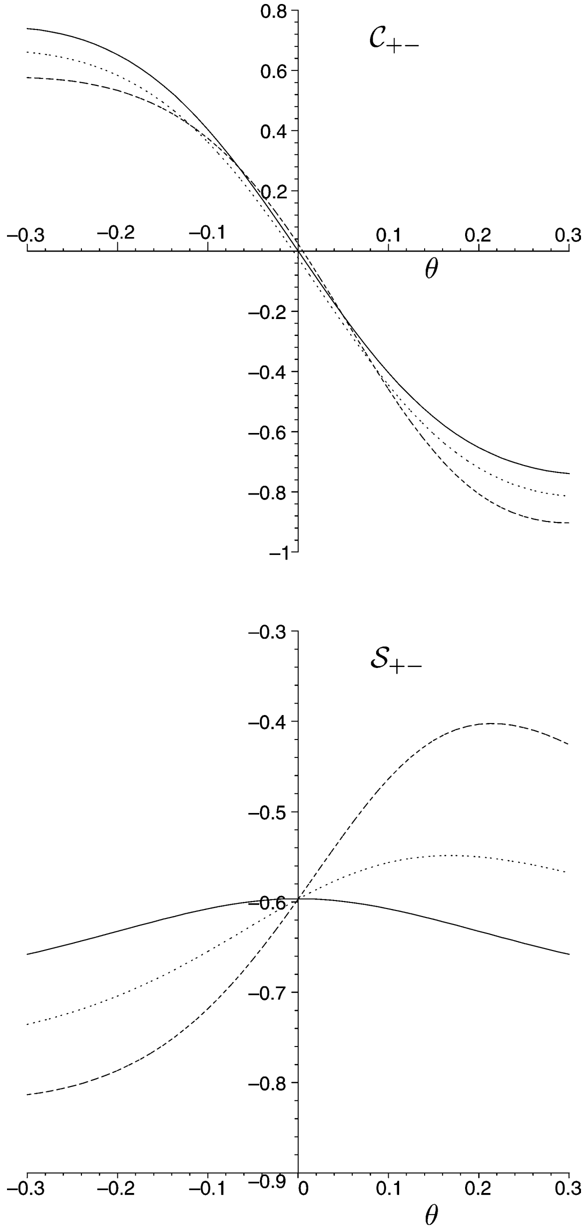

Fig. 1. $\mathcal{C}$ and $\mathcal{S}$ parameters for the decay $B^0(\bar{B}^0) \to \pi^+\pi^-$. Full line is for $\delta_1 = \delta_2 = 0^\circ$, dotted line for $\delta_1 = -10^\circ$, $\delta_2 = -20^\circ$, dashed line for $\delta_1 = 10^\circ$, $\delta_2 = -40^\circ$.

For a typical value $\theta = 0.2$, this ratio has the modulus $|\text{“}P/T\text{”}| \approx 0.77$, and a phase $\arg(\text{“}P/T\text{”}) \approx -70^\circ$. The difference is a consequence of the term a_m in Eq. (19), which reflects the physical final-state

Table 1
Observables for different mixing angles θ and strong-interaction phases δ_1 and δ_2. All branching ratios are given in units of 10^{-6}

Observable	No mixing	With mixing			Data	
		$\theta = 0.2$, $\delta_1 = 0°$, $\delta_2 = 0°$	$\theta = 0.17$, $\delta_1 = -10°$, $\delta_2 = -20°$	$\theta = 0.2$, $\delta_1 = 10°$, $\delta_2 = -40°$		
$\mathcal{C}_{+-}$	± 0.00	-0.65	-0.66	-0.81	$-0.58 \pm 0.15 \pm 0.07$	(Belle [1])
					$-0.30 \pm 0.25 \pm 0.04$	(BaBar [17])
$\mathcal{S}_{+-}$	-0.60	-0.63	-0.55	-0.40	$-1.00 \pm 0.21 \pm 0.07$	(Belle [1])
					$+0.02 \pm 0.34 \pm 0.05$	(BaBar [17])
$\mathrm{Br}(B^0/\bar{B}^0 \to \pi^0\pi^0)$	0.2	1.8	1.7	1.6	$1.7 \pm 0.6 \pm 0.2$	(Belle [4])
					$2.1 \pm 0.6 \pm 0.3$	(BaBar [3])
$\mathrm{Br}(B^0/\bar{B}^0 \to \pi^+\pi^-)$	9.3	12.2	10.5	9.9	$4.4 \pm 0.6 \pm 0.3$	(Belle [18])
					$4.7 \pm 0.6 \pm 0.2$	(BaBar [17])
					$4.5^{+1.4+0.5}_{-1.2-0.4}$	(CLEO [19])
$\mathcal{C}_{00}$	± 0.00	$+0.48$	$+0.51$	$+0.56$		
$\mathcal{S}_{00}$	$+0.73$	-0.65	-0.78	-0.49		

interaction of the $\pi\pi$ system, as implemented in our model through $\pi\pi \leftrightarrow D\bar{D}$ mixing.

2. Branching ratio for $B^0 \to \pi^0\pi^0$ and $B^0 \to \pi^+\pi^-$

The branching ratios (averaged over B^0 and $\bar{B}^0$) may be calculated in our model by taking the absolute square of the $\bar{B}^0$ decay amplitudes in Eq. (12), and the corresponding amplitudes for B^0 decay. The results are shown in Fig. 2. It is remarkable that the empirical branching ratio for $B^0/\bar{B}^0 \to \pi^0\pi^0$ is accurately reproduced, using the same value $\theta \approx 0.2$ which accounts for the asymmetry parameter $\mathcal{C}_{+-}$. We also note that the branching ratio $B^0/\bar{B}^0 \to \pi^+\pi^-$ remains close to its bare value, and can be lowered slightly with the introduction of phases δ_1 and δ_2. Numerical results for $\mathrm{Br}(B^0/\bar{B}^0 \to \pi^0\pi^0)$ and $\mathrm{Br}(B^0/\bar{B}^0 \to \pi^+\pi^-)$ are listed in Table 1.

3. Branching ratio for $B^0 \to D^0\bar{D}^0$

Since our model treats the $\pi\pi$ and $D\bar{D}$ states with $I = 0$ as a coupled system, it also produces predictions for branching ratios and asymmetry parameters in $B^0 \to D^+D^-$ and $B^0 \to D^0\bar{D}^0$. The amplitudes after mixing are

$$\begin{aligned} A_{D^+D^-} &= \frac{1}{2}\Big[i \sin\theta \sqrt{\frac{2}{3}}\,(\sqrt{2}\,\tilde{A}_{\pi^+\pi^-} - \tilde{A}_{\pi^0\pi^0}) \\ &\quad + (\cos\theta + 1)\tilde{A}_{D^+D^-} \\ &\quad + (\cos\theta - 1)\tilde{A}_{D^0\bar{D}^0}\Big], \\ A_{D^0\bar{D}^0} &= \frac{1}{2}\Big[\, i \sin\theta \sqrt{\frac{2}{3}}\,(\sqrt{2}\,\tilde{A}_{\pi^+\pi^-} - \tilde{A}_{\pi^0\pi^0}) \\ &\quad + (\cos\theta - 1)\tilde{A}_{D^+D^-} \\ &\quad + (\cos\theta + 1)\tilde{A}_{D^0\bar{D}^0}\Big]. \end{aligned} \tag{20}$$

Of particular interest is the branching ratio for $B^0/\bar{B}^0 \to D^0\bar{D}^0$, since it vanishes at the level of the bare amplitude ($\tilde{A}_{D^0\bar{D}^0} = 0$), and is induced by mixing with the $\pi\pi$ system. For $\theta = 0.2$, ignoring the phases δ_1, δ_2, our model predicts

$$\mathrm{Br}\big(B^0/\bar{B}^0 \to D^0\bar{D}^0\big) = 1.45 \times 10^{-7}. \tag{21}$$

(At this low level, one must assume that other sources of final-state interaction or a non-zero bare amplitude could raise this branching ratio further.) Direct *CP* violation follows from $A_{D^0\bar{D}^0}$ in Eq. (20): $\mathcal{C}_{D^0\bar{D}^0} = -0.50$ for $\theta = 0.2$. Direct *CP* violation in D^+D^- (and

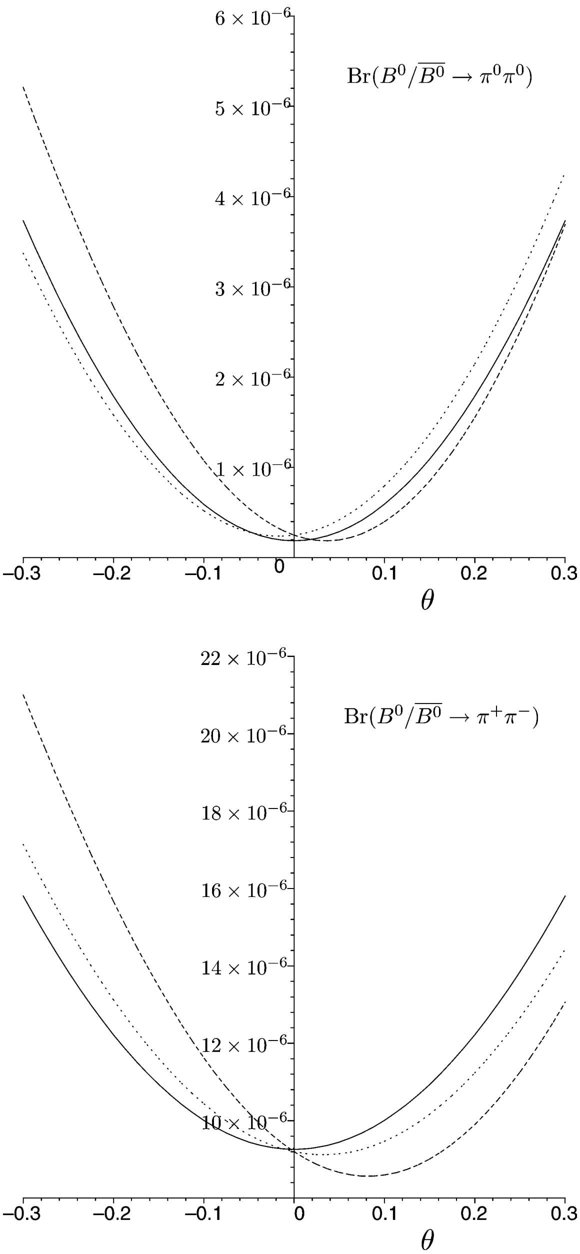

Fig. 2. Average branching ratios for $B^0/\bar{B}^0 \to \pi^0\pi^0$ and $B^0/\bar{B}^0 \to \pi^+\pi^-$. Full line is for $\delta_1 = \delta_2 = 0°$, dotted line for $\delta_1 = -10°$, $\delta_2 = -20°$, dashed line for $\delta_1 = 10°$, $\delta_2 = -40°$.

in $\pi^-\pi^0$) is small, because these decays are dominated by a single amplitude. There is little mixing in $A_{D^+D^-}$ in Eq. (20) (and none in the $I = 2$ amplitude for $\pi^-\pi^0$).

To conclude, we have demonstrated a mechanism of final-state interactions among physical hadrons in $B^0 \to \pi\pi$ decays which predicts a large direct *CP*-violating parameter $\mathcal{C}_{+-}$. The same mechanism enhances the theoretical prediction for the branching ratio of $B^0/\bar{B}^0 \to \pi^0\pi^0$ to the experimentally observed level. Predictions are made for the $\mathcal{C}$ and $\mathcal{S}$ parameters of $B^0(\bar{B}^0) \to \pi^0\pi^0$ decays, and for the branching ratio of $B^0/\bar{B}^0 \to D^0\bar{D}^0$. The model makes essential use of the large empirical ratio $\Gamma(B^0/\bar{B}^0 \to D^+D^-)/\Gamma(B^0/\bar{B}^0 \to \pi^+\pi^-) \approx 50$. Its success in the present context leads to the expectation that sizable direct *CP* violation could be observed in other charmless B decays, in which an amplitude of order λ_u receives a dynamical contribution proportional to λ_c, through mixing with a channel possessing a large branching ratio. The resulting amplitude contains two pieces which are comparable in magnitude and have different weak-interaction and strong-interaction phases. We have treated earlier [15] the charged-particle decays $B^\pm \to \eta\pi^\pm$ (and $B^\pm \to \eta'\pi^\pm$), which are influenced by mixing with the channel $B^\pm \to \eta_c\pi^\pm$, and have predicted significant direct *CP* violation. Evidence for a sizable violation in $B^\pm \to \eta\pi^\pm$ has indeed been reported in one experiment [16], the first ever seen in a charged-particle decay.

References

[1] Belle Collaboration, K. Abe, et al., hep-ex/0401029.
[2] Belle Collaboration, K. Abe, et al., Phys. Rev. D 68 (2003) 012001.
[3] BaBar Collaboration, B. Aubert, et al., Phys. Rev. Lett. 91 (2003) 241801.
[4] Belle Collaboration, S.H. Lee, et al., Phys. Rev. Lett. 91 (2003) 261801.
[5] M. Bauer, B. Stech, M. Wirbel, Z. Phys. C 34 (1987) 103.
[6] M. Neubert, B. Stech, in: A.J. Buras, M. Lindner (Eds.), Heavy Flavours, second ed., World Scientific, Singapore, hep-ph/9705292.
[7] M. Beneke, M. Neubert, hep-ph/0308039, and references therein.
[8] M. Wanninger, L.M. Sehgal, Z. Phys. C 50 (1991) 47;
See also A.N. Kamal, Int. J. Mod. Phys. A 7 (1992) 3515.
[9] Particle Data Group, K. Hagiwara, et al., Phys. Rev. D 66 (2002) 010001.
[10] M. Gronau, D. London, Phys. Rev. Lett. 65 (1990) 3381.
[11] Belle Collaboration, T. Browder, talk at Lepton–Photon Symposium, Fermilab, August 2003.
[12] J.F. Donoghue, et al., Phys. Rev. Lett. 77 (1996) 2178.

[13] M. Suzuki, L. Wolfenstein, Phys. Rev. D 60 (1999) 074019.
[14] A.N. Kamal, C.W. Luo, hep-ph/9702289.
[15] S. Barshay, D. Rein, L.M. Sehgal, Phys. Lett. B 259 (1991) 475.
[16] BaBar Collaboration, B. Aubert, et al., hep-ex/0311016.
[17] BaBar Collaboration, B. Aubert, et al., Phys. Rev. Lett. 89 (2002) 281802.
[18] Belle Collaboration, Y. Chao, et al., hep-ex/0311061.
[19] CLEO Collaboration, A. Bornheim, et al., Phys. Rev. D 68 (2003) 052002.