Sixth International Conference on Sensitivity Analysis of Model Output

# Quality of high dimensional numerical "Space filling designs" for the sensitivity analysis of interference optical filters.

O. Vasseur[a]*, M. Claeys-Bruno[b], M. Sergent[b]

*[a] ONERA DOTA, Chemin de la Hunière, 97761 PALAISEAU Cedex, France*
*[b] Université Paul Cézanne Aix Marseille III, LMRE, 13397 MARSEILLE Cedex 20, France*

**Abstract**

In the field of computer experiments, when the relations between the outputs and the inputs of the computer code are complex, the Space Filling Designs (SFD) are used to study the outputs all over the domain or to build metamodels. We introduce here the results of SFD quality obtained in high dimensional case by the methodology of Minimum Spanning Tree (MST). Then, different SFD are used for the sensitivity analysis of interference optical filters. The intrinsic quality of SFD is assessed by MST criterion and is proved to be in accordance with the results of the optical filter sensitivity analysis. This study can be considered as a first step to connect intrinsic quality and extrinsic quality of SFD designs.

*Keywords:* Computer experiments; Space-Filling Designs; Minimum Spanning Tree; Minimal Spanning Tree; Thin film coatings; Optical filter; Sensitivity analysis; Complex systems.

## 1. Quality analysis of numerical designs "Space filling designs" by minimal spanning tree and sensitivity analysis of interference optical filters.

In the field of computer experiments, when the relations between the outputs and the inputs of the computer code are complex, the Space Filling Designs (SFD) are used to study the outputs all over the domain or to build metamodels. In the case of the dimension of the space less than 10, Franco and al. (2009) pointed out that classical criteria are insufficient to conclude about the uniform distribution of points and proposed to use Minimum Spanning Tree (MST) (Beardwood, 1959) criterion which is more informative than the classical methods. Indeed, Dussert and all. (1986) showed that the construction of a MST on a set of points in dimension 2 allows us to qualify the distribution of the points and Wallet and all. (1998) pointed out its high discrimination power and stability for topographical analysis.

From the MST, the mean m and the standard deviation $\sigma$ of the edges length may be used as characteristics for the corresponding distribution. In the (m, $\sigma$) plane, any distribution of points can be plotted and easily compared with well-characterized distributions (for example, perfectly ordered or random ones). On this graphical representation, it is possible to distinguish different areas and mainly the quasi-regular area which is the best area for

---

* Corresponding author. Tel.: (33) 1 69 93 63 53; fax: (33) 1 69 93 63 45.
*E-mail address*: Olivier.Vasseur@onera.fr

space filling designs due to the best compromise between a high average length of MST branches to fill the space and a small standard deviation to obtain a sufficient regularity: for example random designs have points too closed and grid designs ($\sigma=0$) are too regular.

In dimension higher than 20 a lot of classical designs (Latin Hypercube Designs, Low discrepancy Sequences, Cover designs…) are studied, but the most and the best classical designs are very closed to the random area.

Different SFD with various quality determined with the MST criterion are used to perform sensitivity analyses of optical thin film coatings. By this way, it is possible to connect and compare the intrinsic quality of designs determined by the MST with the extrinsic quality of the results corresponding to this application case.

Due to the interference characteristics of multilayer filters, interference optical coatings give the opportunity to evaluate methods which can explore high dimensional space of parameters and mainly in presence of interactions between some parameters. Each layer is characterized by its refractive index value (n) and its thickness (t): thus, for a filter with p layers we can generate up to 2p-dimensions spaces. Refractive index value errors or thickness errors during the manufacturing of these layers can induce dramatic consequences on the wished optical properties (Macleod, 1986). Our study of the effect of errors on refractive index values of each layer of the coating on the filter optical transmittance was mostly realized with the following 29-layers coating:

$$\text{Glass/HLHL4HLHLH L HLHL4HLHLH L HLHL4HLHLH/air} \qquad (1)$$

where H and L are quarter-wave layers at $\lambda_0 = 1\mu m$ ($n*t=\lambda_0/4$) of high (H) and low (L) refractive index values respectively.

According to the knowledge of the optical property of the studied filters the metamodel coefficients expected as predominant can be compared with those obtained with each SFD. The most critical interactions are well determined by the best intrinsic designs and the intrinsic quality of designs determined by the MST criterion corresponds to the quality of the conclusions obtained in the sensitivity analysis.

Finally, the results of the sensitivity analysis of optical coatings confirm the intrinsic quality of the Space Filling Designs determined by the Minimal Spanning Tree and reveal the correlation between the intrinsic and extrinsic qualities of SFD designs. The employed method will be very useful to assess very quickly the potential robustness of coatings by considering the monitoring strategy, to perform sensitivity analyses of complex filters on waveguide structures (Vasseur, 1998) and to study the parameters interactions of complex optronic systems such that coherent beam combining of fiber lasers (Jolivet and al., 2009).

## 2. References

Beardwood J., Halton J.H., Hammersley J.M., 1959: "The shortest path through many points", *Camb. Philos. Soc. Proc.*, 55, pp. 299-327.

Dussert C., Rasigni G., Rasigni M., Palmari J., 1986: "Minimal spanning tree: A new approach for studying order and disorder", *Physical Review B.*, 34 (5), pp. 3528-3531.

Franco J., Vasseur O., Corre B., Sergent M., 2009: "Minimum Spanning Tree: A new approach to assess the quality of the design of computer experiments", *Chemometrics and Intelligent Laboratory Systems*, 97, pp. 164-169.

Jolivet V., Bourdon P., Bennaï B., Lombard L., Goular G., Pourtal E., Canat G., Jaouën Y., Moreau B., Vasseur O., 2009: "Coherent Combining of Fiber Amplifiers Through Atmospheric Turbulence", *IEEE J. Sel. Topics Quantum Electron.*, 15, pp. 257-268.

Macleod H.A., 1986: *Thin film optical filters*, Adam Hilger ltd, Bristol.

Vasseur O., 1998: "Reflectivity of dielectric coatings deposited on the end facet of a weakly guiding fiber", *J. Opt. Soc. Am. A*, 12, pp. 77-83.

Wallet F., Dussert C., 1998: "Comparison of spatial point patterns and processes characterization methods", *Europhysics Lett.*, 42, pp. 493-498.