Available online at www.sciencedirect.com
ScienceDirect
Computers and Mathematics with Applications 54 (2007) 872–877
An International Journal computers & mathematics with applications
www.elsevier.com/locate/camwa

Mann and Ishikawa iterative processes for multivalued mappings in Banach spaces

Bancha Panyanak

Department of Mathematics, Faculty of Science, Chaing Mai University, Chiang Mai 50200, Thailand

Received 27 July 2006; received in revised form 19 February 2007; accepted 16 March 2007

Abstract

Let K be a nonempty compact convex subset of a uniformly convex Banach space, and $T : K \to \mathcal{P}(K)$ a multivalued nonexpansive mapping. We prove that the sequences of Mann and Ishikawa iterates converge to a fixed point of T. This generalizes former results proved by Sastry and Babu [K.P.R. Sastry, G.V.R. Babu, Convergence of Ishikawa iterates for a multi-valued mapping with a fixed point, Czechoslovak Math. J. 55 (2005) 817–826]. We also introduce both of the iterative processes in a new sense, and prove a convergence theorem of Mann iterates for a mapping defined on a noncompact domain.

Keywords: Multivalued mappings; Fixed points; Mann iterates; Ishikawa iterates; Uniformly convex Banach spaces

1. Introduction

Let K be a nonempty bounded closed convex subset of a Banach space X. A mapping $T : K \to K$ is said to be nonexpansive if

$$\|Tx - Ty\| \leq \|x - y\|, \quad \text{for all } x, y \in K.$$

It has been shown that if X is uniformly convex then every nonexpansive mapping $T : K \to K$ has a fixed point (see Browder [2], cf. also Kirk [3]). In 1974, Ishikawa [4] introduced a new iteration procedure for approximating fixed points of pseudo-contractive compact mappings in Hilbert spaces as follows.

$$x_{n+1} = \alpha_n x_n + (1 - \alpha_n)T[\beta_n x_n + (1 - \beta_n)Tx_n], \quad n \geq 0,$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ are sequence in $[0, 1]$ satisfying certain restrictions. Note that the normal Mann iteration procedure [5],

$$x_{n+1} = \alpha_n x_n + (1 - \alpha_n)Tx_n, \quad n \geq 0,$$

where $\{\alpha_n\}$ is a sequences in $[0, 1]$, is a special case of the Ishikawa one. For a comparison of the two iterative processes in the one-dimensional case, we refer the reader to Rhoades [6]. For more details and literature on the

E-mail address: banchap@chiangmai.ac.th.

0898-1221/$ - see front matter
doi:10.1016/j.camwa.2007.03.012

convergence of Ishikawa and Mann iterates we refer to [7–14]. Recently, Sastry and Babu [1] introduced the analogs of Mann and Ishikawa iterates for multivalued mappings and proved convergence theorems for nonexpansive mappings whose domain is a compact convex subset of a Hilbert space. In this paper, we generalize results of Sastry and Babu to uniformly convex Banach spaces. We also introduce both of the iteration processes in a new sense, and prove a convergence theorem of Mann iterates for a mapping defined on a noncompact domain.

2. Preliminaries

Let X be a Banach space. A subset K is call *proximinal* if for each $x \in X$, there exists an element $k \in K$ such that

$$d(x, k) = \operatorname{dist}(x, K) = \inf\{\|x - y\| : y \in K\}.$$

It is well known that every closed convex subset of a uniformly convex Banach space is proximinal. We shall denote by $\mathcal{P}(K)$ the family of nonempty bounded proximinal subsets of K. Let $H(\cdot, \cdot)$ be the Hausdorff distance on $\mathcal{P}(K)$, i.e.,

$$H(A, B) = \max\left\{\sup_{a\in A} \operatorname{dist}(a, B), \sup_{b\in B} \operatorname{dist}(b, A)\right\}, \quad A, B \in \mathcal{P}(K),$$

where $\operatorname{dist}(a, B) = \inf\{\|a - b\| : b \in B\}$ is the distance from the point a to the set B.

A multivalued mapping $T : K \to \mathcal{P}(K)$ is said to be a *nonexpansive* if

$$H(Tx, Ty) \le \|x - y\| \quad \text{for all } x, y \in K.$$

A point x is called a fixed point of T if $x \in Tx$. The existence of fixed points for multivalued nonexpansive mappings in uniformly convex Banach spaces was proved by Lim [15]. From now on, X stands for a uniformly convex Banach space and $F(T)$ stands for the fixed point set of a mapping T.

Definition 2.1 (*[1]*). Let K be a nonempty convex subset of $X, T : K \to \mathcal{P}(K)$ a multivalued mapping and fix $p \in F(T)$.

(A) *The sequence of Mann iterates* is defined by $x_0 \in K$,

$$x_{n+1} = \alpha_n x_n + (1 - \alpha_n) y_n, \quad \alpha_n \in [0, 1], n \ge 0,$$

where $y_n \in Tx_n$ is such that $\|y_n - p\| = \operatorname{dist}(p, Tx_n)$.

(B) *The sequence of Ishikawa iterates* is defined by $x_0 \in K$,

$$y_n = (1 - \beta_n) x_n + \beta_n z_n, \quad \beta_n \in [0, 1], n \ge 0$$

where $z_n \in Tx_n$ is such that $\|z_n - p\| = \operatorname{dist}(p, Tx_n)$, and

$$x_{n+1} = (1 - \alpha_n) x_n + \alpha_n z'_n, \quad \alpha_n \in [0, 1]$$

where $z'_n \in Ty_n$ is such that $\|z'_n - p\| = \operatorname{dist}(p, Ty_n)$.

The following lemma can be found in [1]; for completeness we will include the proof.

Lemma 2.2. *Let $\{\alpha_n\}, \{\beta_n\}$ be two real sequences such that*

(i) $0 \le \alpha_n, \beta_n < 1$,
(ii) $\beta_n \to 0$ *as* $n \to \infty$ *and*
(iii) $\sum \alpha_n \beta_n = \infty$.

Let $\{\gamma_n\}$ be a nonnegative real sequence such that $\sum \alpha_n \beta_n (1 - \beta_n) \gamma_n$ is bounded. Then $\{\gamma_n\}$ has a subsequence which converges to zero.

Proof. Since $\lim_n \beta_n = 0$ and $\sum \alpha_n \beta_n = \infty$, then $\sum \alpha_n \beta_n (1 - \beta_n) = \infty$. We shall show that $\liminf_n \gamma_n = 0$. Suppose not, i.e. that there exists $\varepsilon > 0$ and $N \in \mathbb{N}$ such that $\gamma_n > \varepsilon$ for all $n \ge N$. This implies

$$\varepsilon \sum_{n=N}^{\infty} \alpha_n \beta_n (1 - \beta_n) \le \sum_{n=N}^{\infty} \alpha_n \beta_n (1 - \beta_n) \gamma_n < \infty,$$

which is a contradiction, and hence the conclusion follows. ■

The following lemma is a characterization of uniform convexity which can be found in [16].

Lemma 2.3. *Let X be a Banach space. Then X is uniformly convex if and only if for any given number $\rho > 0$, the square norm $\|\cdot\|^2$ of X is uniformly convex on B_ρ, the closed ball centered at the origin with radius ρ; namely, there exists a continuous strictly increasing function $\varphi : [0, \infty) \to [0, \infty)$ with $\varphi(0) = 0$ such that*

$$\|\alpha x + (1-\alpha)y\|^2 \le \alpha\|x\|^2 + (1-\alpha)\|y\|^2 - \alpha(1-\alpha)\varphi(\|x-y\|),$$

for all $x, y \in B_\rho, \alpha \in [0, 1]$.

Lemma 2.4 ([7]). *Suppose X is a uniformly convex Banach space. Suppose $0 < a < b < 1$, and $\{t_n\}$ is a sequence in $[a, b]$. Suppose $\{w_n\}$, $\{y_n\}$ are sequences in X such that $\|w_n\| \le 1$, $\|y_n\| \le 1$ for all n. Define $\{z_n\}$ in X by $z_n = (1-t_n)w_n + t_n y_n$. If $\lim_n \|z_n\| = 1$; then $\lim_n \|w_n - y_n\| = 0$.*

3. Main results

The following theorem is a generalization of Theorem 5 in [1].

Theorem 3.1. *Let K be a nonempty compact convex subset of a uniformly convex Banach space X. Suppose that a nonexpansive map $T : K \to \mathcal{P}(K)$ has a fixed point p. Let $\{x_n\}$ be the sequence of Ishikawa iterates defined by* (B). *Assume that*

(i) $0 \le \alpha_n, \beta_n < 1$,
(ii) $\beta_n \to 0$ *and*
(iii) $\sum \alpha_n \beta_n = \infty$. *Then the sequence $\{x_n\}$ converges to a fixed point of T.*

Proof. By using Lemma 2.3, we have

(1)

$$\begin{aligned}\|x_{n+1} - p\|^2 &= \|(1-\alpha_n)x_n + \alpha_n z'_n - p\|^2\\ &\le (1-\alpha_n)\|x_n - p\|^2 + \alpha_n\|z'_n - p\|^2 - \alpha_n(1-\alpha_n)\varphi(\|x_n - z'_n\|)\\ &\le (1-\alpha_n)\|x_n - p\|^2 + \alpha_n H^2(Ty_n, Tp) - \alpha_n(1-\alpha_n)\varphi(\|x_n - z'_n\|)\\ &\le (1-\alpha_n)\|x_n - p\|^2 + \alpha_n\|y_n - p\|^2 - \alpha_n(1-\alpha_n)\varphi(\|x_n - z'_n\|),\end{aligned}$$

(2)

$$\begin{aligned}\|y_n - p\|^2 &= \|(1-\beta_n)x_n + \beta_n z_n - p\|^2\\ &\le (1-\beta_n)\|x_n - p\|^2 + \beta_n\|z_n - p\|^2 - \beta_n(1-\beta_n)\varphi(\|x_n - z_n\|)\\ &\le (1-\beta_n)\|x_n - p\|^2 + \beta_n H^2(Tx_n, Tp) - \beta_n(1-\beta_n)\varphi(\|x_n - z_n\|)\\ &\le (1-\beta_n)\|x_n - p\|^2 + \beta_n\|x_n - p\|^2 - \beta_n(1-\beta_n)\varphi(\|x_n - z_n\|)\\ &= \|x_n - p\|^2 - \beta_n(1-\beta_n)\varphi(\|x_n - z_n\|).\end{aligned}$$

From (1) and (2), we get

(3)

$$\|x_{n+1} - p\|^2 \le \|x_n - p\|^2 - \alpha_n\beta_n(1-\beta_n)\varphi(\|x_n - z_n\|).$$

Therefore

$$\alpha_n\beta_n(1-\beta_n)\varphi(\|x_n - z_n\|) \le \|x_n - p\|^2 - \|x_{n+1} - p\|^2.$$

This implies

$$\sum_{n=1}^{\infty} \alpha_n\beta_n(1-\beta_n)\varphi(\|x_n - z_n\|) \le \|x_1 - p\|^2 < \infty.$$

Hence by Lemma 2.2, there exists a subsequence $\{x_{n_k} - z_{n_k}\}$ of $\{x_n - z_n\}$ such that $\varphi(\|x_{n_k} - z_{n_k}\|) \to 0$ as $k \to \infty$ and hence $\|x_{n_k} - z_{n_k}\| \to 0$, by the continuity and strictly increasing nature of φ. By the compactness of K, we may assume that $x_{n_k} \to q$ for some $q \in K$. Thus

$$\begin{aligned}\operatorname{dist}(q, Tq) &\le \|q - x_{n_k}\| + \operatorname{dist}(x_{n_k}, Tx_{n_k}) + H(Tx_{n_k}, Tq)\\ &\le \|q - x_{n_k}\| + \|x_{n_k} - z_{n_k}\| + \|x_{n_k} - q\| \to 0 \quad \text{as } k \to \infty.\end{aligned}$$

Hence, q is a fixed point of T. Now on taking q in place of p, we get that $\{\|x_n - q\|\}$ is a decreasing sequence by (3). Since $\|x_{n_k} - q\| \to 0$ as $k \to \infty$, it follows that $\{\|x_n - q\|\}$ decreases to 0, so that the conclusion of the theorem follows. ■

The following theorem is a generalization of Theorem 6 in [1]. Since the idea is similar to the one given in Theorem 3.1, we just only state the result without the proof.

Theorem 3.2. *Let K be a nonempty compact convex subset of a uniformly convex Banach space X. Suppose that a nonexpansive map $T : K \to \mathcal{P}(K)$ has a fixed point p. Let $\{x_n\}$ be the sequence of Mann iterates defined by* (A). *Assume that*

(i) $0 \le \alpha_n < 1$ *and*
(ii) $\sum \alpha_n = \infty$. *Then the sequence $\{x_n\}$ converges to a fixed point of T.*

Remark 3.3. In Theorem 3.1 (also Theorem 3.2), the compactness assumption is quite strong, since it is easy to find a sequence in the domain which converges to a fixed point of the mapping. Next, we are going to present a convergence theorem without a compactness assumption. To succeed in this aim, we define the Mann and Ishikawa iterates in a new sense, which is a slightly modification from the one given in Definition 2.1.

Definition 3.4. Let K be a nonempty convex subset of a uniformly convex Banach space $X, T : K \to \mathcal{P}(K)$, and suppose that $F(T)$ is a nonempty proximinal subset of K.

(C) *The sequence of Mann iterates* is defined by $x_0 \in K$,

$$x_{n+1} = \alpha_n x_n + (1 - \alpha_n) y_n, \quad \alpha_n \in [a, b], 0 < a < b < 1, n \ge 0,$$

where $y_n \in Tx_n$ is such that $\|y_n - u_n\| = \operatorname{dist}(u_n, Tx_n)$, and $u_n \in F(T)$ such that $\|x_n - u_n\| = \operatorname{dist}(x_n, F(T))$.

(D) *The sequence of Ishikawa iterates* is defined by $x_0 \in K$,

$$y_n = (1 - \beta_n) x_n + \beta_n z_n, \quad \beta_n \in [a, b], 0 < a < b < 1, n \ge 0,$$

where $z_n \in Tx_n$ is such that $\|z_n - u_n\| = \operatorname{dist}(u_n, Tx_n)$, and $u_n \in F(T)$ such that $\|x_n - u_n\| = \operatorname{dist}(x_n, F(T))$, and

$$x_{n+1} = (1 - \alpha_n) x_n + \alpha_n z'_n, \quad \alpha_n \in [a, b],$$

where $z'_n \in Ty_n$ is such that $\|z'_n - v_n\| = \operatorname{dist}(v_n, Ty_n)$, and $v_n \in F(T)$ such that $\|y_n - v_n\| = \operatorname{dist}(y_n, F(T))$.

Remark 3.5. Convergence of $\{x_n\}$ depends on the choice of initial point x_0, and sequences $\{u_n\}$ and $\{y_n\}$. For example, Let $K = [0, 1]$ and define a nonexpansive map T on K by $Tx = \{0, 1\}$ for all $x \in K$. Then the sequence of Mann iterates defined by (C) (and also the sequence of Ishikawa iterates defined by (D)) converges to 0 if $x_0 < \frac{1}{2}$ and converges to 1 if $x_0 > \frac{1}{2}$. In the case of $x_0 = \frac{1}{2}$, the convergence of $\{x_n\}$ depends on the choice of u_0.

The following definition was introduced by Senter and Dotson [17].

Definition 3.6. A multivalued mapping $T : K \to \mathcal{P}(K)$ is said to satisfy *Condition I* if there is a nondecreasing function $f : [0, \infty) \to [0, \infty)$ with $f(0) = 0$, $f(r) > 0$ for $r \in (0, \infty)$ such that

$$\operatorname{dist}(x, Tx) \ge f(\operatorname{dist}(x, F(T))) \quad \text{for all } x \in K.$$

The following result gives many examples of mappings that satisfy Condition I. We omit the proof because it is similar to the one given in [17, Lemma 1].

Proposition 3.7. *Let K be a bounded closed subset of a Banach space X. Suppose that a nonexpansive map $T : K \to \mathcal{P}(K)$ has a nonempty fixed point set. If $I - T$ is closed, then T satisfies Condition I on K.*

The following theorem is a multivalued version of Theorem 1 in [17]. We may observe that the result can be extended to a slightly more general formulation of a quasi-nonexpansive mapping.

Theorem 3.8. *Let K be a bounded closed convex subset of a uniformly convex Banach space, $X, T : K \to \mathcal{P}(K)$ a nonexpansive mapping that satisfies Condition I, and suppose that $F(T)$ is a nonempty proximinal subset of K. Then the sequence of Mann iterates defined by* (C) *converges to a fixed point of T.*

Proof. By the nonexpansiveness of T, for each $n \in \mathbb{N}$ we have

$$\begin{aligned}(4)\quad \|x_{n+1} - u_n\| &= \|\alpha_n x_n + (1-\alpha_n) y_n - u_n\| \\ &\le \alpha_n \|x_n - u_n\| + (1-\alpha_n)\|y_n - u_n\| \\ &\le \alpha_n \|x_n - u_n\| + (1-\alpha_n) H(Tx_n, Tu_n) \\ &\le \|x_n - u_n\| \\ &= \operatorname{dist}(x_n, F(T)).\end{aligned}$$

This implies

$$\operatorname{dist}(x_{n+1}, F(T)) \le \operatorname{dist}(x_n, F(T)).$$

The sequence $\{\operatorname{dist}(x_n, F(T))\}$ is decreasing and bounded below, so $\lim_n \operatorname{dist}(x_n, F(T))$ exists. We shall show that the limit is equal to zero. Suppose $\lim_n \operatorname{dist}(x_n, F(T)) = b > 0$. By (4), we get

$$\begin{aligned}(5)\quad \|x_{n+1} - u_{n+1}\| &= \operatorname{dist}(x_{n+1}, F(T)) \\ &\le \|x_{n+1} - u_n\| \\ &\le \|x_n - u_n\|.\end{aligned}$$

Again $\lim_n \|x_n - u_n\|$ exists, say b'. For each $n \in \mathbb{N}$, let $a_n = \frac{y_n - u_n}{\|x_n - u_n\|}$ and $b_n = \frac{x_n - u_n}{\|x_n - u_n\|}$. Then the sequences $\{a_n\}$ and $\{b_n\}$ are in the unit ball of X. For n large enough, we have $\|x_n - u_n\| \le 2b'$. This implies that

$$\begin{aligned}\|b_n - a_n\| &= \frac{\|x_n - y_n\|}{\|x_n - u_n\|} \\ &\ge \frac{\operatorname{dist}(x_n, Tx_n)}{\|x_n - u_n\|} \\ &\ge \frac{f(\operatorname{dist}(x_n, F(T)))}{\|x_n - u_n\|} \\ &\ge \frac{f(b/2)}{2b'} > 0 \quad \text{for all large } n.\end{aligned}$$

Therefore $\liminf_n \|b_n - a_n\| > 0$. On the other hand, since $\|x_{n+1} - u_n\| \le \|x_n - u_n\|$ for all $n \in \mathbb{N}$, we have $\limsup_n \|x_{n+1} - u_n\| \le b'$, and by (5) $b' \le \liminf_n \|x_{n+1} - u_n\|$. This implies $\lim_n \|x_{n+1} - u_n\| = b'$. Now,

$$\lim_{n\to\infty} \|(1-\alpha_n) b_n + \alpha_n a_n\| = \lim_{n\to\infty} \frac{\|x_{n+1} - u_n\|}{\|x_n - u_n\|} = \frac{b'}{b'} = 1,$$

by Lemma 2.4, $\lim_n \|b_n - a_n\| = 0$ a contradiction, and hence

$$\lim_{n\to\infty} \operatorname{dist}(x_n, F(T)) = 0.$$

The proof of the remaining part closely follows the proof of Theorem 2 of [17]. For the convenience of the reader, we include the details.

Since $\lim_n \operatorname{dist}(x_n, F(T)) = 0$, given $\varepsilon > 0$ there exists $N_\varepsilon > 0$ and $z_\varepsilon \in F(T)$ such that $\|x_{N_\varepsilon} - z_\varepsilon\| < \varepsilon$, which implies $\|x_n - z_\varepsilon\| < \varepsilon$ for all $n \ge N_\varepsilon$. Thus, if $\varepsilon_k = 1/2^k$ for $k \in \mathbb{N}$, then corresponding to each ε_k there is an $N_k > 0$

and a $z_k \in F(T)$ such that $\|x_n - z_k\| \leq \varepsilon_k/4$ for all $n \geq N_k$. We require $N_{k+1} \geq N_k$ for all $k \in \mathbb{N}$. We have, for all $k \in \mathbb{N}$,

$$\|z_k - z_{k+1}\| = \|z_k - x_{N_{k+1}} + x_{N_{k+1}} - z_{k+1}\| < \varepsilon_k/4 + \varepsilon_{k+1}/4 = 3\varepsilon_{k+1}/4.$$

Let $S(z, \varepsilon) = \{x \in X : \|x - z\| \leq \varepsilon\}$. For $x \in S(z_{k+1}, \varepsilon_{k+1})$ we have

$$\|z_k - x\| = \|z_k - z_{k+1} + z_{k+1} - x\| < 3\varepsilon_{k+1}/4 + \varepsilon_{k+1} < 2\varepsilon_{k+1} = \varepsilon_k.$$

That is, $S(z_{k+1}, \varepsilon_{k+1}) \subset S(z_k, \varepsilon_k)$ for $k \in \mathbb{N}$. Thus, $S(z_k, \varepsilon_k)$ is a decreasing sequence of nonempty bounded closed subsets of X. By the Cantor intersection theorem, $\bigcap_{k\in\mathbb{N}} S(z_k, \varepsilon_k) \neq \emptyset$. Let p be any point in the intersection; then $\|z_k - p\| \leq \varepsilon_k \to 0$ as $k \to \infty$. It is easy to see that $F(T)$ is a closed subset of K. Then $p \in F(T)$. Since $\|x_n - z_k\| < \varepsilon_k/4$ for all $n \geq N_k$, we have $x_n \to p$ as $n \to \infty$. ■

By applying Proposition 3.7 and Theorem 3.8, we obtain the following

Corollary 3.9. *Let K be a bounded closed convex subset of a uniformly convex Banach space X, and $T : K \to \mathcal{P}(K)$ be a nonexpansive mapping. Suppose $F(T)$ is a nonempty proximinal subset of K and $I - T$ is closed. Then the sequence of Mann iterates defined by* (C) *converges to a fixed point of T.*

Finally, we conclude with some fundamental questions as follows.

Question 1. Is Theorem 3.8 true for the Mann iterates defined by (A)?

Question 2. Is Theorem 3.8 true for the Ishikawa iterates defined by (B) and/or (D)?

Acknowledgements

This work was conducted while I was visiting the University of Iowa. I would like to thank the Department of Mathematics for their kind hospitality, especially to Professor W. A. Kirk for his valuable suggestions. The author was supported by Thailand Research Fund.

References

[1] K.P.R. Sastry, G.V.R. Babu, Convergence of Ishikawa iterates for a multi-valued mapping with a fixed point, Czechoslovak Math. J. 55 (2005) 817–826.
[2] F.E. Browder, Nonlinear operators and nonlinear equations of evolution in Banach spaces, in: Proc. Symp. Pure Math., vol. 18, Amer. Math. Soc., Providence, RI, 1976.
[3] W.A. Kirk, A fixed point theorem for mappings which do not increase distance, Amer. Math. Monthly 72 (1965) 1004–1006.
[4] S. Ishikawa, Fixed points by a new iteration method, Proc. Amer. Math. Soc. 44 (1974) 147–150.
[5] W.R. Mann, Mean value methods in iteration, Proc. Amer. Math. Soc. 4 (1953) 506–510.
[6] B.E. Rhoades, Comments on two fixed point iteration methods, J. Math. Anal. Appl. 56 (1976) 741–750.
[7] W.G. Dotson Jr., On the Mann iterative process, Trans. Amer. Math. Soc. 149 (1970) 65–73.
[8] R.L. Franks, R.P. Marzec, A theorem on mean value iterations, Proc. Amer. Math. Soc. 30 (1971) 324–326.
[9] C.W. Groetsch, A note on segmenting Mann iterates, J. Math. Anal. Appl. 40 (1972) 369–372.
[10] S. Ishikawa, Fixed points and iteration of a nonexpansive mapping in a Banach space, Proc. Amer. Math. Soc. 59 (1976) 65–71.
[11] A.K. Kalinde, B.E. Rhoades, Fixed point Ishikawa iterations, J. Math. Anal. Appl. 170 (1992) 600–606.
[12] K.K. Tan, H.K. Xu, Approximating fixed points of nonexpansive mappings by the Ishikawa iteration process, J. Math. Anal. Appl. 178 (1993) 301–308.
[13] M.K. Ghosh, L. Debnath, Convergence of Ishikawa iterates of quasi-nonexpansive mappings, J. Math. Anal. Appl. 207 (1997) 96–103.
[14] C.E. Chidume, S.A. Mutangadura, An example of the Mann iteration method for Lipschitz pseudocontractions, Proc. Amer. Math. Soc. 129 (2001) 2359–2363.
[15] T.C. Lim, A fixed point theorem for multivalued nonexpansive mappings in a uniformly convex Banach spsce, Bull. Amer. Math. Soc. 80 (1974) 1123–1126.
[16] H.K. Xu, Inequalities in Banach spaces with applications, Nonlinear Anal. 16 (1991) 1127–1138.
[17] H.F. Senter, W.G. Dotson, Approximating fixed points of nonexpansive mappings, Proc. Amer. Math. Soc. 44 (1974) 375–380.