Propulsion and Power Research 2016;5(1):55–62

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Propulsion and Power Research

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ORIGINAL ARTICLE

Numerical investigation for a hyperbolic annular fin with temperature dependent thermal conductivity

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Received 19 January 2015; accepted 7 July 2015
Available online 23 February 2016

KEYWORDS

Hyperbolic annular fin;
Thermal performance;
Pseudospectral method

Abstract An annular fin of hyperbolic profile with temperature dependent thermal conductivity is studied by pseudospectral method. Graphs illustrating the effect of fin dimensions, surface convection characteristics and the thermal conductivity parameter on the thermal performance of the fin are presented and discussed. A comparison of the obtained numerical results is made with the closed form analytical solution available in the literature for the case of constant thermal conductivity. This comparison confirms the high accuracy of numerical results. When the thermal conductivity increases with temperature, the effect is to elevate both the temperature distribution in the fin and the fin efficiency. The converse is true when the thermal conductivity decreases with temperature.

1. Introduction

Radial (annular) fins are often attached to the cylindrical surfaces to increase the rate of heat transfer from the cylindrical surface to the ambient. Traditional applications have included internal combustion engines, compressors, heat exchangers and control systems. With the advent of space age, space radiators equipped with annular fins of triangular profiles became popular. For example, Schnurr and Cothran [1] used a finite difference approach to study fin-and-tube radiator fitted with annular fins of triangular and trapezoidal profiles and included these effects in their analysis for the fin-to-fin and fin-to-base radiative exchange.

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Peer review under responsibility of National Laboratory for Aeronautics and Astronautics, China.

http://dx.doi.org/10.1016/j.jppr.2016.01.005

Design charts were produced to allow the calculation of heat dissipation for various values of the radiation parameter and the fin radii ratio. In the last thirty years, the use of fins as heat sinks for cooling electronic devices has gained enormous popularity. An earlier design used a capped hollow tube as a transistor heat sink [2,3] in which the cap serves as a radial fin. The analysis involved simultaneous solution of the two fin equations, one for the hollow tube and the other for the cap. Expressions were derived for the dissipation and fin efficiency of the proposed design. A web based analysis tool has been provided by HYPERLINK http://www.electronics-cooling.com/author/p/_teertstra by P. Teertstra et al. [4]. This tool allows the performance evaluation of a convective heat sink made of annular fins of rectangular profile as a function of the geometry, material properties, and the boundary and ambi ent conditions. More recent designs based on the constructal theory [5] appear in the form of a tree-like structure consisting of radial fins of different outer radii mounted on a tube [6]. This design minimizes the thermal resistance between the base of the fin and the environment. However, such a design is expensive to manufacture and hence difficult to implement in practice. The use of radial fins on condensers in refrigeration units is also very common. Condensers of thermosyphons used to stabilize foundations built on permafrost are equipped with radial fins to enhance dissipation of ground removed heat to the environment [7]. The subject of design and analysis of radial fins is a part of a wider body of knowledge called the extended surface heat transfer with contributions from a vast number of research ers throughout the world. The work done on radial fins until the year 2000 has been comprehensively discussed in a treatise by Kraus et al. [8]. The research work on radial fins has continued unabated in the last 10 years. Mokheimer [9] studied the effect of temperature dependent heat transfer coefficient, h, on the efficiency of annular (radial) fins of rectangular, triangular, concave parabolic and convex para bolic profiles and compared his finite difference results with those of Ullmann and Kalman [10] who used a constant h. Mokheimer concluded that the assumption of constant h leads to a significant underestimation of the fin efficiency. The use of his graphs reduces the amount of fin material required for a given heat transfer duty. Motivated by the savings in material, Kundu and Das [11] considered an annular fin with a step change in thickness and gave analytical results for the thermal performance as well as optimum dimensions of such a fin geometry. They found that a step annular fin provides higher heat dissipation with lesser material when compare with an annular fin of uniform thickness. It was suggested that a multiple step design instead of a single step could be used if further improvement in heat transfer and/or a reduction in the amount of material was necessary. Such a multiple-step design would be more difficult and expensive to fabricate and may not be economically feasible in practice. Soylemez [12] performed a thermo-economic analysis for a pipe with radial fins and derived the optimum length of the pipe that gave the maximum savings from a finned pipe for waste heat recovery applications. Chiu and Chen [13] calculated the thermal stresses induced in an annular fin with variable thermal conductivity due to a periodic variation of its base temperature. They found that the maximum radial stress in the fin occurred at a radial distance of 30% of the fin height (difference between the tip and base radii), while the maximum tangential stress occurred at the base of the fin. Unless fins are used in applications involving large tem perature differences, thermal stress calculations may not be warranted. Kang and Look [14] presented an optimization study of an annular fin of trapezoidal profile. Their results allow the designer to establish the optimum dimensions of the trapezoidal fin for a given volume of the fin material. Papers by Arslanturk [15] and Kang and Look [16] focus on annular fins with different heat transfer coefficients on the top and the bottom and present the performance and optimization results when an annular fin is subjected to asymmetric thermal conditions on its two faces. Such asymmetric thermal conditions can occur in horizontally oriented fins under natural convection conditions. In prac tice, if an average of the heat transfer coefficients on the two faces of the fin is used, it often obviates the need for more complicated analyses given in [15,16]. The works of Kundu [17], Sharqawy and Zubair [18], and Rosario and Rahman [19] deal with the performance and optimization issues when a single or an assembly of annular fins operates under dehumidifying conditions. These analyses involve simulta neous heat and mass transfers from the surface of the fin. For a fully wet fin, Sharqawy and Zubair [18] reported that the overall fin efficiency decreases as the relative humidity increases and as the atmospheric pressure decreases. The efficiency of a wet fin was found to be lower than that of a dry fin. Kundu [17] emphasized that for a given amount of material, an annular fin with a step change in thickness also provides superior thermal performance under wet condi tions when compared with the performance of a uniformly thick fin. Aziz and McFadden [20] used a symbolic algebra package to obtain an analytical solution for the temperature distribution in a heat generating annular fin with a constant base heat flux and an adiabatic fin. The analytical solution which appeared in terms of the modified Bessel functions can be used to compute the unknown base temperatures when other pertinent quantities are specified. In an earlier study, Yovanovich et al. [21] analyzed a heat generating annular fin with convective boundary conditions at both ends of the fin, taking into account the contact resistance at the base of the fin. As expected, the presence of contact resistance and the convection resistances at the two ends of the fin significantly impacts the thermal performance of the fin. The second law based optimum design of an annular fin has been reported by Taufiq et al. [22]. Their analysis included the entropy generation due to heat transfer as well as entropy generation due to fluid friction associated with the external flow over the surface of the fin. An entropy minimization study was performed to determine the opti mum fin thickness. Heggs and Ooi [23] developed design charts for annular fins in terms of performance ratio and maximum effectiveness rather than the fin efficiency. They argued that the fin performance ratio, which is the actual heat flow through the fin relative to the maximum achiev able heat flow through the fin, was a better indicator of the fin performance than the traditional idea of fin efficiency. Despite their assertion about the superiority of their parameters versus the fin efficiency, the latter continues to be used in practice and in the current literature. In an experimental study, Waszkiewicz et al. [24] tested annular fins made of aluminum coated with zeolite for use in adsorption chillers. The use of zeolite coating (2 g on each aluminum fin of 10 mm base radius and 76 mm tip radius) enhanced the specific cooling power of the adsorption chillers. It was also noted that further improvement in the cooling power of the chiller could be realized with the elimination or the reduction of the interfacial resistance between the tube and the fin base. Aziz and Khani [25] used the homotopy analysis method to derive analytic solutions for the thermal performance of rotating annular fins of rectangular and various convex parabolic profiles. The study assumed the convection heat transfer on the rotating fin surface to be a function of the radial coordinate and the angular speed of the shaft. The analytic solution was found to mimic quite accurately the results obtained from a direct numerical solution of the governing differential equation. Recently, six new papers that are relevant to the present discussion have appeared in the literature. The problem considered by Schnurr and Cothran [1] has been solved by Hryshchak and Pohrebyts'ka [26] using a double asympto tic expansion. The method, which combines the Poincare small parameter and the method of phase integrals, reduced the mathematical model of the problem to a nonlinear differential equation of second order with variable coeffi cients. The method of Hryshchak and Pohrebyts'ka [26] is simpler but it is only applicable when the radiation-conduction interaction is weak (small parameter). Kundu and Barman [27] have studied the performance of an annular fin under dehumidifying conditions with a new polynomial relationship between the humidity ratio and the saturation temperature. This model improved on the earlier model of Kundu [17] and its accuracy was shown to be better at higher vapor condensation rates compared with the earlier models [17–19]. Mustafa et al. [28] have looked at an annular fin made of a polymeric composite in which high thermal conductivity carbon fibers are laid out circumferen tially at different radii in a polymer of low thermal conductivity. Such a design has recently been advocated to createlighter heat sinks with thermal characteristics similar to those of metallic heat sinks. The composite fin has a much higher thermal conductivity in the radial direction than that in the longitudinal direction. Mustafa et al. [28] used a two-dimensional heat conduction model

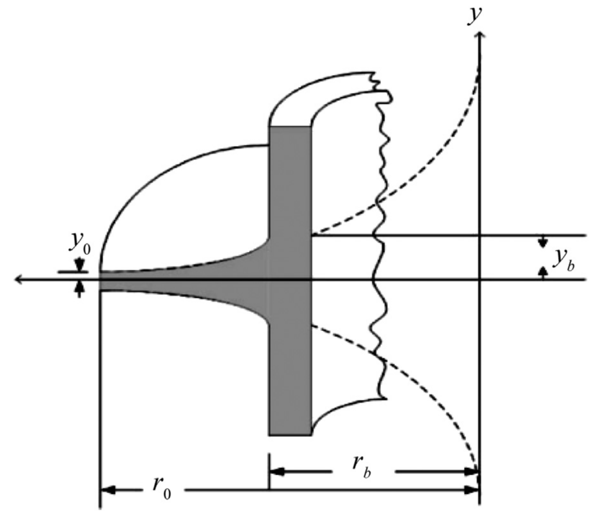

Figure 1 Profile of hyperbolic annular fin.

and developed an analytical solution using the method of separation of variables. The solution took into account the presence of contact resistance at the base of the fin and convection from the tip of the fin. The results can be used to predict the performance of an orthotropic annular fin. An interesting study has been performed by Kang [29] to determine the optimum dimensions of a fixed volume annular fin in which the inside radius of the fin (tube radius) was varied. He found that the optimum heat loss, fin effectiveness, and the fin length increased linearly as the inside radius of the fin (tube radius) increased. In a contemporaneous paper, Kang [30] optimized the annular fin design based on a fixed fin height (the difference between the outer and inner radii of the fin). An important conclusion emerging from this work was that the optimum fin length decreases as the fin base thickness increases. For the same heat transfer duty, annular fins of hyberpolic profile (Figure 1) offer considerable saving of material compared with the annular fin of rectangular profile. Kraus et al. [8] present an analytical solution for a convecting annular hyperbolic fin of constant thermal conductivity in terms of modified Bessel functions. In recent years, Arauzo et al. [31] analyzed the problem considered by Kraus et al. [8] and derived a truncated series solution and claimed the solution to be highly accurate. In another paper,Campo and Cui [32] applied a coordinate transformation to convert the Bessel equation into a differential equation for a straight fin which admits analytical solution in terms of the hyperbolic functions. Recently, Yang et al. [33] used a double decomposition method to analyze an annular fin of hyper bolic profile with temperature dependent (linear) thermal conductivity. The present paper studies an annular fin of hyperbolic profile with temperature dependent thermal conductivity using the pseudospectral method. The method is used to generate results for the temperature distribution and the fin efficiency.

2. Mathematical model and assumption

The mathematical model in the problem is based on the following assumptions:

(1) The fin operates under steady state condition.
(2) The fin material is homogeneous and isotropic.
(3) There is no internal heat generation or absorption in the fin.
(4) The fin loses heat by convection to its surroundings.
(5) The thermal conductivity of the fin is a linear function of temperature.
(6) The convective heat transfer coefficient is a constant.
(7) The temperature of the surrounding fluid remains constant during the heat rejection process.
(8) The base of the fin is maintained at a fixed temperature.
(9) Heat conduction occurs only in the radial direction, that is, there are no temperature gradients along the circumferential or axial directions.
(10) Heat loss from the tip of the fin is negligible.

2.1. *Formulation*

In this paper, we consider the annular fins with hyperbolic profile illustrated in Figure 1. The fin has a base radius r_b and a tip radius r_0. The thicknesses of the fin at the base y_b and y_0, respectively. The fin is exposed to a convective environment at temperature T_a which provides a heat transfer coefficient h on both faces of the fin. The combined area of the top and bottom faces of the fin is A_s. The fin has a thermal conductivity k which is a function of temperature. The thickness y of the fin varies inversely with the radius r i.e. $y \bullet r =$ a constant. The energy equation can be written as follows

$$\frac{d}{dr}\left(k_s A_r \frac{dT}{dr}\right)dr - A_s h(T - T_a) = 0,$$

where $k_s = k_a(1 + \beta(T - T_a))$ is the fin material thermal conductivity, $A_r = 2\pi r y_r$ is the cross-section area, and $A_s = 2\pi r ds$ is the local surface area. Then the energy equation can be rewritten as

$$\frac{d}{dr}\left(k_s 2\pi r y_r \frac{dT}{dr}\right)dr - 2\pi rh\, ds(T - T_a) = 0. \quad (1)$$

For a sufficiently thin fin, the arc length ds is approximately equal to the incremental radius dr which allows Eq. (2) to be written as follows:

$$\frac{d}{dr}\left(k_s r y_r \frac{dT}{dr}\right) - rh(T - T_a) = 0.$$

Utilizing the following dimensionless variables,

$$\mu = \frac{k_b - k_a}{k_a} = \beta(T_b - T_a), \qquad R = \frac{r}{r_0},$$
$$\theta = \tfrac{T - T_a}{T_b - T_a}, \qquad m = r_0\sqrt{\frac{h}{k_a y_b}},$$

the dimensionless energy equation can be expressed as follows:

$$\frac{d^2\theta}{dR^2} + \mu\theta\,\frac{d^2\theta}{dR^2} + \mu\left(\frac{d\theta}{dR}\right)^2 - \frac{R}{R_b}m^2\theta = 0. \quad (2)$$

and the dimensionless boundary conditions are

$$R = R_b:\ \theta = 1, \quad (3)$$

$$R = 1:\ \frac{d\theta}{dR} = 0. \quad (4)$$

If $\mu = 0$, Eq. (2) becomes a linear differential equation and it has an analytic solution in terms of the fractional modified Bessel functions of the first kind [8] as follows

$$\theta(R) = \sqrt{\frac{R}{R_b}}\left(\frac{I_{1/3}\left(\frac{2m}{3\sqrt{R_b}}R^{3/2}\right)I_{2/3}\left(\frac{2m}{3\sqrt{R_b}}\right) - I_{-1/3}\left(\frac{2m}{3\sqrt{R_b}}R^{3/2}\right)I_{-2/3}\left(\frac{2m}{3\sqrt{R_b}}\right)}{I_{1/3}\left(\frac{2m}{3}R_b\right)I_{2/3}\left(\frac{2m}{3\sqrt{R_b}}\right) - I_{-1/3}\left(\frac{2m}{3}R_b\right)I_{-2/3}\left(\frac{2m}{3\sqrt{R_b}}\right)}\right). \quad (5)$$

2.2. *Fin efficiency*

The fin efficiency η is defined as the ratio of the actual heat transfer rate from the fin to the surrounding and the ideal heat transfer rate from a fin of infinite thermal conductivity [8]

$$\eta = \frac{Q}{Q_{ideal}},$$

which may be expressed as

$$\eta = \frac{2}{1 - R_b^2}\int_{R_b}^{1} R\theta dR. \quad (6)$$

3. Numerical method

Eq. (2) is a strong nonlinear ordinary differential equation. To reduce the nonlinearity of Eq. (2), we tried to rearrange it as

$$\left\{(1 + \mu\theta^*)\frac{d^2}{dR^2} - \frac{R}{R_b}m^2\right\}\theta = -\mu\left(\frac{d\theta^*}{dR}\right)^2, \quad (7)$$

where θ^* denotes the last iterative value of dimensionless temperature.

To apply the pseudospectral collocation method, Chebysheve–Gausse–Lobatto collocation points are used for spatial discretization of the dimensionless energy conservation equation

$$\xi_i = -\cos\left[\frac{\pi(i-1)}{N-1}\right], \qquad i = 1, 2, ..., N. \quad (8)$$

The above collocation points take values in the interval $[-1,1]$. A transformation should be adopted to transform any arbitrary ($R : [R_{min}, R_{max}]$) interval into standard interval ($\xi : [-1, 1]$)

$$R = \frac{1}{2}[(R_{max} - R_{min})\xi + (R_{max} + R_{min})] \quad (9)$$

In the theory of the pseudospectral collocation method, the dimensionless temperature can be approximated to dimensionless temperature on collocation points by Lagrange interpolation polynomials, like

$$\theta(\xi) = \sum_{i=1}^{N} \Theta(\xi_i) h_i(\xi), \tag{10}$$

where h_i the Lagrange interpolation polynomial, and its detailed definition is

$$h_i(\xi) = \frac{w_i/(\xi - \xi_i)}{\sum_{j=1}^{N} w_j/(\xi - \xi_j)} \tag{11}$$

where

$$w_i = (-1)^{i-1}\delta_j, \qquad \delta_j = \begin{cases} 1/2, & j = 1, N \\ 1, & j = 2, 3, \ldots, N-1 \end{cases}$$

Substituting Eq. (10) into Eq. (7) and boundary conditions Eqs. (3) and (4) the matrix form of spectral discretized algebraic equation can beobtained as

$$\boldsymbol{A\Theta} = \boldsymbol{B}, \quad i = 1, 2, \ldots, N \tag{12}$$

and

$$\Theta_1 = 1, \ \sum_{j=1}^{N} D_{Nj}^{(1)} \Theta_j = 0, \tag{13}$$

where the element expressions for matrix $\boldsymbol{A}$ and $\boldsymbol{B}$

$$A_{ij} = \begin{cases} (1 + \mu\Theta_i^*)\left(\frac{2}{1-R_b}\right)^2 D_{ij}^{(2)} - \frac{(1+R_b)+(1-R_b)\xi_i}{2R_b} m^2, & i = j; \\ (1 + \mu\Theta_i^*)\left(\frac{2}{1-R_b}\right)^2 D_{ij}^{(2)}, & i \neq j. \end{cases} \tag{14}$$

$$B_i = -\mu\left[\sum_{j=1}^{N}\left(\frac{2}{1-R_b}\right) D_{ij}^{(1)} \Theta_j^*\right]^2 \tag{15}$$

where $D_{ij}^{(1)}$ and $D_{ij}^{(2)}$ are entries of the first order and the second order derivative coefficient matrices, respectively [34,35].

The implementation of pseudospectral collocation method for solving hyperbolic annular fin can be executed through the following routine:

Step 1. Input the number of collocation points, and compute the coordinate value of nodes, the first and second order derivative matrix.
Step 2. Initialize the value of dimensionless temperature θ initial assumptions (zero for example) in all directions except of boundaries.
Step 3. Assemble matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ by Eqs. (14) and (15).
Step 4. Impose boundary conditions (Eq. (13)) in system Eq. (12) and directly solve the new system.
Step 5. Terminate the iteration if the maximum absolute difference of the last and present dimensionless temperature is less than the tolerance (10^{-12} for example), otherwise go back to step 3.

4. Results and discussion

Eq. (2) with the initial and boundary conditions Eqs. (3) and (4) was solved numerically using pseudospectral method [34,35]. In order to test the accuracy of pseudospectral method, we compare the pseudospectral solution for a hyperbolic annular fin of constant thermal conductivity for which an exact solution (Eq. (5)) is available. This comparison is illustrated in Figure 2 for $R_b = 0.25$ and a range of values of the fin parameter m. The numerical solution is seen to be in excellent agreement with the closed form analytical solution. Figure 3 shows the pseudospectral results for a variable thermal conductivity fin with $\mu = 0.1$. The value of $\mu = 0.1$ represents a fin whose thermal conductivity is only mildly sensitive to the temperature variation.

In Figure 4, we have plotted the temperature distributions in a fin with $R_b = 0.25$ and $m = 0.5$ for μ ranging from -0.3 to $+0.3$. It may be noted that $\mu > 0$ indicates that the thermal conductivity of the fin increases with temperature, while $\mu < 0$ corresponds to the case of thermal conductivity decreasing with temperature. When compared with a fin of constant thermal conductivity ($\mu = 0$), the fin experiences higher temperatures when its thermal conductivity increases with temperature. Conversely, the local fin temperature is lower when the thermal conductivity of the fin decreases with temperature. This behavior has been consistently observed in many studies as such as Yang et al. [33] and Kraus et al. [8].

The thermal performance of a fin with $R_b = 0.40$ and a large thermal conductivity-temperature variation ($\mu = 0.6$) is shown in Figure 5. The temperature profiles follow the same pattern as seen in Figure 3 i.e. as the fin

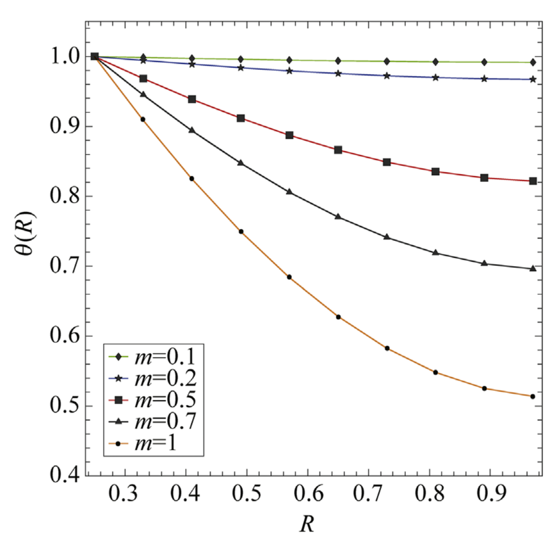

Figure 2 Comparison of the exact solution with numerical solution when $R_b = 1/4$ and $\mu = 0$. Solid line: numerical solution; Symbols: exact solution.

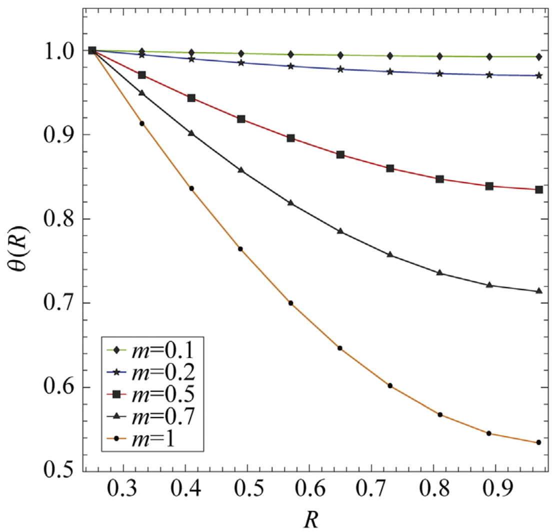

Figure 3 Temperature distribution for $R_b = 1/4$ and $\mu = 1/10$ for different values of m.

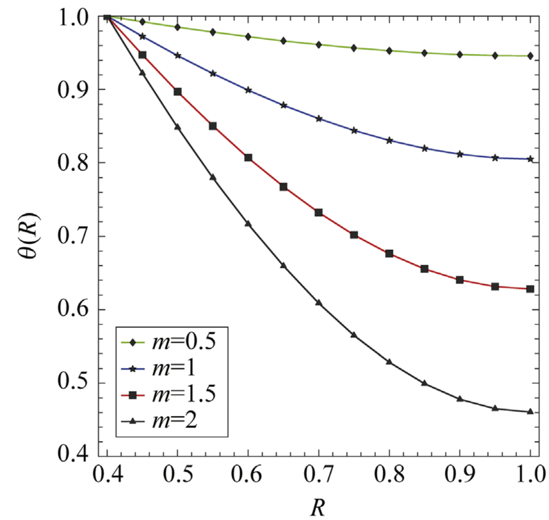

Figure 5 Effect of fin parameter m on the temperature distribution in a fin with $R_b = 4/10$ and $\mu = 6/10$.

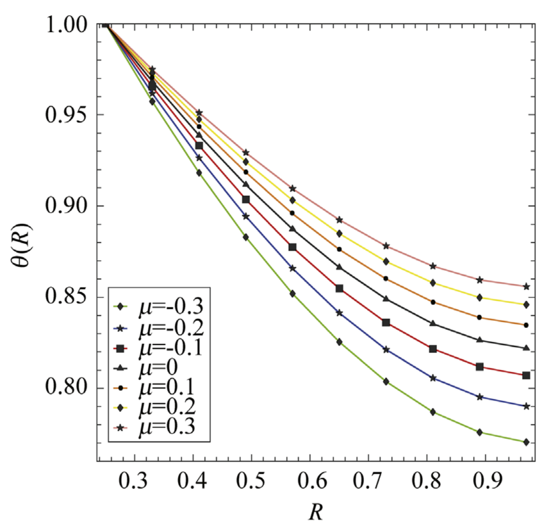

Figure 4 Temperature distribution for $R_b = 1/4$ and $m = 1/2$ for different values of μ.

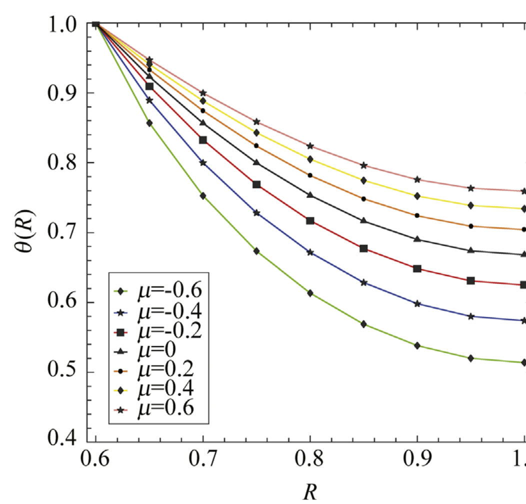

Figure 6 Effect of thermal conductivity parameter μ on the temperature distribution in a fin with $R_b = 6/10$ and $m = 2$.

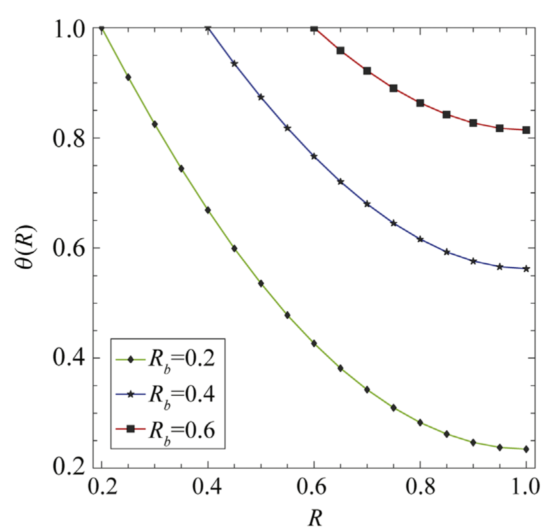

Figure 7 Effect of R_b on the temperature distribution in a fin with $m = 3/2$ and $\mu = 2/10$.

parameter$m$increases, the fin temperature decreases but the magnitude temperature gradient at the base increases indicating that increased heat flow from the base of the fin. Keeping R_b fixed at 0.40, we created Figure 6 to illustrate the effect of thermal conductivity parameter μ on the temperature distribution in the fin. These results parallel those of Figure 4 for a fin with $R_b = 0.25$.

To bring out the effect of varying R_b, we chose three values R_b, namely, 0.2, 0.4, and 0.6, and kept m fixed at 1.5 and μ at 0.2. These computations are displayed in Figure 7. As R_b increases, the local temperature in the fin gets elevated. For example, for $R_b = 0.2$, the dimensionless tip temperature is about 0.25 but for $R_b = 0.6$, this value is about 0.85. This behavior may be explained as follows. As R_b increases, the dimensionless fin height, $1 - R_b$, gets

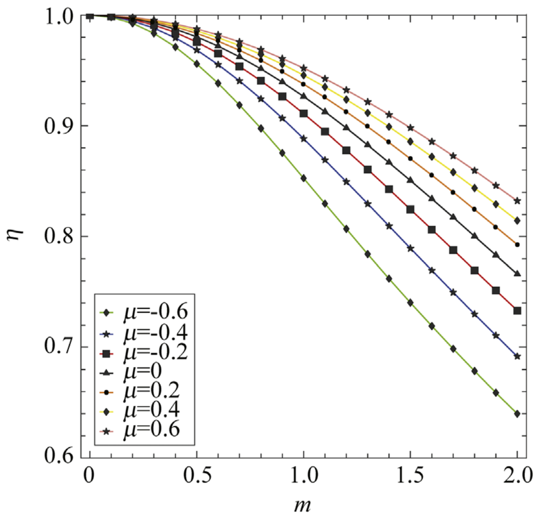

Figure 8 Fin efficiency as a function of m for different μ for a fin with $R_b = 6/10$.

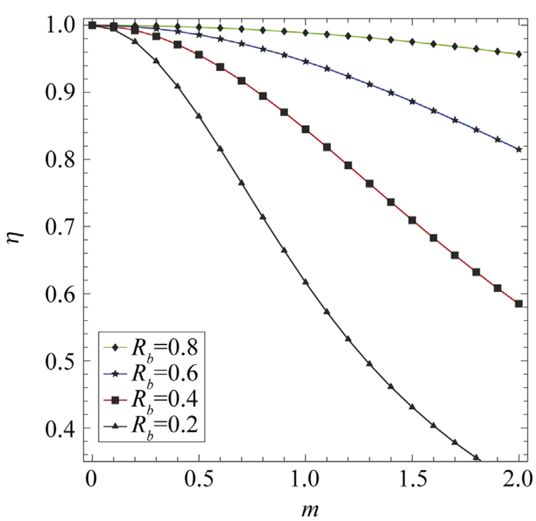

Figure 9 Fin efficiency as a function of fin parameter m for different values of R_b for $\mu = 4/10$.

smaller and the fin temperature in a shorter fin under the specified conditions must be higher than the corresponding temperatures in a longer fin.

Figure 8 shows the fin efficiency η as a function of fin parameter m for values of μ ranging from -0.6 to 0.6. This figure pertains to a fin with $R_b = 0.6$. As expected from the basic fin theory [8], the fin efficiency decreases as the in parameter m increases. For a given m, the fin efficiency increases as the thermal conductivity parameter μ increases. This can be explained if one thinks in terms of the average thermal conductivity. The higher the value of μ, the higher the average thermal conductivity of the fin. Considering the definition of parameter m, the higher average thermal conductivity gives a lower value of m. It was noted earlier in this paragraph, the lower value of m, the higher the fin efficiency. The discussion in this paragraph is consistent with the discussion in Aziz and Khani [25], Yang et al. [33] and many other studies in which fins with temperature dependent thermal conductivity are studied.

Finally, Figure 9 shows the fin efficiency as a function of fin parameter m for $R_b = 0.2$, 0.4, 0.6, and 0.8. For each geometry, the fin efficiency decreases as m increases as was seen in Figure 8. For a fixed m, the fin delivers higher efficiency as R_b increases, that is, as the dimensionless fin height, $1 - R_b$, decreases. Thus a shorter fin has a higher efficiency than a longer fin under identical operating conditions.

References

[1] N.M. Schnurr, C.A. Cothran, Radiation from an array of gray circular fins of trapezoidal profile, AIAA J. 12 (11) (1974) 1476–1480.

[2] A.D. Kraus, The efficiency of a transistor cap as a heat dissipater, Proceedings of 2nd National Heat Transfer Conference, Chicago, IL, ASME Paper No. 60-HT-15.

[3] A. Aziz, Advanced heat conduction theory with a symbolic algebra package, Computational Methods, In: G.R. Liu et al. (Ed.), Springer, 2006, pp. 829–848.

[4] Natural convection modeling of heat sinks using web based tools, Electronics Cooling, Posts by J. Richard Culham, http://www.electronics-cooling.com/author/j/_richard/_culham/ [cited September 2000].

[5] A. Bejan, S. Lorente, Design with Constructal Theory, John Wiley, NY, 2008.

[6] M.A. Almogbel, Constructal tree-shaped fins, Int. J. Therm. Sci. 44 (2005) 342–348.

[7] J.P. Zarling, F.D. Haynes, Thermosyphon-based designs and applications for foundations built on permafrost, Int Artic Tech Conf, Anchorage, AK, May 29–31, 1991. Paper No. 10.2118/22114-MS.

[8] A.D. Kraus, A. Aziz, J.R. Welty, Extended Surface Heat Transfer, John Wiley, NY, 2001.

[9] E.M.A. Mokheimer, Performance of annular fins with different profiles subject to variable heat transfer coefficient, Int. J. Heat Mass Transf. 45 (2002) 3631–3642.

[10] A. Ullman, H. Kalman, Efficiency and optimized dimensions of annular fins of different cross-section shapes, Int. J. Heat Mass Transf. 32 (1989) 1105–1110.

[11] B. Kundu, P.K. Das, Performance analysis and optimization of annular fin with a step change in thickness, ASME J. Heat Transf. 123 (2001) 601–604.

[12] M.S. Soylemez, Optimum length of finned pipe for waste heat recovery, Energy Convers. Manag. 49 (2008) 96–100.

[13] C.H. Chiu, C.K. Chen, Thermal stresses in annular fins with temperature dependent conductivity under periodic boundary condition, J. Therm. Stress. 25 (2002) 475–492.

[14] H.S. Kang, D.C. Look, Optimization of a trapezoidal profile annular fin, Heat Transf. Eng. 30 (5) (2009) 359–367.

[15] C. Arslanturk, Performance analysis and optimization of a thermally non-symmetric annular fin, Int. Commun. Heat Mass Transf. 31 (8) (2004) 1143–1153.

[16] H.S. Kang, D.C. Look, Optimization of a thermally asymmetric convective and radiating annular fin, Heat Transf. Eng. 28 (4) (2007) 310–320.

[17] B. Kundu, Analysis of thermal performance and optimization of concentric circular fins under dehumidifying conditions, Int. J. Heat Mass Transf. 52 (2009) 2646–2659.

[18] M.H. Sharqawy, S.M. Zubair, Efficiency and optimization of an annular fin with combined heat and mass transfer-an analytical solution, Int. J. Refrig. 30 (5) (2007) 751–757.
[19] L. Rosario, M.M. Rahman, Overall efficiency of a radial fin assembly under dehumidifying conditions, ASME J. Energy Resour. Technol. 120 (4) (1998) 299–305.
[20] A. Aziz, G. McFadden, Some new solutions for extended surface heat transfer using symbolic algebra, Heat Transf. Eng. 26 (9) (2005) 30–40.
[21] M.M. Yovanovich, J.R. Culham, T.F. Lemczyk, Simplified solutions to circular annular fins with contact resistance and end cooling, AIAA J. Thermophys. Heat Transf. 2 (1988) 152–157.
[22] B.N. Taufiq, H.H. Masjuki, T.M. Mahlia, R. Saidur, M.S. Faizul, E. Niza Mohamad, Second law analysis for optimal design of radial fin geometry by convection, Appl. Therm. Eng. 27 (8) (2007) 1363–1370.
[23] P.J. Heggs, T.H. Ooi, Design charts for radial rectangular fins in terms of performance ratio and maximum effectiveness, Appl. Therm. Eng. 24 (8–9) (2004) 1341–1351.
[24] S.D. Waszkiewicz, M.J. Tierney, H. Saidani Scott, Development of coated annular fins for adsorption chillers, Appl. Therm. Eng. 29 (11–12) (2009) 2222–2227.
[25] A. Aziz, F. Khani, Analytic solutions for a rotating radial fin of rectangular and various convex parabolic profiles, Commun. Nonlinear Sci. Numeric. Simul. 15 (6) (2010) 1565–1574.
[26] V.Z. Hryshchak, H.M. Pohrebyts'ka, Double asymptotic expansion in the problem of radiation heat exchange of annular fins of trapezoidal shape, J. Math. Sci. 171 (5) (2010) 603–701.
[27] B. Kundu, D. Barman, Analytical study on design analysis of annular fins under dehumidifying conditions with polynomial relationship between humidity ration and saturation temperature, Int. J. Heat Fluid Flow 31 (4) (2010) 722–733.
[28] M.T. Mustafa, S.M. Zubair, A.F.M. Arif, Thermal analysis of orthotropic annular fins with contact resistance: a closed-form analytical solution, Appl. Therm. Eng. 31 (2011) 937–945.
[29] H.S. Kang, Optimization of an annular fin with a pipe of variable inner radius for fixed fin volume, Trans. Korean Soc. Mech. Eng. 34 (3) (2010) 229–235.
[30] H.S. Kang, Optimization of a rectangular profile annular based fin height, J. Mech. Sci. Technol. 23 (2009) 3124–3131.
[31] I. Arauzo, A. Campo, C. Cortes, Quick estimate of the heat transfer characteristics of annular fins of hyperbolic profile with the power series method, Appl. Therm. Eng. 25 (4) (2005) 623–634.
[32] A. Campo, T. Cui, Temperature/Heat transfer analysis of annular fins of hyperbolic profile relying on the simple theory for straight fins of uniform profile, ASMW J. Heat Transf. 130 (5) (2008) 054502–054502-5.
[33] Y.T. Yang, C.C. Chang, C.K. Chen, A double decomposition method for solving the annular hyperbolic profile fins with variable thermal conductivity, Heat Transf. Eng. 31 (4) (2010) 1165–1172.
[34] M.T. Darvishi, S. Kheybari, F. Khani, Spectral collocation method and Darvishi's preconditionings to solve the generalized Burgers–Huxley equation, Commun. Nonlinear Sci. Numeric. Simul. 13 (2008) 2091–2103.
[35] M.T. Darvishi, S. Kheybari, F. Khani, A numerical solution of the Korteweg-de Vries equation by pseudospectral method using Darvishi's preconditioning, Appl. Math. Comput. 182 (2006) 98–105.