



# Leading power SCET analysis of $e^+e^- \rightarrow J/\psi gg$

Zhi-Hai Lin <sup>a</sup>, Guohuai Zhu <sup>b</sup>

<sup>a</sup> *Theory Group, KEK, Tsukuba, Ibaraki 305-0801, Japan*

<sup>b</sup> *Centre de Physique Théorique, Centre National de la Recherche Scientifique, UMR 7644, Ecole Polytechnique, 91128 Palaiseau cedex, France*

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## Abstract

Recently, Belle and BaBar Collaborations observed surprising suppression in the endpoint  $J/\psi$  spectrum, which stimulates us to examine the endpoint behaviors of the  $e^+e^- \rightarrow J/\psi gg$  production. We calculate the  $J/\psi$  momentum and angular distributions for this process within the framework of the soft-collinear effective theory (SCET). The decreasing spectrum in the endpoint region is obtained by summing the Sudakov logarithms. We also find a large discrepancy between the NRQCD and SCET spectrum in the endpoint region even before the large logarithms are summed, which is probably due to the fact that only the scalar structure of the two-gluon system is picked out in the leading power expansion. A comparison with the process  $\Upsilon \rightarrow \gamma gg$  is made.

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## 1. Introduction

Heavy quarkonium system plays an important role in the development of quantum chromodynamics (QCD). The scale of the heavy quark mass guarantees the applicability of perturbative QCD, meanwhile the nonperturbative physics presents itself through hadronization effects. In the past few years, one of developments in heavy quarkonium physics, called nonrelativistic quantum chromodynamics (NRQCD) [1] which generalizes and improves the conventional

color-singlet model (CSM), has provided a successful explanation of the surprising excesses of  $J/\psi$  and  $\psi'$  productions at the Tevatron [2] by introducing color octet contributions.

NRQCD factorization should be further examined in other collider facilities, in particular,  $e^+e^-$  colliders which provide a clean testing ground. SLAC and KEK  $e^+e^- B$  factories are now running at or below the  $\Upsilon(4s)$  resonance. At this energy, it was expected in NRQCD that the inclusive  $J/\psi gg$  process should be dominant [3,4] and in the upper endpoint region of the  $J/\psi$  momentum spectrum, there may exist a sharp peak as a clean signal of the color-octet  $c\bar{c}g$  contribution [5].

*E-mail addresses:* [linzh@post.kek.jp](mailto:linzh@post.kek.jp) (Z.-H. Lin), [guohuai.zhu@cpt.polytechnique.fr](mailto:guohuai.zhu@cpt.polytechnique.fr) (G. Zhu).

Recently, BaBar [6] and Belle [7,8] Collaborations published their measurements on prompt  $J/\psi$  productions in  $e^+e^-$  collision at center-of-mass (c.m.) energy  $\sqrt{s} = 10.58$  GeV. It is really surprising to observe that, according to Belle's data [7,8], it is the  $J/\psi c\bar{c}$  process that dominates the inclusive  $J/\psi$  production at  $B$  factories

$$\begin{aligned} \sigma(e^+e^- \rightarrow J/\psi c\bar{c})/\sigma(e^+e^- \rightarrow J/\psi X) \\ = 0.67 \pm 0.12, \end{aligned} \quad (1)$$

while the momentum distribution of the inclusive  $J/\psi$  production shows a suppression, instead of an (expected) enhancement, in the upper endpoint region.

For the unexpected  $J/\psi c\bar{c}$  dominance, it is argued in Ref. [4] that a large renormalization  $K$  factor might be the answer. Recent investigation [9] also reveals that the color-octet contribution to  $J/\psi$  spectrum can be broadened significantly by the large perturbative corrections and enhanced nonperturbative effects so as not to conflict with the surprising suppression in the endpoint region observed by BaBar and Belle. However a leading-order NRQCD calculation shows that, in the endpoint region, the color-singlet  $J/\psi gg$  contribution is not small at all, which seems to be still in contradiction with the experimental observations. In this work, we are stimulated to investigate the endpoint behaviors of the  $e^+e^- \rightarrow \gamma^* \rightarrow J/\psi gg$  production.

We note that, at the amplitude level,  $\gamma^* \rightarrow J/\psi gg$  is very similar to the decay  $\Upsilon \rightarrow \gamma gg$ . It has been known several years ago that, at the endpoint of the photon spectrum in radiative  $\Upsilon$  decays, NRQCD is not applicable due to the breakdown of both the perturbative expansion and the operator product expansion (OPE) [10]. The same arguments should also apply for the case of the  $J/\psi$  production. This is because NRQCD only contains soft degrees of freedom at low energy, but at the endpoint of the photon and/or  $J/\psi$  spectrum, the gluon jet should be almost collinear. To fix this problem, Fleming et al. proposed a combination of NRQCD for the heavy degrees of freedom and the soft-collinear effective theory (SCET) [11] for the light degrees of freedom. With this method, the radiative  $\Upsilon$  decays were investigated in a series of papers [12–14] which show an improved agreement with the CLEO data [15]. Lately the same method was applied to the color-octet contribution to the inclusive  $J/\psi$  production  $e^+e^- \rightarrow J/\psi + X$  [9]. By the use of the re-

summation of Sudakov logarithms and the nonperturbative shape functions, the color-octet  $J/\psi$  spectrum, which is a sharp peak at maximal energy in leading order calculations, could be significantly broadened and shifted to lower energies. This therefore would resolve the discrepancy between the color-octet  $J/\psi$  production and the experimental observations. According to the spirit of the Sudakov suppression in the endpoint region, the authors in Ref. [4] adopted a phenomenological approach to obtain an appropriate endpoint spectrum for the  $J/\psi gg$  process instead of performing a complete calculation in SCET.

In this Letter, we shall follow the same way of Refs. [9,13], namely SCET combined with NRQCD, to examine the endpoint behavior of the color-singlet  $J/\psi gg$  mechanism.

## 2. Leading order SCET calculation

Several scales are involved in this process: the center-of-mass energy  $\sqrt{s}$ , the  $J/\psi$  mass  $M_\psi$ , and the nonperturbative QCD scale  $\Lambda_{\text{QCD}}$ . In this Letter we will only consider the case where the ratio  $M_\psi/\sqrt{s}$  is kept finite in the limit of infinite  $\sqrt{s}$ . In this point of view,  $J/\psi$  can be taken as a heavy particle. In the kinematic endpoint region of  $J/\psi$  spectra, the failure of NRQCD factorization and the relevance of SCET has been explained clearly in Refs. [9,13]. In brief, the hadronic jet recoiling against  $J/\psi$  is not highly virtual,  $m_X \sim \sqrt{\sqrt{s}\Lambda_{\text{QCD}}}$ , compared with its large momentum of order  $\sqrt{s}$ . This results in the OPE breaking down, and therefore a new effective theory, the so-called SCET, is developed by including collinear degrees of freedom.

In SCET, it is convenient to write a momentum in light-cone coordinates. Working in the  $e^+e^-$  c.m. frame, we define the incoming electron and positron moving along light-cone directions  $n_e^\mu = (1, 0, 0, -1)$  and  $\bar{n}_e^\mu = (1, 0, 0, 1)$ . The produced  $J/\psi$  meson is chosen to move in the  $x-z$  plane with momentum  $P_\psi^\mu = Mv^\mu = (E, |\vec{P}|\sin\theta, 0, |\vec{P}|\cos\theta)$  ( $M$  is  $J/\psi$  mass), and hence the light-cone vectors for two gluons can be defined as  $n^\mu = (1, -\sin\theta, 0, -\cos\theta)$  and  $\bar{n}^\mu = (1, \sin\theta, 0, \cos\theta)$ . Throughout this Letter, we adopt a dimensionless variable  $z = |\vec{P}_\psi|/P_\psi^{\text{max}}$ , where  $P_\psi^{\text{max}}$  denotes the maximum value of the  $J/\psi$  momen-

tum, namely  $P_\psi^{\max} = \sqrt{s}(1-r)/2 \sim 4.9$  GeV. Here  $r = M^2/s \sim 0.08$ . The  $J/\psi$  velocity  $v$  can be expressed as

$$v^\mu = (v_0, |\vec{v}| \sin \theta, 0, |\vec{v}| \cos \theta) = \left( \sqrt{\frac{(1-r)^2}{4r} z^2 + 1}, \frac{1-r}{2\sqrt{r}} z \sin \theta, 0, \frac{1-r}{2\sqrt{r}} z \cos \theta \right). \tag{2}$$

For the process  $e^+e^- \rightarrow \gamma^* \rightarrow J/\psi X$ , the hadronic jet has the momentum  $p_X^\mu = l^\mu - Mv^\mu - k^\mu$ , where  $l^\mu = (\sqrt{s}, 0, 0, 0)$  is the momentum of the virtual photon and  $k^\mu$  is the residual momentum of the  $c\bar{c}$  pair within  $J/\psi$ . In the endpoint region, since the hadronic jet is collinear along the light-cone direction  $n^\mu$ , we can write  $p_X \sim \sqrt{s}(1, \lambda^2, \lambda)$  in the  $n-\bar{n}$  light-cone coordinate. When  $E_\psi^{\max} - E_\psi \sim \Lambda_{\text{QCD}}$ ,  $p_X^2$  is of order  $2\sqrt{s}(E_\psi^{\max} - E_\psi) \sim 2\sqrt{s}\Lambda_{\text{QCD}}$  which implies NRQCD factorization breaks down in this kinematic region. Therefore SCET becomes relevant in the endpoint region  $1-z \sim \Lambda_{\text{QCD}}/M \sim v^2$ , and correspondingly the expansion parameter  $\lambda$  is of order  $\sqrt{1-z}$  in this process.

Before going into details, it is helpful to notice the similarity between  $e^+e^- \rightarrow \gamma^* \rightarrow J/\psi gg$  and  $\Upsilon \rightarrow \gamma gg$ . In fact the cross section of  $J/\psi gg$  production can be related to the ‘‘decay width’’ of the transversely polarized virtual photon  $\gamma^*$  as follows [16]

$$d\sigma(e^+e^- \rightarrow J/\psi gg) = 4\pi\alpha_s^{-3/2} d\Gamma(\gamma^* \rightarrow J/\psi gg), \tag{3}$$

where the polarization vector of the virtual photon satisfies the following equation

$$\epsilon^\mu \epsilon^{*\nu} = -g^{\mu\nu} + \frac{n_e^\mu \bar{n}_e^\nu + n_e^\nu \bar{n}_e^\mu}{2} \equiv -g_{\perp e}^{\mu\nu}. \tag{4}$$

It is then clear that, at the amplitude level, the effective operator of  $\gamma^* \rightarrow J/\psi gg$  should be formally the same as that of  $\Upsilon \rightarrow \gamma gg$ . Therefore the proof of SCET factorization for the former process is almost the same as that of the latter one, which has been elaborated in Ref. [13]. All of our following calculations will be in parallel with those for  $\Upsilon \rightarrow \gamma gg$  in Ref. [13].

To proceed, we shall first match from NRQCD onto SCET. Considering the gauge and reparametrization invariance, the leading SCET color-singlet  $^3S_1$  operator is given by [13]<sup>1</sup>

$$\begin{aligned} \mathcal{O}(1, ^3S_1) &= \psi_{\mathbf{p}}^\dagger \Lambda \cdot \sigma^\delta \chi_{-\mathbf{p}} \\ &\quad \times \text{Tr}\{B_\perp^\alpha \Gamma_{\alpha\beta\delta\mu}^{(1,^3S_1)}(-n \cdot v \bar{\mathcal{P}}, -n \cdot v \bar{\mathcal{P}}^\dagger) B_\perp^\beta\} \\ &= \psi_{\mathbf{p}}^\dagger \Lambda \cdot \sigma^\delta \chi_{-\mathbf{p}} \\ &\quad \times \text{Tr}\left\{B_\perp^\alpha \Gamma_{\alpha\beta\delta\mu}^{(1,^3S_1)}\left(\frac{M(1-r)}{r}, -n \cdot v \mathcal{P}_-\right) B_\perp^\beta\right\}. \end{aligned} \tag{5}$$

where  $\psi_{\mathbf{p}}$  and  $\chi_{-\mathbf{p}}$  are the heavy quark and antiquark fields from NRQCD, and  $B_\perp$  is the leading piece of the collinear-gauge invariant gluon field strength [13]. The operator  $\bar{\mathcal{P}}$  ( $\bar{\mathcal{P}}^\dagger$ ) projects out the large light-cone momentum components of the collinear fields to the right (left). The second line of the above equation is obtained by using the identity  $B_\perp^\alpha n \cdot v (\bar{\mathcal{P}} + \bar{\mathcal{P}}^\dagger) B_\perp^\beta = -M(1-r)/r B_\perp^\alpha B_\perp^\beta$  and the definition  $\mathcal{P}_- = \bar{\mathcal{P}} - \bar{\mathcal{P}}^\dagger$ . From the matching shown in Fig. 1, we obtain the coefficient

$$\begin{aligned} \Gamma_{\alpha\beta\delta\mu}^{(1,^3S_1)}\left(\frac{M(1-r)}{r}, n \cdot v \bar{n} \cdot q_-\right) &= -\frac{4g_s^2 e e_c}{\sqrt{6}} \frac{r}{M(1-r)} g_{\alpha\beta}^\perp \left(g_{\mu\delta} + \frac{1-r}{2r} n_\mu \bar{n}_\delta\right), \end{aligned} \tag{6}$$

where  $g_{\alpha\beta}^\perp = g_{\alpha\beta} - (n_\alpha \bar{n}_\beta + n_\beta \bar{n}_\alpha)/2$ ,  $\bar{n} \cdot q_- = \bar{n} \cdot q - \bar{n} \cdot q'$ .  $q$  and  $q'$  are the momenta of two gluons.

According to the optical theorem, the  $J/\psi$  momentum spectrum and angular distribution can be expressed as

$$\begin{aligned} \frac{d\Gamma(\gamma^* \rightarrow J/\psi gg)}{dz d\cos\theta} &= \frac{(P_\psi^{\max})^3 z^2}{8\pi^2 \sqrt{s} \sqrt{M^2 + (P_\psi^{\max})^2 z^2}} \text{Im} T(z, \theta), \end{aligned} \tag{7}$$

<sup>1</sup> Here ‘leading’ means that the leading-order power expansion in terms of the small parameter  $\lambda$  in SCET.

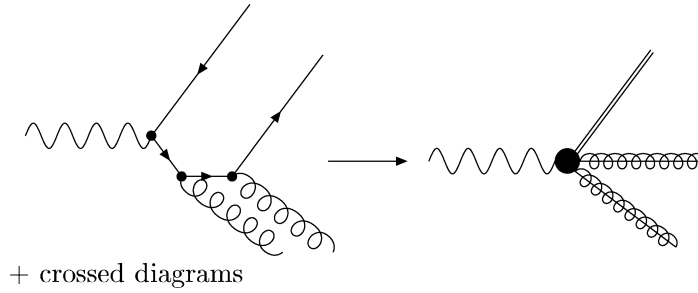


Fig. 1. Matching the amplitude for  $\gamma^* \rightarrow J/\psi gg$  process in QCD and SCET.

where the forward scattering amplitude is

$$T(z, \theta) = -i \int d^4x e^{-il \cdot x} \sum_X \langle 0 | J_v^\dagger(x) | J/\psi + X \rangle \times \langle J/\psi + X | J_\mu(0) | 0 \rangle g_\perp^{\mu\nu}. \quad (8)$$

In SCET, the following factorization formula can be proved in the endpoint region

$$\begin{aligned} \text{Im } T(z, \theta) &= \sum_\omega H\left(\frac{M(1-r)}{r}, \omega, z, \theta, \mu\right) \\ &\times \int dk^+ S(k^+, \mu) \\ &\times \text{Im } J_\omega\left(k^+ + \sqrt{s} - P_\psi^{\text{max}} z - \sqrt{M^2 + (P_\psi^{\text{max}})^2 z^2}, \mu\right), \end{aligned} \quad (9)$$

where  $H$ ,  $S$  and  $J_\omega$  are the hard function, ultrasoft function and jet function, respectively. In order to obtain the above formula, we match the QCD current  $J_\mu$  in Eq. (8) to the leading SCET color-singlet operator Eq. (5)

$$J_\mu(x) = \sum_\omega e^{-i(Mv - \bar{\mathcal{P}}(n/2) \cdot x)} i \Gamma_{\alpha\beta\delta\mu}^{(1,3S_1)}(\omega) \times \psi_{\mathbf{p}}^\dagger \Lambda \cdot \sigma^\delta \chi_{-\mathbf{p}} \text{Tr}\{B_\perp^\alpha \delta_{\omega, \mathcal{P}_-} B_\perp^\beta\}, \quad (10)$$

where the operator  $\bar{\mathcal{P}}$  in the phase factor will sum the label momentum of the two collinear fields  $B_\perp$  and thus can be replaced by  $-\sqrt{s}(1-r)$ . The matching coefficient  $\Gamma_{\alpha\beta\delta\mu}^{(1,3S_1)}(\omega)$  is given in Eq. (6). Since collinear fields in SCET are decoupled from ultrasoft gluons by field redefinition [11] and  $J/\psi$  meson has no collinear freedom, the forward scattering amplitude in Eq. (8) can then be factorized by separating

the heavy quark fields into ultrasoft functions and the collinear gluon fields into jet functions. Specifically, the jet function is defined from the vacuum matrix element of the collinear fields, which is exactly the same as that of the color-singlet radiative  $\Upsilon$  decay [13]

$$\begin{aligned} &\langle 0 | T \text{Tr}[B_\perp^{0\alpha} \delta_{\omega, \mathcal{P}_-} B_\perp^{0\beta}](x) \text{Tr}[B_\perp^{0\alpha'} \delta_{\omega', \mathcal{P}_-} B_\perp^{0\beta'}](0) | 0 \rangle \\ &\equiv \frac{i}{2} (g_\perp^{\alpha\alpha'} g_\perp^{\beta\beta'} + g_\perp^{\alpha\beta'} g_\perp^{\alpha'\beta}) \delta_{\omega, \omega'} \\ &\times \int \frac{d^4k}{(2\pi)^4} e^{-ik \cdot x} J_\omega(k^+, \mu), \end{aligned} \quad (11)$$

where  $B_\perp^0$  is the redefinition of the collinear field to decouple from the ultrasoft gluons. To calculate the jet function, one may directly evaluate the vacuum matrix element of the collinear fields, which is the left-hand side of Eq. (11). Actually the jet function which is independent of the heavy quark fields, should be the same for both  $\Upsilon \rightarrow \gamma gg$  and  $\gamma^* \rightarrow J/\psi gg$  processes, so it can be obtained directly from Ref. [13]. For our purpose, only the imaginary part of the jet function is relevant, and at the lowest order in  $\alpha_s$ , it is

$$\text{Im } J_\omega(k^+, \mu) = \frac{1}{8\pi} \Theta(k^+) \int_{-1}^1 d\xi \delta_{\omega, \sqrt{s}(1-r)\xi}. \quad (12)$$

Following Ref. [13], the ultrasoft function for this process can be written as

$$\begin{aligned} S(k^+, \mu) &= \int \frac{dx^-}{4\pi} e^{-(i/2)k^+x^-} \\ &\times \langle 0 | \chi_{-\mathbf{p}}^\dagger \sigma_i \psi_{\mathbf{p}}(x^-) a_\psi^+ a_\psi \psi_{\mathbf{p}}^\dagger \sigma_i \chi_{-\mathbf{p}}(0) | 0 \rangle \\ &= \langle 0 | \chi_{-\mathbf{p}}^\dagger \sigma_i \psi_{\mathbf{p}} a_\psi^+ a_\psi \delta(in \cdot \partial - k^+) \psi_{\mathbf{p}}^\dagger \sigma_i \chi_{-\mathbf{p}} | 0 \rangle, \end{aligned} \quad (13)$$

while the leading order hard function is computed as

$$\begin{aligned}
 H(\omega, z, \theta, \mu) &= \frac{2}{3} \left( \frac{4g_s^2 ee_c r}{\sqrt{6}M(1-r)} \right)^2 g_{\perp e}^{\mu\nu} \left( g_{\mu\delta} + \frac{1-r}{2r} n_\mu \bar{n}_\delta \right) \\
 &\quad \times \left( g_{\nu\lambda} + \frac{1-r}{2r} n_\nu \bar{n}_\lambda \right) (g^{\delta\lambda} - v^\delta v^\lambda) \\
 &= \frac{32\pi^2}{3} \left( \frac{4\alpha_s ee_c r}{\sqrt{6}M(1-r)} \right)^2 F(z, \theta), \tag{14}
 \end{aligned}$$

where the explicit expression for  $F(z, \theta)$  is

$$\begin{aligned}
 F(z, \theta) &= 2 - \sin^2 \theta \\
 &\quad + \frac{\sin^2 \theta}{4r^2} [(1+r)v_0 - (1-r)|\vec{v}|]^2. \tag{15}
 \end{aligned}$$

Here  $v$  is the  $J/\psi$  velocity given in Eq. (2).

With these functions in hand, we obtain the explicit form for the imaginary part of the forward scattering amplitude in Eq. (9) as follows

$$\begin{aligned}
 \text{Im } T(z, \theta) &= \Theta(\sqrt{s} - P_\psi - E_\psi) \frac{16\pi}{3M} \left( \frac{4\alpha_s ee_c r}{\sqrt{6}M(1-r)} \right)^2 \\
 &\quad \times F(z, \theta) \langle 0 | \chi_{-\mathbf{p}}^\dagger \sigma_i \psi_{\mathbf{p}} a_\psi^\dagger a_\psi \psi_{\mathbf{p}}^\dagger \sigma_i \chi_{-\mathbf{p}} | 0 \rangle \\
 &= \Theta(\sqrt{s} - P_\psi - E_\psi) \frac{8N_c |R(0)|^2}{3M} \\
 &\quad \times \left( \frac{4\alpha_s ee_c r}{\sqrt{6}M(1-r)} \right)^2 F(z, \theta), \tag{16}
 \end{aligned}$$

where  $R(0)$  denotes the radial wave function of  $J/\psi$  at the origin. Using the above equation, we arrive at the differential cross section in the tree-level SCET calculation<sup>2</sup>

$$\begin{aligned}
 \frac{d\sigma_{\text{trSCET}}}{dz d\cos\theta} &= \Theta(\sqrt{s} - P_\psi - E_\psi) \frac{32(\alpha_s \alpha e_c)^2 N_c |R(0)|^2}{9s M^3} \\
 &\quad \times \frac{r^2(1-r)z^2}{\sqrt{4r + (1-r)^2 z^2}} F(z, \theta). \tag{17}
 \end{aligned}$$

<sup>2</sup> ‘The tree-level SCET calculation’ here is referred to the leading power calculation in SCET before the resummation over large logarithms.

Since OPE breaks down and large logarithms arise as  $z$  approaches to 1, resummation over large logarithms is indispensable before comparing to the experimental observations. In SCET, these logarithms can be resummed using renormalization group equations (RGE). To do this, one has to first calculate the anomalous dimension of the effective operator (Eq. (5)). Fortunately, this effective operator is formally the same as that which appears in the color-singlet radiative  $\Upsilon$  decays, therefore we can directly read the anomalous dimension from Ref. [13]

$$\begin{aligned}
 \gamma(\eta) &= \frac{2}{\beta_0} \left\{ C_A \left[ \frac{11}{6} + (\eta^2 + (1-\eta)^2) \right. \right. \\
 &\quad \left. \left. \times \left( \frac{\ln \eta}{1-\eta} + \frac{\ln(1-\eta)}{\eta} \right) \right] - \frac{n_f}{3} \right\}. \tag{18}
 \end{aligned}$$

With this anomalous dimension, one can then resum the large logarithms using RGE from the matching (hard) scale to the collinear scale. The collinear scale should roughly be the invariant mass of the jet, namely  $\mu_c(z) = \sqrt{2\sqrt{s}(E_\psi^{\text{max}} - E_\psi(z))}$ . However there is no obvious clue what the matching scale should be. In NRQCD calculations, this scale is often chosen at quarkonium mass  $M$ , but in SCET it is found that, at least the hard scale for color-octet  $J/\psi$  production should be about  $-n \cdot v \bar{P} = M(1-r)/r$  [9], according to the logarithm that appears in the anomalous dimension calculations. As we know that, there is no large logarithm far from the endpoint region, which means that the collinear scale, which is of the order of  $\sqrt{s}$  for small  $z$ , should be comparable to the hard scale. Therefore in the following, we will naively choose the hard scale as  $\mu_h = \sqrt{s}(1-r)$ , which is simply the large light-cone component of the gluon jet momentum. Finally, the result for the resummed differential cross section is

$$\begin{aligned}
 \frac{d\sigma_{\text{resum}}}{dz d\cos\theta} &= \Theta(\sqrt{s} - P_\psi - E_\psi) \frac{32(\alpha_s(\mu_h) \alpha e_c)^2 N_c}{9s} \\
 &\quad \times \frac{|R(0)|^2}{M^3} \frac{r^2(1-r)z^2}{\sqrt{4r + (1-r)^2 z^2}} \\
 &\quad \times F(z, \theta) \int_0^1 d\eta \left( \frac{\alpha_s(\mu_c(z))}{\alpha_s(\mu_h)} \right)^{2\gamma(\eta)}. \tag{19}
 \end{aligned}$$

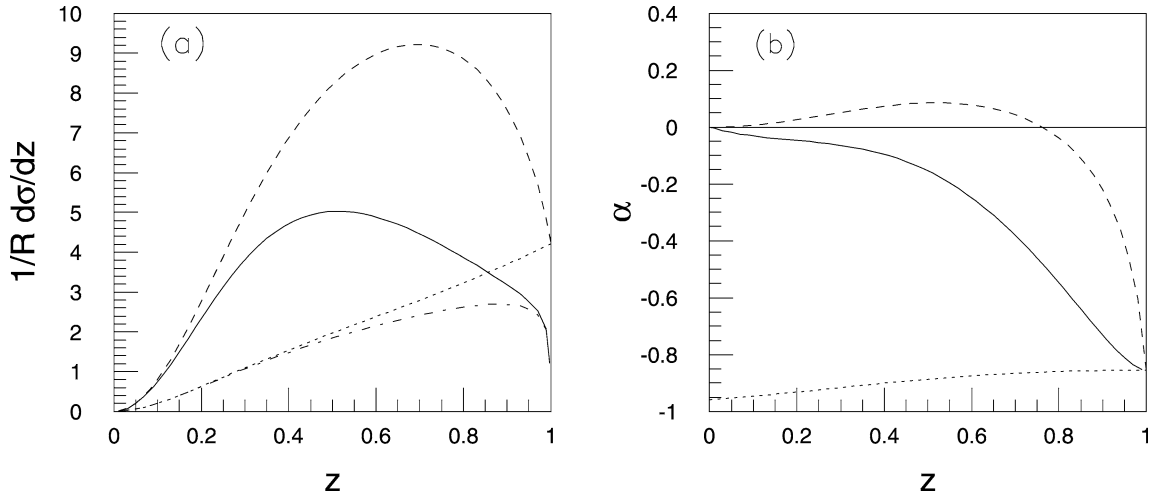


Fig. 2. The momentum distribution (a) and angular coefficient  $\alpha(z)$  (b) for  $e^+e^- \rightarrow J/\psi gg$  process. The dashed, dotted and dot-dashed curves correspond to the NRQCD, tree-level SCET and resummed SCET calculations, respectively. The solid curve is for the interpolated resummed results.

### 3. Results and discussions

It is understood that SCET is only valid at the large  $z$  region, while NRQCD should be fine in the small and medium  $z$  region. Therefore in order to obtain a formula which can describe the  $J/\psi$  production in the whole kinematic region, one shall interpolate smoothly between the NRQCD and resummed SCET results. Here we propose an interpolating formula

$$\frac{d\sigma_{\text{int}}}{dz d\cos\theta} = (1-z) \frac{d\sigma_{\text{NRQCD}}}{dz d\cos\theta} + z \frac{d\sigma_{\text{resum}}}{dz d\cos\theta}. \quad (20)$$

Obviously the NRQCD contribution vanishes in the limit of  $z = 1$ , and only the resummed contribution survives, while at the small  $z$ , the NRQCD contribution dominates. In addition, if one does not do any expansion and resummation in SCET,  $\sigma_{\text{resum}}$  should be replaced by  $\sigma_{\text{NRQCD}}$  and hence Eq. (20) will reproduce the NRQCD result.

The differential cross section for the  $J/\psi gg$  production are restricted by unitarity, parity, and angular momentum considerations. Its polar angle dependence can be parametrized into the form [3]

$$\frac{d\sigma}{dz d\cos\theta} = S(z) [1 + \alpha(z) \cos^2\theta], \quad (21)$$

where the angular coefficient  $\alpha(z)$  is generally limited in the interval  $-1 \leq \alpha(z) \leq 1$ . This general form

has been confirmed directly by the calculations in the framework of NRQCD [3,4,16]. From Eqs. (15), (17) and (19), it is easy to find that the tree-level and resummed SCET results also keep the form of Eq. (21), and furthermore, have the same coefficient  $\alpha(z)$ . As a natural result, the interpolated resummed cross section in Eq. (20) follows the same behavior.

In our numerical estimation we use  $\sqrt{s} = 10.58$  GeV and  $M = 2m_c = 3.0$  GeV. For simplicity, we also normalize the cross section to a dimensionless quantity by a factor  $R = (128/3)\alpha_s(\mu_h)^2\alpha^2 e_c^2 M |R(0)|^2/s^3$ .

In Fig. 2(a), we show the momentum distribution of the process  $e^+e^- \rightarrow J/\psi gg$ . The dashed, dotted, dot-dashed and solid curves correspond to the NRQCD, tree-level SCET, resummed SCET calculations and the interpolated resummed result, respectively. The NRQCD result is taken from Ref. [16], while the tree-level SCET, resummed SCET and interpolated resummed results are obtained by integrating over the polar angle  $\cos\theta$  in Eqs. (17), (19) and (20). As a cross-check of Eq. (17), one can see that the tree-level SCET result coincides with the NRQCD one in the limit  $z \rightarrow 1$ . Comparing to the NRQCD calculation, the interpolated resummed momentum distribution is suppressed significantly not only in the large  $z$  region but also in the medium  $z$  region. For example, at  $z = 0.9$  the ratio of the interpolated resummed

cross section and the NRQCD cross section is about 0.4, while at  $z = 0.5$  the ratio is still 0.6 which is not quite close to unit. However, the large suppression might be overestimated. This is because of the large discrepancy between the NRQCD and tree-level SCET results. Although at  $z = 1$ , the NRQCD and tree-level SCET results are exactly the same, which is guaranteed by the matching procedure, the tree-level SCET spectrum deviates very quickly from the NRQCD one as  $z$  departs from one. For instance, at  $z = 0.9$ , the tree-level SCET cross section is only half of the NRQCD one. This indicates that the tree-level SCET calculation may not be a good expansion of the NRQCD calculation even in the large  $z$  region. The resummed SCET cross section is entirely based on the tree-level SCET calculation, as shown in Eqs. (17) and (19), and therefore the over-suppression occurs after interpolating between the NRQCD and the resummed SCET contributions.

The discrepancy between the NRQCD and tree-level SCET results can be further investigated by the  $J/\psi$  angular distribution. In Fig. 2(b), we show the angular coefficient  $\alpha$  defined in Eq. (21) as the function of  $z$ . The dashed, dotted and solid curves are for the NRQCD, tree-level SCET and the interpolated resummed results, respectively. The resummed SCET result has the same  $\alpha(z)$  as that of the tree-level SCET.  $\alpha$  in the NRQCD calculation is around zero in the region  $z < 0.85$ , while falls off rapidly as  $z > 0.85$ . At  $z = 1$ ,  $\alpha$  is about  $-0.85$ . In contrast,  $\alpha$  in the tree-level or resummed SCET result almost does not change with  $z$ . This behavior provides some hint about why the tree-level SCET result does not match with the NRQCD one very well at large  $z$ . It was known that at the end point  $z = 1$ , only the scalar component of the gluon–gluon system is allowed, which gives  $\alpha = -0.85$  [17]. Apart from the end point, other spin components should be involved and might give dominant contributions which increase  $\alpha$  fast to be around zero with decreasing  $z$ . The leading SCET expansion in the small parameter  $\lambda \sim \sqrt{1-z}$ , which gives rise to a scalar operator for the gluon–gluon system (Eqs. (5) and (6)), cannot describe the contributions from other spin components. This implies that the power counting rules of SCET might break down due to some yet unknown reasons. One possibility is that part of the power suppressed contributions might be kinematically enhanced significantly. If this were true, one

would have to match onto SCET to the next-to-leading order in  $\lambda$ , obtain the power suppressed operators and their coefficients, and then perform the resummation procedure. However this complicated calculation goes far beyond the purpose of this Letter.

It is well known that the scale of  $J/\psi$  is a little awkward for the application of NRQCD, thus one might worry whether the discrepancy between the NRQCD and tree-level SCET results is just an illusion. The key observation here is that, the only dimensionless parameter in this process is  $r = M^2/s$ . Therefore the normalized cross section  $\sigma/R$ , which is dimensionless, should only depend on  $r$ . That means even in a model world in which  $J/\psi$  could be chosen to be very heavy (for example 30 GeV), the  $J/\psi$  momentum spectrum would still be the same as that showed in Fig. 2 if  $r = M^2/s$  is taken to be fixed by increasing the c.m. energy  $\sqrt{s}$  correspondingly. That is to say, even in a model world in which the application of NRQCD is guaranteed by very massive  $J/\psi$  and the perturbative treatment of jet function is guaranteed by the larger c.m. energy  $\sqrt{s}$ , the large discrepancy between the NRQCD and tree-level SCET calculations would still be there.

As we have emphasized before the similarity between the radiative  $\Upsilon$  decay  $\Upsilon \rightarrow \gamma gg$  and the inclusive  $J/\psi$  production  $\gamma^* \rightarrow J/\psi gg$ , one might naturally ask whether there is similar trouble for the former case. As shown in Ref. [13], for the photon momentum spectrum, there is no significant discrepancy between the NRQCD and tree-level SCET results for the radiative  $\Upsilon$  decay. However Ref. [13] has not investigated the angular distribution of photons. Considering the process  $e^+e^- \rightarrow \Upsilon \rightarrow \gamma gg$  at CLEO,  $\Upsilon$  is transversely polarized in the c.m. frame. Accordingly the ultrasoft matrix element, which is proportional to  $v^\delta v^{\delta'} - g^{\delta\delta'}$  in Ref. [13] ( $v$  is the  $\Upsilon$  velocity), now should change to be proportional to  $-g_{\perp e}^{\delta\delta'}$  (Eq. (4)) by using vacuum-saturation approximation. Performing an analogous calculation as what we have done in the last section, we obtain the differential  $\Upsilon$  decay rate in the tree-level SCET

$$\frac{d\Gamma_{\text{trSCET}}}{\Gamma_0 dz d\cos\theta} = \frac{3}{8}z(1 + \cos^2\theta), \quad (22)$$

where  $z = 2E_\gamma/M_\Upsilon$  and  $\theta$  is the scattering angle between the momentum of the outgoing photon and the electron beamline in the c.m. frame. Here  $\Gamma_0$  is a nor-

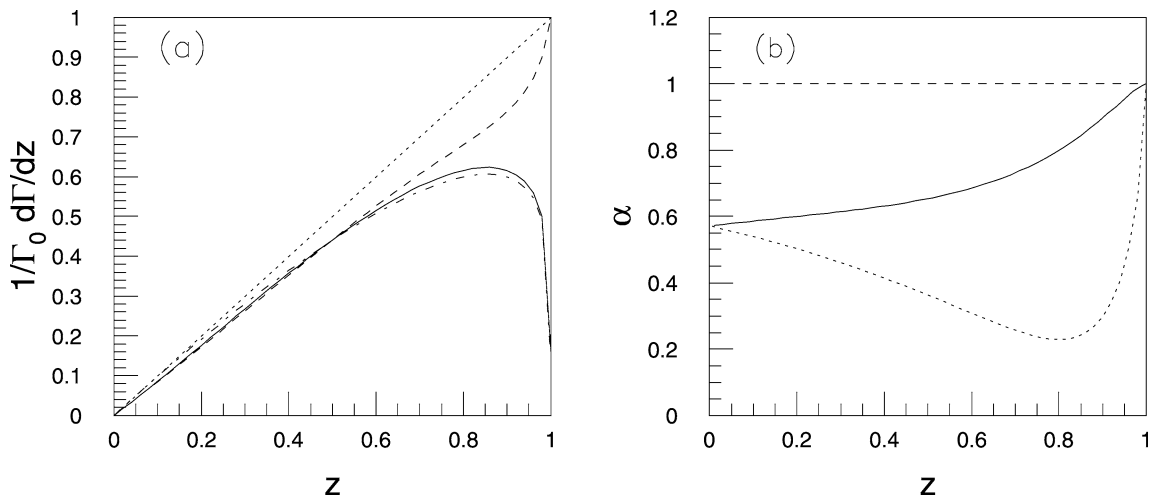


Fig. 3. The momentum distribution (a) and angular coefficient  $\alpha(z)$  (b) for  $e^+e^- \rightarrow \gamma \rightarrow \gamma gg$  process. The dashed, dotted and dot-dashed curves correspond to the NRQCD, tree-level SCET and resummed SCET calculations, respectively. The solid curve is for the interpolated resummed results.

malization constant. Eq. (22) indicates that the angular coefficient  $\alpha(z)$  defined in Eq. (21) is always equal to unity at any  $z$  in the SCET calculation.

The resummed SCET momentum distribution is the same as that in Ref. [13], while we choose the interpolation way as in Eq. (20) in order to give an interpolated resummed result for both momentum and angular distribution.

In Fig. 3, we show the momentum distributions and the coefficient  $\alpha$  of the radiative  $\gamma$  decay as the function of  $z$  in the NRQCD (the dashed line), the tree-level SCET (the dotted line), the resummed SCET (the dot-dashed line) and the interpolated resummed calculations. The NRQCD result is taken from Ref. [18]. It is clear that although the momentum distribution in the tree-level SCET calculation is close to that in NRQCD, the large difference of  $\alpha(z)$  still exists in the endpoint region. Similar with the  $J/\psi gg$  production, the gluon–gluon system here also leaves only scalar component at the endpoint  $z = 1$  [19]. Therefore the NLO matching onto SCET might also play an important role in this process.

In this Letter, we studied the collinear suppression effect in the process  $e^+e^- \rightarrow J/\psi gg$  and performed a leading power calculation in SCET. We obtained the decreasing  $J/\psi$  spectrum in the endpoint region, which comes from the Sudakov logarithms suppression, and combine our SCET result with the NRQCD

calculation. We then showed the momentum and angular distributions for  $J/\psi$  in SCET and compared with the NRQCD results. Surprisingly, we found that, even before the resummation over large logarithms, there already exists a large discrepancy between the SCET and NRQCD results in the endpoint region of  $J/\psi$  spectrum. A similar discrepancy is also found in the angular distribution of the radiative  $\gamma$  decay. Therefore it should be highly interesting to have further investigations, for example, including the power suppressed contributions, on these processes.

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