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Analytic resolution of puzzle in $B \to K\pi$ decays

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ABSTRACT

We present a systematic method to extract each Standard Model (SM)-*like* hadronic parameter as well as new physics parameters in analytic way for $B \to K\pi$ decays. Using the analytic method to the currently available experimental data, we find two possible solutions physically equivalent: one showing the large SM-*like* color-suppressed tree contribution and the other showing the large SM-*like* electroweak penguin contribution. The magnitude of new physics (NP) amplitude and its weak phase are quite large. For instance, we find $|P^{\mathrm{NP}}/P| = 0.39 \pm 0.13$, $\phi^{\mathrm{NP}} = 91^\circ \pm 15^\circ$ and $\delta^{\mathrm{NP}} = 8^\circ \pm 27^\circ$, which are the ratio of the NP to the SM penguin contribution, the weak and the relative strong phase of the NP amplitude, respectively.

1. Introduction

Brilliant progress of the B factory experiments sheds light on the study of rare B decays, which are crucial for testing the Standard Model (SM) and detecting any hints beyond the SM. Especially, $B \to K\pi$ decays are of great importance not only for investigating new physics (NP) due to the property of penguin dominance but also examining one of angles of Cabibbo–Kobayashi–Maskawa (CKM) unitarity triangle [1–3]. Many elaborate theoretical calculations based on QCDF [4], PQCD [5,6] and SCET [7] have been done for physical observables within the SM. But, some experimental data have shown considerable discrepancy from the theoretical estimation, inspiring searching NP in $B \to K\pi$ decays.

The ratios

$$R_c \equiv 2\frac{\mathcal{B}(B^+ \to K^+\pi^0)}{\mathcal{B}(B^+ \to K^0\pi^+)} \quad \text{and} \quad R_n \equiv \frac{1}{2}\frac{\mathcal{B}(B^0 \to K^+\pi^-)}{\mathcal{B}(B^0 \to K^0\pi^0)} \tag{1}$$

are expected to satisfy $R_c \approx R_n$ within the SM [3]. Before ICHEP-2006, those experimental values had shown a significant discrepancy, but as time passes they were getting closer to each other [8]. Current data updated by March 2007 in HFAG [9] show $R_c = 1.12 \pm 0.07$ and $R_n = 0.98 \pm 0.08$, which are consistent with the SM expectation. On the other hand, the CP asymmetry measurements still show a disagreement with the SM prediction. The SM naively expects $\mathcal{A}_{\mathrm{CP}}(B^0 \to K^+\pi^-) \approx \mathcal{A}_{\mathrm{CP}}(B^+ \to K^+\pi^0)$ for the direct CP asymmetry and $(\sin 2\beta)_{K_S\pi^0} \approx (\sin 2\beta)_{c\bar{c}s} = 0.68$ for the mixing-induced CP asymmetry. But the current experimental data show

$$\mathcal{A}_{\mathrm{CP}}(B^+ \to K^+\pi^0) - \mathcal{A}_{\mathrm{CP}}(B^0 \to K^+\pi^-) = 0.15 \pm 0.03, \tag{2}$$

$$(\sin 2\beta)_{K_S\pi^0} - (\sin 2\beta)_{c\bar{c}s} = -0.30 \pm 0.19. \tag{3}$$

The recent PQCD result for the difference of the above direct CP asymmetries is 0.08 ± 0.09, which is actually consistent with the data. However, the PQCD prediction $\mathcal{A}_{\mathrm{CP}}(B^+ \to K^+\pi^0)_{\mathrm{PQCD}} = -0.01^{+0.03}_{-0.05}$ still has 1.5σ difference from the current experimental data $\mathcal{A}_{\mathrm{CP}}(B^+ \to K^+\pi^0)_{\mathrm{EXP}} = 0.050 \pm 0.025$. Moreover, the difference of the mixing-induced CP asymmetry from the PQCD prediction is 0.065 ± 0.04, which shows about 2σ off the data.

Searching for NP via the electroweak penguin (EWP) processes in the $B \to K\pi$ decays has drawn lots of attention for a long time, especially based on various specific NP scenarios such as SUSY models [10], flavor-changing Z' models [11], four generation models [12], and so on. On the other hand, numerous model-independent attempts have been also made in search of NP within the quark diagram approach [13–18]. According to reparametrization invariance (RI) which was recently proposed in Refs. [19,20], any NP contribution can be absorbed into the SM amplitudes always in pair: for example, both the color-suppressed tree and the EWP amplitude. Thus, we would like to point out that the large enhancement of the color-suppressed tree amplitude and the EWP amplitude can be simultaneously understood by the single NP contribution with a non-zero NP weak phase within the model-independent analysis.

Our main goal in this work is to propose a systematic method for extracting each hadronic parameters in the presence of the single NP contribution under the consideration of RI. It will be shown that the parametrization with the additional NP contribution can be modified into the same form of the parametrization of the SM.

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The complete analytic solution for each hadronic parameters in this SM-*like* parametrization will be given in terms of the experimental data, and also their numerical values. Therefore, once the experimental data are given, one can pinpoint the hadronic parameters and will be able to directly compare to the theoretical estimations. For the extraction of NP parameters, the additional theoretical inputs are needed. To this end, we adopt two different schemes, one is flavor SU(3) symmetry and the other is PQCD prediction. It is discussed that how this NP contribution depends on the weak phase γ.

2. Parametrization and reparametrization invariance

In the quark diagram approach [21,22], the decay amplitudes of four $B \to K\pi$ modes are described as

$$A(B^+ \to K^0\pi^+) = \mathcal{P} + \mathcal{A}, \tag{4}$$

$$A(B^0 \to K^+\pi^-) = -\mathcal{P} - \mathcal{P}^C_{\rm EW} - \mathcal{T}, \tag{5}$$

$$\sqrt{2}A(B^+ \to K^+\pi^0) = -\mathcal{P} - \mathcal{P}_{\rm EW} - \mathcal{P}^C_{\rm EW} - \mathcal{T} - \mathcal{C} - \mathcal{A}, \tag{6}$$

$$\sqrt{2}A(B^0 \to K^0\pi^0) = \mathcal{P} - \mathcal{P}_{\rm EW} - \mathcal{C}, \tag{7}$$

under the redefinition of

$$\mathcal{P} + \mathcal{EP} - \frac{1}{3}\mathcal{P}^C_{\rm EW} - \frac{1}{3}\mathcal{EP}^C_{\rm EW} \to \mathcal{P}, \tag{8}$$

$$\mathcal{A} + \mathcal{EP}^C_{\rm EW} \to \mathcal{A}. \tag{9}$$

Each topological parameter represents strong penguin ($\mathcal{P}$), electroweak penguin ($\mathcal{P}_{\rm EW}$), exchange penguin ($\mathcal{EP}$), tree ($\mathcal{T}$), color-suppressed tree ($\mathcal{C}$) and annihilation ($\mathcal{A}$) topologies, respectively. The superscript C on the penguin parameters denotes a color-suppressed process. It is understood that each parameter includes both the weak phase and the strong phase in it. Each penguin parameters are involved in three terms associated with the internal quark exchanges. They can be manipulated by

$$\mathcal{P} \equiv V^*_{tb}V_{ts}\tilde{\mathcal{P}}_{tc} + V^*_{ub}V_{us}\tilde{\mathcal{P}}_{uc} \equiv \mathcal{P}_{tc} + \mathcal{P}_{uc}$$

using unitarity of the CKM matrix. Note that the CKM factors relevant to each parameter are $V^*_{tb}V_{ts}$ for the $\mathcal{P}_{tc}, \mathcal{P}_{\rm EW}, \mathcal{P}^C_{\rm EW}$ and $V^*_{ub}V_{us}$ for the $\mathcal{T}, \mathcal{C}, \mathcal{A}$, $\mathcal{P}_{uc}$. The relative sizes among these parameters are roughly estimated within the SM [22] as

$$\begin{aligned}
&1: && |\mathcal{P}_{tc}|,\\
&\mathcal{O}(\lambda): && |\mathcal{T}|, |\mathcal{P}_{\rm EW}|,\\
&\mathcal{O}(\lambda^2): && |\mathcal{C}|, |\mathcal{P}^C_{\rm EW}|,\\
&\mathcal{O}(\lambda^3): && |\mathcal{A}|,
\end{aligned} \tag{10}$$

where $\lambda \sim 0.2$ from the Wolfenstein parametrization [23]. For the relative size of $|\mathcal{P}_{uc}|$, one can roughly estimate that

$$\left|\frac{\mathcal{P}_{uc}}{\mathcal{P}_{tc}}\right| = \left|\frac{V^*_{ub}V_{us}\tilde{\mathcal{P}}_{uc}}{V^*_{tb}V_{ts}\tilde{\mathcal{P}}_{tc}}\right| \sim \lambda^2 \left|\frac{\tilde{\mathcal{P}}_{uc}}{\tilde{\mathcal{P}}_{tc}}\right|. \tag{11}$$

Note that $\tilde{\mathcal{P}}_u$ and $\tilde{\mathcal{P}}_c$ are smaller than $\tilde{\mathcal{P}}_t$ [24], and more precisely it can be estimated that $0.2 < |\tilde{\mathcal{P}}_{uc}/\tilde{\mathcal{P}}_{tc}| < 0.4$ within the perturbative calculation [25]. Therefore, we assume $|\mathcal{P}_{uc}/\mathcal{P}_{tc}| \sim \mathcal{O}(\lambda^3)$ for our analysis. It has been generally argued that the NP effects, if present, are the size of the EWP amplitude or smaller in $B \to K\pi$ decays. Thus we neglect all the minor contributions smaller than $|\mathcal{C}|$, such as $\mathcal{A}$ and $\mathcal{P}_{uc}$, for simplicity. (We also neglect $\mathcal{P}^C_{\rm EW}$, since the $|\mathcal{P}^C_{\rm EW}|$ is expected to be smaller than $|\mathcal{C}|$ [14,16].) Therefore, in our analysis the limit of NP sensitivity would be the order of $|\mathcal{C}|(\sim \lambda^2\mathcal{P}_{tc})$ at most.

Explicitly showing the weak phase γ and the strong phases δ, the decay amplitudes can be rewritten as

$$A(B^+ \to K^0\pi^+) = -P, \tag{12}$$

$$A(B^0 \to K^+\pi^-) = P\left(1 - r_T e^{i\gamma}e^{i\delta_T}\right), \tag{13}$$

$$\begin{aligned}
&\sqrt{2}A(B^+ \to K^+\pi^0)\\
&\quad = P\left(1 - r_T e^{i\gamma}e^{i\delta_T} - r_C e^{i\gamma}e^{i\delta_C} + r_{\rm EW}e^{i\delta_{\rm EW}}\right),
\end{aligned} \tag{14}$$

$$\sqrt{2}A(B^0 \to K^0\pi^0) = P\left(-1 - r_C e^{i\gamma}e^{i\delta_C} + r_{\rm EW}e^{i\delta_{\rm EW}}\right), \tag{15}$$

where $P \equiv |\mathcal{P}_{tc}|$, $r_T \equiv |\mathcal{T}/\mathcal{P}_{tc}|$, $r_C \equiv |\mathcal{C}/\mathcal{P}_{tc}|$, $r_{\rm EW} \equiv |\mathcal{P}_{\rm EW}/\mathcal{P}_{tc}|$, which are defined to be positive. We set the strong phase of the penguin contribution P to be zero so that all the other strong phases are relative to it. It is also used that $V^*_{tb}V_{ts} = -|V^*_{tb}V_{ts}|$. We assume that the weak phase γ can be measured from elsewhere. Then the number of unknown parameters in the above decay amplitudes is 7 ($P, r_T, r_C, r_{\rm EW}$, $\delta_T, \delta_C, \delta_{\rm EW}$) within the SM. We again emphasize that this approximated parametrization is the most efficient way to probe new physics up to the order of $|\mathcal{C}|$.

Now we introduce a single NP contribution coming through the EWP (or the color suppressed tree) contribution such as

$$P^N e^{i\phi^N}e^{i\delta^N}, \tag{16}$$

where P^N is defined to be positive, and ϕ^N and δ^N are weak and strong phase of the NP term, respectively. Then the two decay amplitudes in Eqs. (14) and (15) are modified by simply adding the NP term in the EWP contribution:

$$\begin{aligned}
&\sqrt{2}A(B^+ \to K^+\pi^0)\\
&\quad = P\left(1 - r_T e^{i\gamma}e^{i\delta_T} - r_C e^{i\gamma}e^{i\delta_C} + r_{\rm EW}e^{i\delta_{\rm EW}} + r^N e^{i\phi^N}e^{i\delta^N}\right),
\end{aligned} \tag{17}$$

$$\begin{aligned}
&\sqrt{2}A(B^0 \to K^0\pi^0)\\
&\quad = P\left(-1 - r_C e^{i\gamma}e^{i\delta_C} + r_{\rm EW}e^{i\delta_{\rm EW}} + r^N e^{i\phi^N}e^{i\delta^N}\right),
\end{aligned} \tag{18}$$

where $r^N \equiv P^N/P$. It has been introduced that any single decay amplitude can be separated into two decay amplitudes which have arbitrary weak phases θ and η, respectively, unless θ and η are equal or modulo π [19]. Since any physical results should not be changed, it is called reparametrization invariance (RI). More explicitly, any phase term $e^{i\phi}$ can be separated as

$$e^{i\phi} = \frac{\sin(\phi - \eta)}{\sin(\theta - \eta)}e^{i\theta} - \frac{\sin(\phi - \theta)}{\sin(\theta - \eta)}e^{i\eta}, \tag{19}$$

where the phases θ and η are arbitrarily chosen, satisfying $\theta - \eta \neq 0 \pmod{\pi}$. This is a simple algebraic identity. Due to this identity, the NP amplitude can be re-expressed as

$$r^N e^{i\phi^N}e^{i\delta^N} = r^N\frac{\sin\phi^N}{\sin\gamma}e^{i\gamma}e^{i\delta^N} - r^N\frac{\sin(\phi^N - \gamma)}{\sin\gamma}e^{i\delta^N}. \tag{20}$$

Here, the weak phases γ and 0 are chosen in order to match with the weak phases of the color-suppressed tree and EWP amplitudes. Then those two terms can be absorbed into the parameters of the color-suppressed tree and EWP leading to the following parametrization

$$\begin{aligned}
&\sqrt{2}A(B^+ \to K^+\pi^0)\\
&\quad = P\left(1 - r_T e^{i\gamma}e^{i\delta_T} - r^M_C e^{i\gamma}e^{i\delta^M_C} + r^M_{\rm EW}e^{i\delta^M_{\rm EW}}\right),
\end{aligned} \tag{21}$$

$$\begin{aligned}
&\sqrt{2}A(B^0 \to K^0\pi^0)\\
&\quad = P\left(-1 - r^M_C e^{i\gamma}e^{i\delta^M_C} + r^M_{\rm EW}e^{i\delta^M_{\rm EW}}\right),
\end{aligned} \tag{22}$$

which have the same form of the SM parametrization with the following modified parameters

$$r^M_C e^{i\delta^M_C} \equiv r_C e^{i\delta_C} - r^N\frac{\sin\phi^N}{\sin\gamma}e^{i\delta^N}, \tag{23}$$

$$r^M_{\rm EW}e^{i\delta^M_{\rm EW}} \equiv r_{\rm EW}e^{i\delta_{\rm EW}} - r^N\frac{\sin(\phi^N - \gamma)}{\sin\gamma}e^{i\delta^N}. \tag{24}$$

The NP amplitude is now absorbed into the SM parameters of the color-suppressed tree and EWP. Therefore, color-suppressed tree amplitude in these SM-*like* parametrization can be affected by the NP contribution of EWP unless $\phi^N = 0$ as shown in Eq. (23).

3. Analytic solutions for the SM-like parameters

In this section, we present the analytic solutions for the SM-*like* parameters [26]. For the first step, we rewrite Eqs. (12), (13), (21), and (22) as

$$A^{0+}e^{i\alpha^{0+}} \equiv A^{0+}e^{i\pi} = -P, \tag{25}$$

$$A^{+-}e^{i\alpha^{+-}} = P\left(1 - r_T e^{i\gamma} e^{i\delta_T}\right), \tag{26}$$

$$\sqrt{2}A^{+0}e^{i\alpha^{+0}} = P\left(1 - r_T e^{i\gamma} e^{i\delta_T} - r_C^M e^{i\gamma} e^{i\delta_C^M} + r_{\mathrm{EW}}^M e^{i\delta_{\mathrm{EW}}^M}\right), \tag{27}$$

$$\sqrt{2}A^{00}e^{i\alpha^{00}} = P\left(-1 - r_C^M e^{i\gamma} e^{i\delta_C^M} + r_{\mathrm{EW}}^M e^{i\delta_{\mathrm{EW}}^M}\right), \tag{28}$$

where A^{ij} denote magnitudes of the decay amplitudes of $B \to K^i\pi^j$ and α^{ij} represent their complex phases ($ij = \{0+, +-, +0, 00\}$). We put a *bar* on top of the amplitude parameters in case of the CP conjugate modes. It should be noted that these SM-*like* parametrization is including the NP contribution, namely the one coming into EWP sector, via RI. Table 1 shows current experimental data for the $B \to K\pi$ decays [27,28]. We use the notation for the branching ratios and CP asymmetries compatible with HFAG [9]:

$$\mathcal{B}^{ij} \propto \tau_{B^{(+,0)}} \frac{A^{ij^2} + \bar{A}^{ij^2}}{2}, \tag{29}$$

$$\mathcal{A}_{\mathrm{CP}}^{ij} \equiv -\frac{A^{ij^2} - \bar{A}^{ij^2}}{A^{ij^2} + \bar{A}^{ij^2}}, \tag{30}$$

$$S_f \equiv \eta_f \frac{2\,\mathrm{Im}\,\lambda_f}{1 + |\lambda_f|^2}, \tag{31}$$

where $\tau_{B^{(+,0)}}$ is the life time of a $B^{(+,0)}$ meson. The λ_f is defined by $\lambda_f = e^{-2i\beta}\bar{A}/A$ and η_f is the CP eigenvalue of the final state f. We also use the following numerical values from PDG [29]:

$$\sin 2\beta = 0.687, \qquad \gamma = 63^\circ, \qquad \frac{\tau_{B^+}}{\tau_{B^0}} = 1.071. \tag{32}$$

The number of parameters is 7 ($P, r_T, r_C^M, r_{\mathrm{EW}}^M, \delta_T, \delta_C^M, \delta_{\mathrm{EW}}^M$), while 9 observables are available in $B \to K\pi$ decays. Since $\mathcal{A}_{\mathrm{CP}}^{0+}$ automatically vanishes in our parametrization, we discard the data. Setting aside the mixing induced CP asymmetry data $S_{K_S\pi^0}$, we use the remaining 7 experimental data in order to determine the 7 parameters. From Eq. (25) we easily get the solution for P in terms of the observable by taking into account the phase space factor:

$$P = A^{0+} = (49.9 \pm 1.1)\ \mathrm{eV}. \tag{33}$$

Combining Eqs. (25) and (26), one finds [16] that

$$R = 1 + r_T^2 - 2r_T \cos\delta_T \cos\gamma, \tag{34}$$

$$-\mathcal{A}_{\mathrm{CP}}^{+-} R = 2r_T \sin\delta_T \sin\gamma, \tag{35}$$

where R is given [2] by

$$R \equiv \frac{\mathcal{B}^{+-}}{\mathcal{B}^{0+}} \frac{\tau_{B^+}}{\tau_{B^0}} = 0.90 \pm 0.05. \tag{36}$$

The analytic solutions for δ_T and r_T are obtained in terms of the observables from the above equations are

$$\cot\delta_T = \frac{\sin 2\gamma}{(-\mathcal{A}_{\mathrm{CP}}^{+-})R} \times \left[1 \pm \sqrt{1 + \frac{1}{\cos^2\gamma}\left(R - 1 - \left(\frac{-\mathcal{A}_{\mathrm{CP}}^{+-} R}{2\sin\gamma}\right)^2\right)}\right], \tag{37}$$

$$r_T = \sqrt{R\left(1 - \mathcal{A}_{\mathrm{CP}}^{+-}\cot\gamma\cot\delta_T\right) - 1}. \tag{38}$$

Using the experimental data given in Table 1, we obtain numerical values of r_T and δ_T. As shown in Fig. 1, the following two solutions are found:

$$r_T = 0.14 \pm 0.07, \qquad \delta_T = 20^\circ \pm 12^\circ, \quad \text{or} \tag{39}$$

$$r_T = 0.78 \pm 0.07, \qquad \delta_T = 3.6^\circ \pm 0.5^\circ. \tag{40}$$

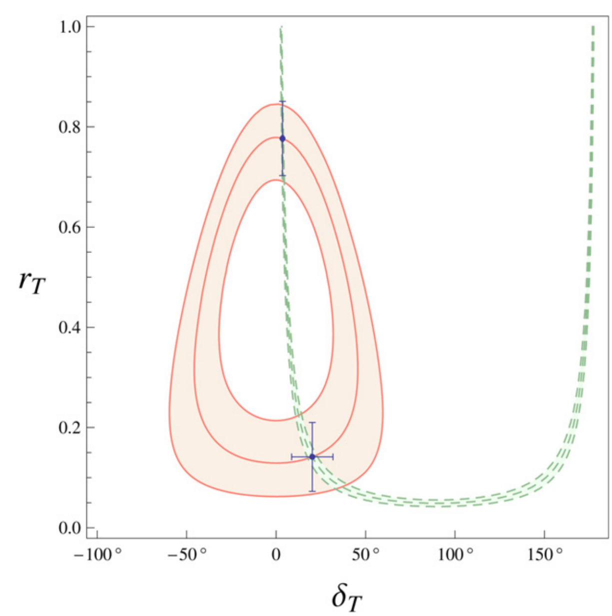

Fig. 1. Contour plot corresponding to the 1σ range of R and $\mathcal{A}_{\mathrm{CP}}^{+-}$ in Eqs. (34) and (35) in the r_T–δ_T plane. The solid lines are from Eq. (34) and the dashed lines are from Eq. (35). The two intersection regions show the two different solutions for r_T and δ_T. The two solutions are marked with error bars.

Table 1
Current experimental data for $B \to K\pi$. The branching ratios are in 10^{-6}. The average values are given by HFAG, updated by September 2007 [9]

Measurement	BaBar	Belle	CLEO	Average
$\mathcal{B}(K^0\pi^+)$	$23.9 \pm 1.1 \pm 1.0$	$22.8^{+0.8}_{-0.7} \pm 1.3$	$18.8^{+3.7+2.1}_{-3.3-1.8}$	23.1 ± 1.0
$\mathcal{B}(K^+\pi^0)$	$13.6 \pm 0.6 \pm 0.7$	$12.4 \pm 0.5 \pm 0.6$	$12.9^{+2.4+1.2}_{-2.2-1.1}$	12.9 ± 0.6
$\mathcal{B}(K^+\pi^-)$	$19.1 \pm 0.6 \pm 0.6$	$19.9 \pm 0.4 \pm 0.8$	$18.0^{+2.3+1.2}_{-2.1-0.9}$	19.4 ± 0.6
$\mathcal{B}(K^0\pi^0)$	$10.3 \pm 0.7 \pm 0.6$	$9.2 \pm 0.7^{+0.6}_{-0.7}$	$12.8^{+4.0+1.7}_{-3.3-1.4}$	9.9 ± 0.6
$\mathcal{A}_{\mathrm{CP}}(K^0\pi^+)$	$-0.029 \pm 0.039 \pm 0.010$	$0.03 \pm 0.03 \pm 0.01$	$0.18 \pm 0.24 \pm 0.02$	0.009 ± 0.025
$\mathcal{A}_{\mathrm{CP}}(K^+\pi^0)$	$0.030 \pm 0.039 \pm 0.010$	$0.07 \pm 0.03 \pm 0.01$	$-0.29 \pm 0.23 \pm 0.02$	0.050 ± 0.025
$\mathcal{A}_{\mathrm{CP}}(K^+\pi^-)$	$-0.107 \pm 0.018^{+0.007}_{-0.004}$	$-0.093 \pm 0.018 \pm 0.008$	$-0.04 \pm 0.16 \pm 0.02$	-0.097 ± 0.012 [a]
$\mathcal{A}_{\mathrm{CP}}(K^0\pi^0)$	$-0.24 \pm 0.15 \pm 0.03$	$-0.05 \pm 0.14 \pm 0.05$		-0.14 ± 0.11
$S_{K_S\pi^0}$	$0.40 \pm 0.23 \pm 0.03$	$0.33 \pm 0.35 \pm 0.08$		0.38 ± 0.19

[a] This average also includes the CDF result: $-0.086 \pm 0.023 \pm 0.009$.

Since the second value of r_T is unreasonably larger than the SM expectation, which is around 0.15, we safely choose the first one as our solution.

The next step is to determine α^{00} and $\bar{\alpha}^{00}$ in terms of the experimental data. After subtracting Eq. (28) from Eq. (27) and also considering their CP conjugate modes, we get the following equations:

$$\sqrt{2}\left(A^{+0}e^{i\alpha^{+0}} - A^{00}e^{i\alpha^{00}}\right) = Pxe^{i\zeta}, \tag{41}$$

$$\sqrt{2}\left(\bar{A}^{+0}e^{i\bar{\alpha}^{+0}} - \bar{A}^{00}e^{i\bar{\alpha}^{00}}\right) = P\bar{x}e^{i\bar{\zeta}}, \tag{42}$$

where

$$xe^{i\zeta} \equiv 2 - r_T e^{i\gamma} e^{i\delta_T}, \tag{43}$$

$$\bar{x}e^{i\bar{\zeta}} \equiv 2 - r_T e^{-i\gamma} e^{i\delta_T}. \tag{44}$$

It is easy to find α^{00} and $\bar{\alpha}^{00}$ from these equations:

$$\alpha^{00} = \zeta \pm \mathrm{ArcCos}\left(\frac{2{A^{+0}}^2 - 2{A^{00}}^2 - P^2x^2}{2\sqrt{2}A^{00}Px}\right), \tag{45}$$

$$\bar{\alpha}^{00} = \bar{\zeta} \pm \mathrm{ArcCos}\left(\frac{2{\bar{A}^{+0}}{}^2 - 2{\bar{A}^{00}}{}^2 - P^2\bar{x}^2}{2\sqrt{2}\bar{A}^{00}P\bar{x}}\right). \tag{46}$$

There occurs a two-fold ambiguity for α^{00} and also for $\bar{\alpha}^{00}$. We call them $[\alpha^{00}_{(1)}, \alpha^{00}_{(2)}]$ and $[\bar{\alpha}^{00}_{(1)}, \bar{\alpha}^{00}_{(2)}]$, respectively. Consequently, the solution for α^{00} and $\bar{\alpha}^{00}$ has a four-fold ambiguity in total due to their combinations. For convenience, we represent each case as Cases 1, 2, 3, and 4, respectively, corresponding to the combinations of $(\alpha^{00}_{(1)}, \bar{\alpha}^{00}_{(1)})$, $(\alpha^{00}_{(1)}, \bar{\alpha}^{00}_{(2)})$, $(\alpha^{00}_{(2)}, \bar{\alpha}^{00}_{(1)})$, and $(\alpha^{00}_{(2)}, \bar{\alpha}^{00}_{(2)})$. However, in reality for any given $\alpha^{00}_{(1,2)}$ (or $\bar{\alpha}^{00}_{(1,2)}$), there exist only two possible cases: for instance, for given $\alpha^{00}_{(1)}$, only Cases 1 and 2 are possible solutions which indicates a two-fold ambiguity.

It is instructive to represent the phases α^{00} and $\bar{\alpha}^{00}$ geometrically as in Fig. 2. The figure shows the famous isospin quadrangle in a complex plane depicting the isospin relation among the decay amplitudes for $B \to K\pi$:

$$\begin{aligned} &A\left(B^+ \to K^0\pi^+\right) + \sqrt{2}A\left(B^+ \to K^+\pi^0\right) \\ &\quad = A\left(B^0 \to K^+\pi^-\right) + \sqrt{2}A\left(B^0 \to K^0\pi^0\right). \end{aligned} \tag{47}$$

The notation $A(B \to K^i\pi^j) \equiv A^{ij}e^{i\alpha^{ij}}$ is used in the figure and $A^{ij}_{(1,2)}$ corresponds to the case of $\alpha^{00}_{(1,2)}$. The isospin quadrangle can be geometrically constructed as follows. The two complex values of $A(B^+ \to K^0\pi^+)$ and $A(B^0 \to K^+\pi^-)$ in the complex plane are fixed from the solutions shown above. Subsequently, the value $xe^{i\zeta}$ is determined, where $xe^{i\zeta} \equiv A(B^0 \to K^+\pi^-) - A(B^+ \to K^0\pi^+) = \sqrt{2}A(B^+ \to K^+\pi^0) - \sqrt{2}A(B^0 \to K^0\pi^0)$ as defined in Eq. (43). Since the magnitudes A^{+0} and A^{00} are directly determined from the measurements, we find two distinct solutions for $A(B^+ \to K^+\pi^0)$ and $A(B^0 \to K^0\pi^0)$ which are expressed as $A^{+0}_{(1,2)}$ and $A^{00}_{(1,2)}$ in Fig. 2. Two sides of the quadrangles denoted by the diamond marks (and the circle marks) are equal in length to each other. The quadrangles in the figure have been constructed by using the present experimental data. We recall that the weak phase γ has been used as an input in Eq. (32).

Next, we find analytic solutions for $r^M_C, r^M_{\mathrm{EW}}, \delta^M_C, \delta^M_{\mathrm{EW}}$ in terms of the observables, using α^{00} and $\bar{\alpha}^{00}$ determined in Eqs. (45) and (46). To this end, we use Eq. (28) and its CP conjugate version. They can be rewritten as

$$-r^M_C e^{i\gamma} e^{i\delta^M_C} + r^M_{\mathrm{EW}} e^{i\delta^M_{\mathrm{EW}}} = ye^{i\eta}, \tag{48}$$

$$-r^M_C e^{-i\gamma} e^{i\delta^M_C} + r^M_{\mathrm{EW}} e^{i\delta^M_{\mathrm{EW}}} = \bar{y}e^{i\bar{\eta}}, \tag{49}$$

where

$$ye^{i\eta} \equiv \sqrt{2}\frac{A^{00}}{P}e^{i\alpha^{00}} + 1, \tag{50}$$

$$\bar{y}e^{i\bar{\eta}} \equiv \sqrt{2}\frac{\bar{A}^{00}}{P}e^{i\bar{\alpha}^{00}} + 1. \tag{51}$$

It is straightforward to find the solutions for $r^M_C, r^M_{\mathrm{EW}}, \delta^M_C, \delta^M_{\mathrm{EW}}$ as a function of $y, \bar{y}$ and $\eta, \bar{\eta}$ from Eqs. (48) and (49):

$$r^M_C = \frac{1}{2\sin\gamma}\sqrt{|y|^2 + |\bar{y}|^2 - 2y\bar{y}\cos(\bar{\eta} - \eta)}, \tag{52}$$

$$r^M_{\mathrm{EW}} = \frac{1}{2\sin\gamma}\sqrt{|y|^2 + |\bar{y}|^2 - 2y\bar{y}\cos(2\gamma + \bar{\eta} - \eta)}, \tag{53}$$

$$\delta^M_C = \mathrm{ArcTan}\left(-\frac{y\cos\eta - \bar{y}\cos\bar{\eta}}{y\sin\eta - \bar{y}\sin\bar{\eta}}\right), \tag{54}$$

$$\delta^M_{\mathrm{EW}} = \mathrm{ArcTan}\left(-\frac{y\cos(\eta - \gamma) - \bar{y}\cos(\bar{\eta} + \gamma)}{y\sin(\eta - \gamma) - \bar{y}\sin(\bar{\eta} + \gamma)}\right). \tag{55}$$

We note that there occurs no ambiguity in the above equations. Therefore, we have found the analytic solutions for the 7 parameters: (P, r_T, δ_T) without ambiguity, and $(r^M_C, r^M_{\mathrm{EW}}, \delta^M_C, \delta^M_{\mathrm{EW}})$ with a four-fold discrete ambiguity which stems from α^{00} and $\bar{\alpha}^{00}$ given in (45) and (46).

Now we substitute the values of experimental data into our analytic solutions in order to get the numerical values of $r^M_C, r^M_{\mathrm{EW}}, \delta^M_C$, and δ^M_{EW}. Table 2 shows the result for each case. The prediction for $S_{K_S\pi^0}$ is also given for each case. Provided precise measurement of $S_{K_S\pi^0}$, one can choose consistent solutions with the data of $S_{K_S\pi^0}$ among these 4 cases. Then, as mentioned before, one can analyze each hadronic parameters of the solutions, comparing to given theoretical estimation such as PQCD and QCDF. The Cases 1 and 3 solutions are discarded because their predictions for $S_{K_S\pi^0}$ are quite different from the current data. The solutions for Cases 2 and 4 are our favorites because their predictions for $S_{K_S\pi^0}$ are consistent with the data within 1σ error. The Case 2 solution shows large color-suppressed tree than the typical SM estimation, while the Case 4 solution presents large EWP, where both cases suggest considerable NP contribution.

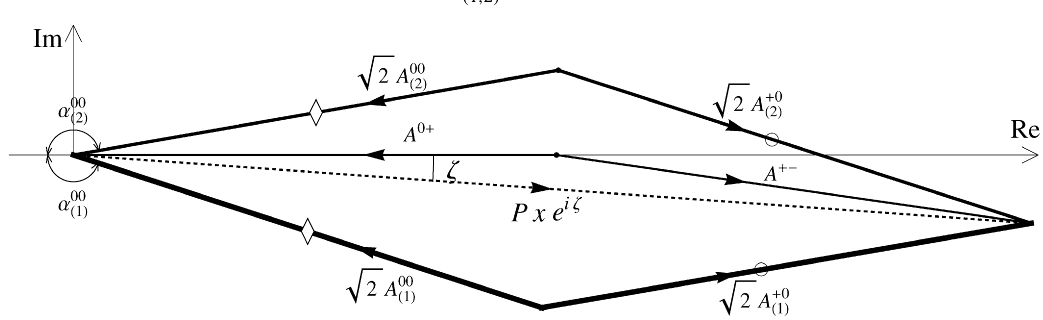

Fig. 2. The isospin quadrangles in a complex plane displaying the isospin relation among the decay amplitudes for $B \to K\pi$. $A^{ij}_{(1,2)}$ corresponds to the case of $\alpha^{00}_{(1,2)}$.

Table 2
Four possible solutions for $r_C^M, r_{EW}^M, \delta_C^M, \delta_{EW}^M$ and the prediction for $S_{K_S\pi^0}$ in each case. Current experimental value for $S_{K_S\pi^0}$ is 0.38 ± 0.19

	r_C^M	r_{EW}^M	δ_C^M	δ_{EW}^M	$S_{K_S\pi^0}$
Case 1	0.085 ± 0.080	0.25 ± 0.11	$226° \pm 81°$	$77° \pm 16°$	0.69 ± 0.14
Case 2	0.36 ± 0.13	0.068 ± 0.064	$192° \pm 11°$	$202° \pm 78°$	0.08 ± 0.26
Case 3	0.24 ± 0.13	0.17 ± 0.10	$-20° \pm 15°$	$-6.4° \pm 26°$	0.92 ± 0.07
Case 4	0.12 ± 0.11	0.29 ± 0.13	$235° \pm 45°$	$-80° \pm 15°$	0.55 ± 0.16

Please note that many authors uncovered that the anomalous behaviors of the experimental data could be accommodated with the enhancement of the EWP amplitude [13,14] as well as an additional weak phase in the electroweak sector [15–17], and a few authors have also found that the color-suppressed tree amplitude would be the main source of NP in the $B \to K\pi$ modes [17,18]. Due to our analytic approach, we can find two solutions physically equivalent: one showing the large SM-*like* color-suppressed tree contribution and the other showing the large SM-*like* EWP contribution.

4. Extracting new physics parameters and discussion

Finally, we would like to solve Eqs. (23) and (24) for the NP parameters r^N, δ^N and ϕ^N. The left-hand side of Eqs. (23) and (24) has 4 parameters which can be obtained from the analytic solution shown above. But the number of unknown parameters on the right-hand side is 7 ($r_C, \delta_C, r_{EW}, \delta_{EW}, r^N, \delta^N, \phi^N$). Thus there is no model independent way to extract NP parameters without additional theoretical inputs. We need at least 3 additional inputs in the color-suppressed tree and the EWP sector in order to determine NP parameters. Here, we adopt two different schemes for the additional theoretical inputs: one is flavor SU(3) symmetry, and the other is recent PQCD calculation.

Using the flavor SU(3) symmetry, we estimate the color-suppressed tree amplitude from the $B \to \pi\pi$ decays amplitudes, following Ref. [16],

$$C = \frac{\lambda}{1-\lambda^2/2} C_{\pi\pi} = (3.8 \pm 0.4)\ \text{eV}, \tag{56}$$

$$\delta_C = -12° \pm 15°, \tag{57}$$

where the $C_{\pi\pi}$ is color-suppressed tree amplitude of $B \to \pi\pi$ decays. The EWP amplitude is also associated with the tree and color-suppressed amplitudes under flavor SU(3) symmetry [30] as

$$r_{EW} e^{i\delta_{EW}} = -\frac{3}{2}\frac{c_9+c_{10}}{c_1+c_2}\frac{1}{\lambda^2 R_b}\left(r_T e^{i\delta_T} + r_C e^{i\delta_C}\right). \tag{58}$$

The parameters r_T and δ_T can be obtained within the $B \to K\pi$ modes as in Eq. (39). And the parameters r_C and δ_C are given by Eqs. (56) and (57) combined with Eq. (33). Subsequently r_{EW} and δ_{EW} can be obtained from the above equation. We summarize the result:

$$\begin{aligned} \text{SU(3)}: \quad & r_C e^{i\delta_C} = (0.076 \pm 0.008) e^{i(-12\pm15)°}, \\ & r_{EW} e^{i\delta_{EW}} = (0.14 \pm 0.04) e^{i(9\pm10)°}. \end{aligned} \tag{59}$$

On the other hand, The recent PQCD calculation for the $B \to K\pi$ decays gives [6]

$$\begin{aligned} \text{PQCD}: \quad & r_C e^{i\delta_C} = (0.039) e^{-i61°}, \\ & r_{EW} e^{i\delta_{EW}} = (0.12) e^{i22°}. \end{aligned} \tag{60}$$

We use these two different schemes for the values of SM parameters in order to extract NP parameters. Actually, only 3 additional inputs are enough to extract the NP parameters. Nevertheless, we adopt above 4 additional inputs in order to get rid of discrete ambiguity. We define the following quantities:

Table 3
Numerical values of the new physics parameters after using the additional inputs of the SM parameters from the flavor SU(3) symmetry and PQCD result, respectively. The result is shown for the Cases 2 and 4

		r^N	ϕ^N	δ^N
SU(3) symmetry	Case 2	0.39 ± 0.13	$91° \pm 15°$	$8° \pm 27°$
	Case 4	0.29 ± 0.19	$150° \pm 24°$	$29° \pm 17°$
PQCD	Case 2	0.34 ± 0.13	$93° \pm 15°$	$7° \pm 28°$
	Case 4	0.31 ± 0.30	$162° \pm 21°$	$36° \pm 14°$

$$\Delta r_C e^{i\Delta\delta_C} \equiv r_C^M e^{i\delta_C^M} - r_C e^{i\delta_C}, \tag{61}$$

$$\Delta r_{EW} e^{i\Delta\delta_{EW}} \equiv r_{EW}^M e^{i\delta_{EW}^M} - r_{EW} e^{i\delta_{EW}}. \tag{62}$$

The parameters of Δr_C, $\Delta\delta_C$, Δr_{EW}, and $\Delta\delta_{EW}$ can be extracted using above additional theoretical inputs. Then, we can easily see from Eqs. (23) and (24) that the following relation should be satisfied:

$$\Delta\delta_C = \Delta\delta_{EW}\ (\text{mod}\,\pi) = \delta^N\ (\text{mod}\,\pi). \tag{63}$$

And, we find the solutions of NP parameters as

$$\delta^N = \Delta\delta_{EW} \quad \text{or} \quad \Delta\delta_{EW} - \pi, \tag{64}$$

$$\frac{\sin\phi^N}{\sin(\phi^N-\gamma)} = \frac{\Delta r_C}{\Delta r_{EW}}, \tag{65}$$

$$r^N = \frac{\sin\gamma}{\sin\phi^N}\Delta r_C. \tag{66}$$

For the δ^N, two different solutions are possible as shown in Eq. (64). Since the strong phase of NP contribution is expected to be small, we choose the one with close to the δ_{EW}. The numerical values for the solution with the two different schemes of theoretical inputs are shown in Table 3. We can see that the result is consistent each other for both schemes of theoretical input. Note that in both cases, for both schemes of theoretical input, the magnitude of the NP amplitude is quite large and its weak phase is also sizable.

Since the experimental value of γ still has large uncertainties, we investigate how our NP solutions depend on these experimental results. We perform a minimum χ^2 analysis to get the NP solution in order to simply see the dependence. After employing four additional inputs of $r_C, \delta_C, r_{EW}, \delta_{EW}$ from flavor SU(3) symmetry, the number of unknown parameters is 6 ($P, r_T, \delta_T, r^N, \phi^N, \delta^N$) while we can use 8 available experimental data excluding $\mathcal{A}_{CP}^{0+}$. The fitting result as a function of γ is shown in Fig. 3. As we can see, the NP contribution is not much sensitive to γ.

5. Conclusions

In this work, we present complete analytic method for analyzing the hadronic parameters with the single NP contribution under consideration of reparametrization invariance. It is shown that any single NP contribution in the color-suppressed tree sector or EWP sector can affect both the SM parameters of color-suppressed tree and EWP. We show the analytic solution for every parameters of SM-*like* parametrization, and also for the NP parameters. Therefore one can pinpoint each hadronic parameters and compare them to the theoretical estimations once the precise experimental data are

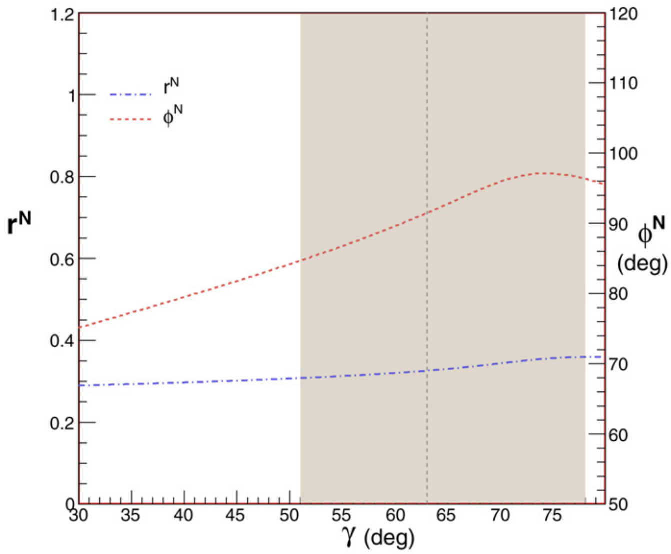

Fig. 3. The χ^2 fitting result for the new physics parameters r^N and ϕ^N as a function of γ, using the SU(3) symmetry input. The shaded area is the experimentally allowed region of γ given in PDG 2006.

given. There were 4 possible solutions for the SM-*like* parameters which can be chosen rightfully by considering mixing induced CP asymmetry data. Consequently, it could be understood simultaneously that the two different intriguing solutions occur: one is large color-suppressed tree and the other is large EWP. We obtain the solution for the NP parameters after adopting additional theoretical input. The solution shows quite large NP contribution and sizable weak phase of it.

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