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Estimation of period of vibration for concrete shear wall buildings

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Abstract Most seismic design codes generally provide formulas to be used for the estimation of the base shear and lateral loads. For the determination of the lateral loads, it is required to estimate first the fundamental vibration period of the building theoretically or experimentally (Uniform Building Code (UBC-1997); Structural Engineers Association of California (SEAOC-1996); Egyptian Code for Computation of Loads and Forces in Structural and Building Work (EGC-2012); Applied Technological Council, 1978).

In various codes such as the current United States (US) and Egyptian building codes and also in the recommendations of many researches, empirical period formulas for concrete shear wall buildings relate the building fundamental period of vibration (*T*) to the building overall height (*H*). In this paper, using the available data for the fundamental vibration period of reinforced concrete shear wall buildings measured from their motions recorded during eight California earthquakes, improved formulas for estimating the fundamental period of vibration (*T*) of concrete shear wall buildings are developed by regression analysis of the measured period data. The results indicate that the value of coefficient C_t in the current US and Egyptian building codes formula should be decreased from its present value 0.02 to 0.014. Also, factors to limit the period calculated by rational analysis, such as Rayleigh's method, are recommended in this paper. Comparisons between the periods determined using the proposed formula and the measured values show good agreement.

Introduction

The fundamental vibration period of a building appears in the equation specified in building codes to calculate the design base shear and lateral forces. Building design codes provide empirical formulas that depend on the building material [steel, reinforced concrete (RC), etc.], building type (frame, shear wall, etc.), and overall dimensions. The fundamental vibration

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period of buildings has a significant influence on the seismic induced lateral forces.

The period of formulas in the 1997 Uniform Building Code (UBC 1997) [1], the 1996 Structural Engineering Association of California (SEAOC) recommendation [2] and the recent Egyptian Code (EGC 2012) [3] is derived from those developed in 1975 as part of the ATC3-06 project [Applied Technological Council (ATC) 1978] [4] largely based on periods of buildings measured from their motions recorded during the 1971 San Fernando earthquake. Goel and Chopra [5–8] developed improved empirical formulas to estimate the fundamental vibration period of RC moment resisting frame and RC shear wall buildings for use in equivalent lateral force analysis specified in building codes using motions of many buildings recorded during earthquakes. Data used in [8] have been combined from the motions of buildings recorded during the 1971 San Fernando, 1984 Morgan Hill, 1986 Mt. Lewis and Palm Spring, 1987 Whitter, 1989 Loma Prieta, 1990 Upland, 1991 Sierra Madre and 1994 Northridge earthquakes.

By observation of the measured periods of shear wall buildings compared with the building codes formula, Kwon and Kim [9] are suggested to reduce the factor C_t to be 0.015 instead of 0.02.

Salama [10] studies the effect of the floor height in the period of vibration for concrete moment resisting frame buildings.

The objective of this paper is evaluating the present formulas and developing improved empirical formulas to estimate the fundamental vibration period of concrete shear wall buildings based on data given in [8] for use in equivalent lateral force analysis specified in building codes. Also, factors to limit the period calculated by rational analysis, such as Rayleigh's method, are recommended in this paper.

Period database

Data that are used in the regression analysis in this paper are that used by Goel and Chopra (1997) data [8]. This database contains data for buildings measured from their motions recorded during eight California earthquakes, starting with the 1971 San Fernando earthquake and ending with 1994 Northridge earthquake.

Table 1 shows the subset of this database pertaining to 16 concrete shear wall buildings (27 data points). The number of data points exceeds the number of buildings because the period of some buildings was determined from their motions recorded during more than one earthquake or was reported by more than one investigator for the same earthquake.

Code formulas

The empirical formulas for the fundamental vibration period of concrete shear wall buildings in most design codes such as U.S. building codes (UBC-97, ATC 1978, SEAOC-96, and NEHRP 1994) and the recent Egyptian code (EGC 2012) are of the form

$$T = C_t H^{0.75} \tag{1}$$

where H = overall height of the building in feet above the base; and C_t is a numerical coefficient related to the lateral-force-resisting system. The values of C_t specified in these codes are: 0.02 for concrete shear wall buildings.

In some codes such as the UBC-97 and SEAOC-96 permit an alternative value for C_t to be calculated from

$$C_t = 0.1/\sqrt{A_c} \tag{2}$$

where A_c, the combined effective area (in square feet) of the shear walls, is defined as

$$A_c = \sum_{i=1}^{NW} A_i \left[0.2 + \left(\frac{D_i}{H}\right)^2\right] \tag{3}$$

In which A_i = the horizontal cross-sectional area (in square feet); D_i = the dimension in the direction under consideration (in feet) of the ith shear wall of the first story of the structure; and NW = the total number of shear walls. The value of D_i/H in (3) should not exceed 0.90.

ATC3-06 and the earlier versions of other U.S. seismic codes and Egyptian code (EGC 1993) [11], an alternative formula for concrete shear wall buildings

$$T = \frac{0.05H}{\sqrt{D}} \tag{4}$$

where D = the dimension, in feet, of the building at its base in the direction under consideration.

Most of current codes specify that the design base shear should be calculated from

$$V = CW \tag{5}$$

where W = total seismic dead load; and C = seismic coefficient depended on the soil profile, seismic zone factor; important factor; the fundamental period T; and the numerical coefficient representative of the inherent over strength and global ductility capacity of the lateral-load-resisting system.

The fundamental period T, calculated using the empirical formulas in (1) or (4), should be smaller than the true period to obtain conservative estimate for base shear. Therefore, code formulas are internationally calibrated to underestimate the period by approximately 10–20% at first yield of the building.

The codes permit calculation of the period by a rational analysis [10], such as Rayleigh's method, but specify that the resulting value should not be longer than that estimated from the empirical formula by a certain factor to safeguard against unreasonable assumptions in the rotational analysis.

Evaluations of code formulas

In order to evaluate the code period formulas, the measured building period's records are compared with those obtained from the empirical code formula (Eq. (1)) in Fig. 1 where they are plotted against the building height (H).

The measured periods are shown by solid circles, whereas code periods are shown by curve denoted as T. Also, curves for 1.2T and 1.4T are included representing restrictions on the period from rational analysis imposed by various US and Egyptian codes.

From Fig. 1 for all concrete shear wall buildings, we can observe the following

- The code formula leads to periods are longer than measured periods for about fifty percent of buildings.
- The longer period from the code formula leads to seismic coefficient smaller than the value based on the measured period.

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Table 1 Period data for concrete shear wall buildings [8].

No.	Location	ID number	No. of stories	Overall height (ft.)	Floor height (ft.)	Earthquake	Period T (s)	
							Longitudinal	Transverse
1	Belmont	C58262	2	28	10	Lorna Prieta	0.13	0.2
2[a]	Burbank	C24385	10	88	13.33	Northridge	0.6	0.56
3[a]	Burbank	C24385	10	88	11.43	Whittier	0.57	0.51
4	Hayward	C58488	4	50	12.77	Lorna Prieta	0.15	0.22
5	Long Beach	C14311	5	71	13.75	Whittier	0.17	0.34
6	Los Angeles	ATC 3	12	159	8.714	San Fernando	1.15	MRF
7[a]	Los Angeles	C24468	8	127	9.714	Northridge	1.54	1.62
8[a]	Los Angeles	C24601	17	149.7	13.25	Northridge	1.18	1.05
9	Los Angeles	C24601	17	149.7	10.36	Sierra Madre	1	1
10[a]	Los Angeles	N253-5	12	161.5	11.27	San Fernando	1.19	1.14
11[a]	Los Angeles	N253-5	12	161.5	9.286	San Fernando	1.07	1.13
12	Palm Desert	C12284	4	50.2	9.5	Palm Spring	0.5	0.6
13	Pasadena	N264-5	10	142	10.63	Lytle Creek	0.71	0.52
14[a]	Pasadena	N264-5	10	142	23.8	San Fernando	0.98	0.62
15[a]	Pasadena	N264-5	10	142	23.8	San Fernando	0.97	0.62
16	Piedmont	C58334	3	36	18.27	Lorna Prieta	0.18	0.18
17	Pleasant Hill	C58348	3	40.6	15.67	Lorna Prieta	0.38	0.46
18	San Bruno	C58394	9	104	9.84	Lorna Prieta	1.2	1.3
19	San Bruno	C58394	9	104	9.84	Lorna Prieta	1	1.45
20	San Jose	C57355	10	124	9.84	Lorna Prieta	MRF	0.75
21	San Jose	C57355	10	124	9.286	Morgan Hill	MRF	0.61
22	San Jose	C57355	10	124	9.286	Mount Lewis	MRF	0.61
23	San Jose	C57356	10	96	8.45	Lorna Prieta	0.73	0.43
24	San Jose	C57356	10	96	8.45	Lorna Prieta	0.7	0.42
25	San Jose	C57356	10	96	15	Morgan Hill	0.65	0.43
26	San Jose	C57356	10	96	15	Mount Lewis	0.63	0.41
27[a]	Watsonville	C47459	4	66.3	13	Lorna Prieta	0.24	0.35

Note: MRF implies moment-resisting frames from the lateral-load resisting system; number followed by "C" or "N" indicates the station number and by "ATC" indicates the building number in ATC3-06 report.

[a] Denotes buildings with $\ddot{u}_{g0} \geqslant 0.15$.

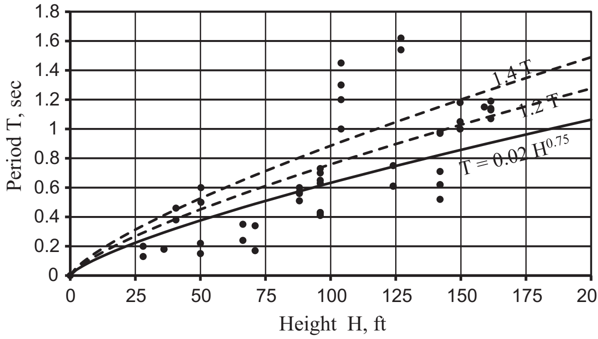

Fig. 1 Comparison of measured and code periods for concrete shear wall buildings.

- The limits of 1.2T or 1.4T for the period calculated from a rational analysis are obviously inappropriate.

From the previous observations the coefficient $C_t = 0.020$ in current codes may be too exaggerated and should be reduced.

Regression analysis method

From the code formulas and recommended formulas in the recent researches, the suggested formula which is adopted in the present paper is of the form

$$T = \alpha H^{\beta} \tag{6}$$

In which constants α and β depend on building properties. This form is adopted in the present paper and constants α and β are determined by regression analysis of the measured period data.

For the purpose of regression analysis, it is useful to recast (4) as

$$y = a + \beta x \tag{7}$$

In which $y = \log(T)$, $a = \log(\alpha)$, and $x = \log(H)$.

The database given in Table 1 is used in the regression analysis. These data represent the measured fundamental period of concrete shear wall buildings (T) and the corresponding overall height (H) for each building.

Using computer software, multiple regression analysis technique is developed for the suggested form (6) to obtain the constants a, and β of the line represented by (7). Then α was back-calculated from the relation $a = \log(\alpha)$. The regression analysis technique depends on minimizing the squared error between the measured and computed periods.

This procedure leads to values of α and β for (6) to represent the best-fit to the measured period data using least-squares method.

The standard error of estimate is

$$s_e = \sqrt{\frac{\sum_{i=1}^{n}[y_i - (a + \beta x_i)]^2}{n-2}} \tag{8}$$

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Table 2 Comparison of results from regression analysis for concrete shear wall buildings.

Regression analysis type	Period formula			
	Best-fit	Best-fit – 0.84 σ	Best-fit + 0.84 σ	s_e
Unconstrained	$T = 0.0045\ H^{1.076}$	$T_L = 0.0032\ H^{1.076}$	$T_L = 0.0063H^{1.076}$	0.391
Constrained with $\beta = 1.1$	$T = 0.0040\ H^{1.1}$	$T_L = 0.0029\ H^{1.1}$	$T_L = 0.0056\ H^{1.1}$	0.392
Constrained with $\beta = 1.0$	$T = 0.0064\ H^{1.0}$	$T_L = 0.0046\ H^{1.0}$	$T_L = 0.0088\ H^{1.0}$	0.393
Constrained with $\beta = 0.75$	$T = 0.020\ H^{0.75}$	$T_L = 0.014\ H^{0.75}$	$T_L = 0.028\ H^{0.75}$	0.424

where $y_i = \log\ (T_i)$ = observed value (with T_i = measured period); $(a + \beta x_i) = [\log\ (\alpha) + \beta \log(H_i)]$ = computed value of the ith; and n = total number of data points. The s_e represents scatter in the data and approaches, for large n, the standard deviation σ of the measured periods from the best-fit equation.

For code applications, the formula should provide lower values of the period, and this was obtained by lowering the best-fit line [see (7)] by 0.84 s_e (corresponding to 80% of standard normal distribution area) without changing its slope. Thus α_L, the lower value of α, is computed from

$$\log(\alpha_L) = \log(\alpha) - 0.84\ s_e \tag{9}$$

This bound implies that only 20% of the measured periods would fall below the lower bound line.

Also, codes specify an upper limit on the period calculated by rational analysis. This limit was obtained by raising the best-fit line [see (7)] by 0.84 s_e without changing its slopes. Thus α_U, the upper value of α, is computed from

$$\log(\alpha_U) = \log(\alpha) + 0.84\ s_e \tag{10}$$

This bound implies that only 20% of the measured periods would fall upper the higher bound line.

Results of regression analysis

The theoretical form of Eq. (6) was adopted in the present investigation and constants α and β were considered as variables. This unconstrained regression analysis led to the best possible fit and thus the minimum possible error between the measured and calculated periods (s_e = 0.391). The obtained formula is

$$T = 0.0045\,H^{1.07} \tag{11}$$

In another regression analysis only α was considered as variable; β was fixed at 1.0 which concords to some early codes formulas. As expected, this constrained regression analysis led to negligible higher errors (s_e = 0.393). The adjustment formula is

$$T = 0.0064\,H^{1.0} \tag{12}$$

The formula that is of interest for code-type application is the one that provides a lower bound to the measured data, denoted as T_L, correspondence to the best fit – 0.84 σ. The upper limit on the period is calculated by rational analysis, denoted as T_U, correspondence to the best fit + 0.84 σ.

Eqs. (9) and (10) were used in the last regression analysis [Eq. (12)] to obtain α_L = 0.021 and α_U = 0.032 Leading to

$$T_L = 0.0046\,H^{1.0} \tag{13}$$

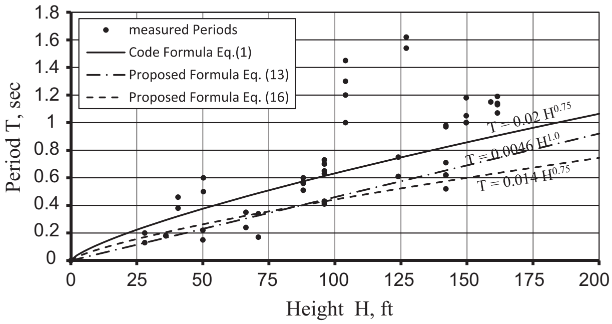

Fig. 2 Comparison of measured, code formula and proposed formula periods for concrete shear wall buildings.

and

$$T_U = 0.0088\,H^{1.0} \tag{14}$$

This indicates that (13) is suitable for estimating, conservatively, the fundamental period and (14) for limiting the period computed from rational analysis. This period should not be longer than $1.9T_L$; the factor 1.90 is determined as the ratio 0.0088:0.0046, rounded-off to one digit after the decimal point.

Using the same procedure used in the previous regression, another regression analysis is performed by taking β fixed at 0.75 which concords to the most codes formulas. The obtained formulas are

$$T = 0.020\,H^{0.75} \tag{15}$$

$$T_L = 0.014\,H^{0.75} \tag{16}$$

and

$$T_U = 0.028\,H^{0.75} \tag{17}$$

All the resulted formulas and the corresponding error from previous regression analyses, implemented using the data given in Table 1, are given in Table 2.

Also, Fig. 2 shows comparison between the measured building period's records with those obtained from the proposed Eqs. (13), (16) and those obtained from the empirical code formula (1). This comparison clears that the resulting building periods from the proposed formulas are closer to the measured building periods than those obtained by the empirical formula using in most design codes (1).

Discussion of the results

Most design codes such as U.S. codes and the recent Egyptian code (EGC 2012) specify that the design base shear (V) of concrete shear wall buildings should be calculated from

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$$V = S_d(T)\,W/g \tag{18}$$

where W = total seismic dead load and $S_d\,(T)$ = seismic coefficient which is a function in the time period T.

As an example, for the buildings with a total height less than 150 ft. which represent most Egyptian buildings, the coefficient $S_d\,(T)$ is inversely proportional to the building time period T

$$S_d(T) \propto 1/T \tag{19}$$

From the previous equations, the base shear calculated by using the empirical formula for the fundamental vibration period of concrete shear wall buildings that are used in most design codes (1) is equal to approximately 70% of that calculated according to the suggested coefficient C_t given in (16) (i.e. the coefficient C_t = 0.020 in current codes may be too exaggerated).

Conclusions and recommendations

Based on analysis of the available data for the fundamental vibration period of 16 concrete shear wall buildings (27 data points) measured from their motions recorded during earthquakes, the current formula in the U.S. and Egyptian codes may be inappropriate where the coefficient C_t = 0.020 in current codes may be too exaggerated and should be decreased from its present value 0.02 to 0.014 (16).

As an alternative, (13) is recommended for estimating the period of concrete shear wall buildings. This formula provides the best fit of (6) to the available data. The fit is better than possible Eq. (1) in most current codes. Furthermore, the period from rational analysis should not be allowed to be exceed the value from the recommended equation by a factor larger than 1.9.

Regression analysis that led to the recommended formula should be repeated periodically on larger data sets including buildings in other parts of the world where building design practice is significantly different than California.

Conflict of interest

The authors declare that there are no conflicts of interest.

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