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Cold electroweak baryogenesis with Standard Model CP violation

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ABSTRACT

We study a mechanism that generates the baryon asymmetry of the Universe during a tachyonic electroweak phase transition. We utilize as sole source of CP violation an operator that was recently obtained from the Standard Model by integrating out the quarks.

1. Introduction

The requirements for the dynamical creation of the baryon asymmetry in the Universe are stated by the Sakharov conditions: Violation of baryon number conservation, violation of charge conjugation (C) and charge-parity conjugation (CP) symmetry, and departure from equilibrium [1]. While there exists a plethora of possible explanations for the baryon asymmetry, for a review see e.g. [2], all of them have to introduce physics beyond the Standard Model (SM) in order to provide all these ingredients. Besides, most of them operate at energy ranges that are not testable by experiments in the near future, and hence become difficult to falsify. In principle, the SM contains all three necessary ingredients mentioned before. Baryon number violation is supplied by the weak anomaly [3] and weak interactions violate C maximally and violate CP through the Kobayashi–Maskawa mechanism [4]. Finally, departure from equilibrium could occur due to expanding Higgs bubbles during a first-order electroweak phase transition, and the corresponding mechanism is called electroweak baryogenesis [5]. However, the experimental lower bound on the Higgs mass implies that there did not occur a strongly first-order phase transition as required but a crossover in the SM [6]. Even if the phase transition would be first-order, the CP violation from the CKM matrix is not strong enough in the Higgs bubble walls to generate a substantial baryon asymmetry. While this rules out electroweak baryogenesis in the SM, there are extensions of the SM which overcome these shortcomings while still remaining close to the SM. This includes, e.g. the MSSM with CP violation in the chargino [7,8] or neutralino sector [9] or singlet extensions of the MSSM [10,11].

Another scenario is given by the so-called cold electroweak baryogenesis where the electroweak phase transition is tachyonic and initiated at the end of (inverted) low-scale hybrid inflation. In this case, not the temperature dependence of the free energy but the inflation field turns the effective squared Higgs mass parameter negative [12–17]. The electroweak phase transition occurs basically at zero temperature, and because of the spinodal instability induced on the Higgs field, its low momentum modes grow exponentially [16,18,19]. This allows for a classical treatment of the dynamics.

The generation of the baryon asymmetry in this scenario has been simulated on the lattice [17], where the source of CP violation was assumed to be of the form

$$\frac{\kappa_{CP}}{M^2}\phi^\dagger\phi\,\epsilon^{\mu\nu\lambda\sigma}\,\mathrm{tr}(W_{\mu\nu}W_{\lambda\sigma}), \tag{1}$$

where W denotes the $SU(2)_L$ field strength. A term of this form could originate in an effective action from a more fundamental theory at higher energies or from integrating out heavy fermions. In the present work we report on the simulation of cold electroweak baryogenesis using a CP-violating operator that was recently obtained by integrating out the quarks of the Standard Model [20] and reads

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$$\frac{\kappa_{CP}}{M^2}\epsilon^{\mu\nu\lambda\sigma}\left(Z_\mu W^+_{\nu\lambda}W^-_\alpha\left(W^+_\sigma W^-_\alpha + W^+_\alpha W^-_\sigma\right) + \text{c.c.}\right). \tag{2}$$

Our main concern is if an operator of this form can successfully bias the Chern–Simons number and through the anomaly, baryon number, during tachyonic preheating and hence explain the observed baryon asymmetry via cold electroweak baryogenesis.

It has often been argued in the literature that CP violation in the SM is too small to be able to generate the baryon asymmetry and we review this argument and the results of Ref. [20] in the next section. In Section 3 we present numerical results of the lattice simulation of cold electroweak baryogenesis. Finally, in Section 4 we conclude.

2. CP violation in the Standard Model

It is often stated in the literature that the CP violation present in the SM is insufficient to explain the observed baryon asymmetry. These claims rest usually on the so-called Jarlskog determinant [21] and we review this argument in the following. The basic observation is that physical observables cannot depend on the flavor basis chosen for the quarks; in particular transformations of the right-handed quarks leave the Lagrangian invariant since the weak interactions are chiral. Besides, the quark fields can be redefined absorbing one complex phase. The last fact implies that all CP-odd observables in the SM have to be proportional to

$$J = s_1^2 s_2 s_3 c_1 c_2 c_3 \sin(\delta) = (3.0 \pm 0.3) \times 10^{-5}, \tag{3}$$

with the Jarlskog invariant J given in terms of the Kobayashi–Maskawa parametrization of the CKM matrix V with a CP-violating phase δ as defined in Refs. [21,22]. In addition, if two up- or down-type quark masses were degenerate, there would be no CP violation in the Standard Model since flavor basis transformation can in this case be used to remove the complex phase of the CKM matrix altogether from the Lagrangian.

If one further assumes that the observable under consideration is polynomial in the quark masses, the simplest dimensionless expression that fulfills these constraints is found to be the Jarlskog determinant that has the form

$$\begin{aligned}\Delta_{CP} &= v^{-12}\,\mathrm{Im}\,\mathrm{Det}\left[m_u m_u^\dagger, m_d m_d^\dagger\right]\\ &= J v^{-12}\prod_{i<j}\left(\tilde m_{u,i} - \tilde m^2_{u,j}\right)\prod_{i<j}\left(\tilde m^2_{d,i} - \tilde m^2_{d,j}\right) \simeq 10^{-19},\end{aligned} \tag{4}$$

where v is the Higgs vacuum expectation value and $\tilde m^2_{u/d}$ denotes the diagonalized mass matrices according to

$$m_d m_d^\dagger = D\tilde m_d^2 D^\dagger, \qquad m_u m_u^\dagger = U\tilde m_u^2 U^\dagger. \tag{5}$$

The identity in Eq. (4) results then from the following relation of the CKM matrix (summation over indices is only performed as explicitly shown)

$$\mathrm{Im}\left[V_{ab}V^\dagger_{bc}V_{cd}V^\dagger_{da}\right] = J\sum_{e,f}\epsilon_{ace}\epsilon_{bdf}, \qquad V = U^\dagger D. \tag{6}$$

According to this argument CP violation in the SM seems to be too small to explain the observed baryon asymmetry that is of order $\eta \sim 10^{-10}$ and several proposals in the literature aim at avoiding this bound. For example, it has been argued that at temperatures of the electroweak scale the CP violation might be only suppressed by the temperature rather than by the Higgs vev v as given in Eq. (4), but this still is insufficient to be significant in a baryogenesis mechanism [23,24]. In the context of electroweak baryogenesis coherent scattering at the bubble wall has been suggested in the same work [23,24] but finally dismissed [25]. Furthermore construction of rephasing invariants containing derivatives of the Higgs field have proved to be incapable of being relevant in electroweak baryogenesis [26].

The approach in Refs. [20,27] is based on the effective action obtained by integrating out the quarks of the SM in the gradient expansion (more precisely an expansion in Lorentz indices, i.e. the covariant derivative expansion). The resulting effective action can potentially contain factors of the form

$$\frac{m_i^2 - m_j^2}{m_i^2 + m_j^2}, \quad \log\left[m_i^2/m_j^2\right]. \tag{7}$$

These expressions vanish for degenerate quarks but are unlike the factors in the Jarlskog determinant not suppressed for small Yukawa couplings. The appearance of terms of this form was already argued in the context of cold electroweak baryogenesis in Ref. [28]. Indeed when the quarks of the SM are integrated out, one finds in the effective action in next-to-leading order of the gradient expansion the operator [20]

$$\begin{aligned}&\frac{i}{8(4\pi)^2}\frac{N_c}{16}\frac{J\kappa_{CP}}{\tilde m_c^2}\epsilon^{\mu\nu\lambda\sigma}\\ &\quad\times\int d^4x\left(\frac{v}{\phi}\right)^2\left(Z_\mu W^+_{\nu\lambda}W^-_\alpha\left(W^+_\sigma W^-_\alpha + W^+_\alpha W^-_\sigma\right) + \text{c.c.}\right),\end{aligned} \tag{8}$$

where J is the Jarlskog invariant given in Eq. (3) and

$$\kappa_{CP} \approx 9.87. \tag{9}$$

As required, the operator is proportional to J and would vanish for degenerate quark masses. However, the latter information is hidden in the numerical coefficient κ_{CP} that is a function of the six quark masses.[1,2] The function is symmetric under the exchange of two families but finite in the limit of one family becoming massless. The scale of the operator is hence given by the second heaviest family.

This result was obtained in unitary gauge and the action can be rewritten in $SU(2)_L$ gauge invariant quantities. The charged gauge fields can be rewritten as

$$\begin{aligned}W^+_{\mu\nu} &= \frac{\phi^\dagger W_{\mu\nu}\tilde\phi}{\phi^\dagger\phi}, \qquad W^-_{\mu\nu} = \frac{\tilde\phi^\dagger W_{\mu\nu}\phi}{\phi^\dagger\phi},\\ W^+_\mu &= \frac{\phi^\dagger\mathcal{D}_\mu\tilde\phi}{\phi^\dagger\phi}, \qquad W^-_\mu = \frac{\tilde\phi^\dagger\mathcal{D}_\mu\phi}{\phi^\dagger\phi},\end{aligned} \tag{10}$$

and similarly for the uncharged quantity

$$Z_\mu = W^3_\mu - B_\mu = \frac{\phi^\dagger\mathcal{D}_\mu\phi - \tilde\phi^\dagger\mathcal{D}_\mu\tilde\phi}{2\phi^\dagger\phi}. \tag{11}$$

In the following we employ the operator (8) in a modified form in the lattice study of cold electroweak baryogenesis. This is necessary, since the gradient expansion used to obtain the result is only strictly valid in the case

[1] There is a recent claim [29] that an alternative method to the one used in Ref. [20] leads to no CP violation in the imaginary part of the Euclidean effective action in next-to-leading order. Moreover, using this method all operators considered by the authors (all CP-odd and several CP-even operators) of the imaginary part apparently vanish to this order in four dimensions. On the other hand, the next-to-leading order result in two dimensions (as presented in [27] and confirmed with a different method in [30]) does not vanish. A vanishing result in four dimensions seems implausible. We also stress that the results in [20] are obtained on par with a large number of consistency checks and computed with the help of a computer algebra program.

[2] The same work [29] also reported on CP violation in the real part of the Euclidean effective action that is however subdominant for cold electroweak baryogenesis due to parity conservation.

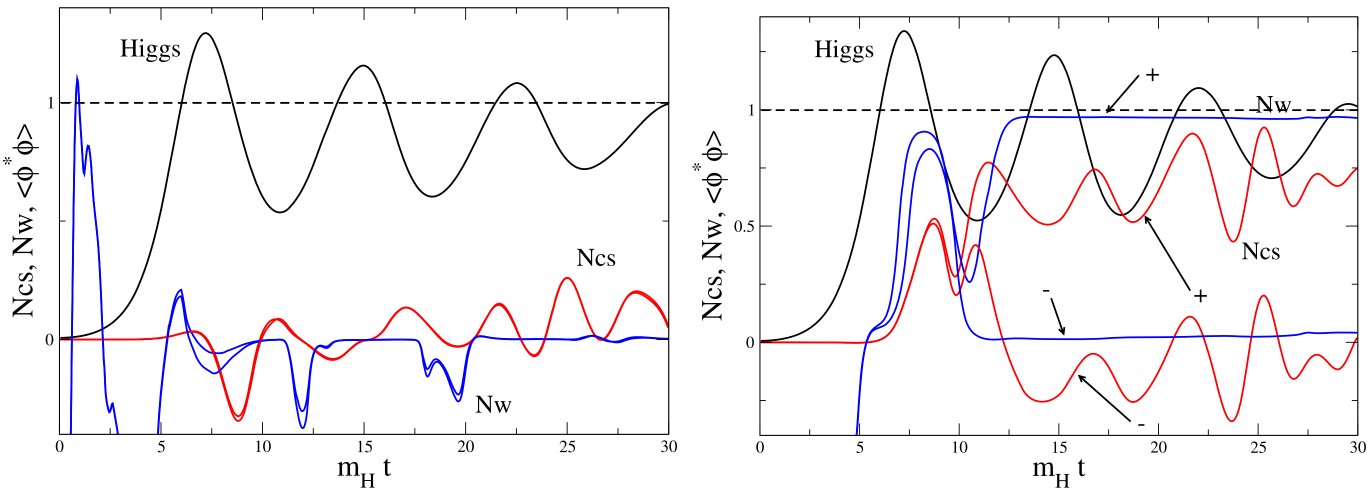

Fig. 1. Two example configuration: The right configuration results in a difference in the Higgs winding number for CP conjugate initial conditions, while the left does not. The Higgs field (straight curve) is normalized to v^2. The red and blue lines denote N_{cs} and N_w respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

$$Z_\mu, W^\pm_\mu \ll m, \qquad Z_{\mu\nu}, W^\pm_{\mu\nu} \ll m^2. \tag{12}$$

As mentioned earlier, the operator turns out to be finite for vanishing up and down quark masses and one finds that the next-to-leading order operator is suppressed by the charm mass as indicated in the notation of Eq. (8). It is plausible that this is the appropriate mass scale that has to be used in the inequalities (12). Still, the vev of Higgs field vanishes at certain points during the tachyonic phase transition, thus invalidating the expansion.

In order to avoid that the dynamics is dominated by those points, we introduce a cutoff. The purpose of the cutoff is to suppress the CP violation in the region where the operator (8) is artificially large due to the break down of the gradient expansion. For the mechanism of cold electroweak baryogenesis, it is not essential that CP violation is active for Higgs configurations with almost vanishing vev. Two configurations with CP conjugate initial conditions will slowly drift apart, eventually leading to a difference in Higgs winding number, even if CP violation is only active in the regions of large Higgs vev.

Instead of an expansion in gradients, an expansion in inverse gradients is feasible in the regions of small Higgs vev and higher order operators should come with a suppression of order

$$\Lambda^2 \approx \frac{(\partial_\mu \phi)^2}{v^2} \lesssim \frac{V_{pot}}{v^2} \approx \frac{m_H^2}{8}. \tag{13}$$

In practice, we insert Λ whenever the Higgs field appears in the denominator

$$\frac{1}{\phi^\dagger\phi} \to \frac{1}{c(\phi^\dagger\phi + \Lambda^2)}, \qquad c\left(\frac{v^2}{2} + \Lambda^2\right) = \frac{v^2}{2}, \tag{14}$$

and c is just a constant fixed to satisfy the last condition. Even after introduction of the cutoff, the gradient expansion could be jeopardized by too large gauge fields. Means of justifying the gradient expansion could be to analyze the dependence of our results on the cutoff and the impact of the CP-violating operator *a posteriori*.

3. Numerical analysis

Following the analysis of Refs. [17,31–34], the $SU(2)$-Higgs model is discretized on a space–time lattice, and the classical equations of motion derived in a straightforward way. The implementation of the CP-violating operator hereby requires symmetrization in space and time, leading to implicit equations of motion in time, again in a similar fashion as for the operator (1) used in [17]. The equations are solved by iteration and convergence of this iteration procedure imposes certain restrictions on the coefficient κ_{CP}, the timestep dt and the cutoff Λ, in order for the CP-violating force to not be too large. Ultimately, we are interested in the Higgs winding number and the Chern–Simons number that read

$$N_w = \frac{1}{24\pi^2}\epsilon_{ijk}\int dx^3\,(\phi^\dagger\partial_i\phi)(\phi^\dagger\partial_j\phi)(\phi^\dagger\partial_k\phi), \tag{15}$$

$$N_{cs} = \frac{1}{32\pi^2}\epsilon_{ijk}\int dx^3\left(W_i^a W_{jk}^a - \frac{1}{3}\epsilon_{abc}W_i^a W_j^b W_k^c\right), \tag{16}$$

where we used the temporal gauge, $W_0^a = 0$, and $W_{\mu\nu}^a$ denotes the corresponding field strength.

The simulation generates random configurations on a $n_x^3 = 64^3$ lattice, with lattice spacing $am_H = 0.35$. These are evolved using the equations of motion to $m_H t = 30$, with timestep $dt = 0.0125$. Each configuration is run twice, using $\kappa_{CP} = \pm 50$, and subtracting the resulting N_{cs} and N_w values. This way, the ensemble of initial conditions is CP symmetric and the asymmetry is strictly zero at $\kappa_{CP} = 0$. This drastically reduces statistical noise, allowing a much clearer signal [33]. Correspondingly, only instances with integer difference in the Chern–Simons number between $\pm\kappa_{CP}$ contribute. Statistics of this sort of observable is binomial such that the standard deviation is given by

$$\sigma = \frac{\sqrt{\frac{N_{\text{jumps}}}{2^2 N_{\text{configs}}} - \left(\frac{N_{\text{jumps}}}{2N_{\text{configs}}}\right)^2}}{\sqrt{N_{\text{configs}} - 1}}. \tag{17}$$

It will turn out that the asymmetry for all pairs of configurations is non-negative and that the result is overall not consistent with zero.

As parameters we use $m_H = 2m_W$ and a cut-off of $c\Lambda^2 = 50^2\ \text{GeV}^2$. The value of the cutoff corresponds to the estimate in (13). In the simulation, we observe that the baryon asymmetry scales roughly with the fourth power of the cutoff what introduces a sizable uncertainty. A more sophisticated estimate of the cutoff and a more extensive presentation of the data will be given elsewhere [35].

Fig. 1 shows two example configurations, one of which contributes to the asymmetry. The Higgs field falls into the broken

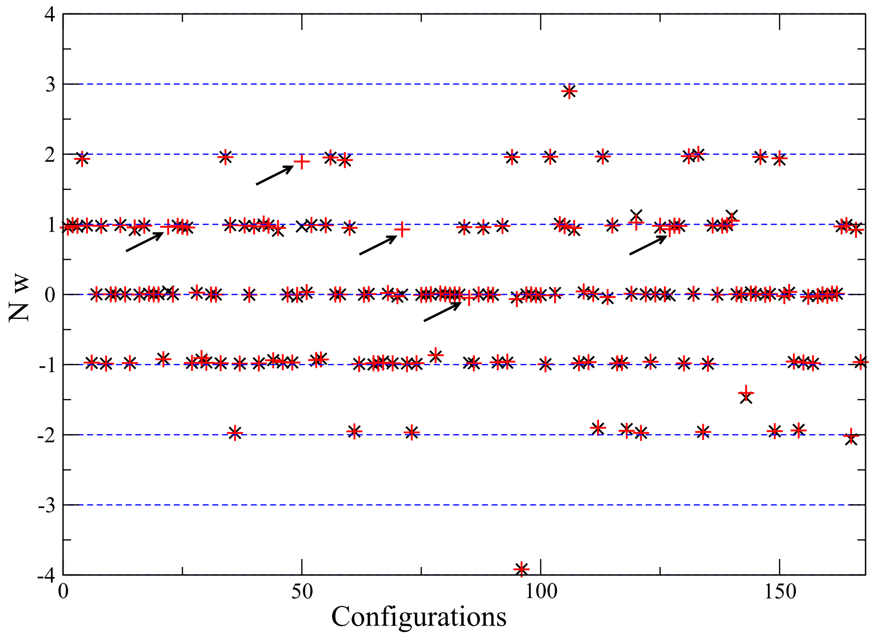

Fig. 2. The final value of Higgs winding number for all 167 pairs of configurations. Black ×-marks correspond to $+\kappa_{CP}$, red +-marks to $-\kappa_{CP}$. In total 5 pairs of configurations contribute to a net baryon asymmetry. Deviations from integer values are rather small and due to lattice discretization errors. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

phase and performs a damped oscillation. In the meantime, energy is transferred into the gauge and Higgs fields, and their modes grow under influence of the CP violation.

A first difference between the CP conjugate configurations occurs typically around $tm_H \approx 6$. At this time already a substantial amount of energy is transformed into the gauge fields and the Higgs vev is non-vanishing what is essential for the CP-violating operator (2) to be relevant. At the first minimum of the Higgs oscillation, where many zeros of the Higgs field are created [32] winding number is potentially generated. The presence of CP violation can hereby lead to a net baryon number between configurations with CP conjugate initial conditions. At late times winding number and Chern–Simons number agree and in the case at hand the Chern–Simons number follows the previously generated winding number. At the end, winding number approaches an integer valued vacuum while Chern–Simons number, containing thermal noise, oscillates for a longer time before it eventually settles into the same integer value [17].

Notice that this behavior is quite different to the mechanism based on the operator (1). For a non-vanishing Higgs field this operator can be interpreted as a chemical potential for the Chern–Simons number. Accordingly, Chern–Simons number is already generated during the first roll-off of the Higgs field and in the first minimum of the Higgs field the winding accommodates to the Chern–Simons number instead of vice-versa [33].

The final value of winding number is shown for an ensemble of 167 configurations in Fig. 2. We note that all final values are very close to integers, showing that the lattice discretization errors are well under control. Only for one pair of configurations the winding number did not settle yet in an integer value at the end of the simulation. There are five configurations with a net winding number, indicated by arrows. Hence the asymmetry is

$$\delta N_w = \frac{5}{334} \pm \frac{5}{334}\sqrt{\left(\frac{1}{5} - \frac{1}{167}\right)\frac{167}{166}} = 0.015 \times (1 \pm 0.44). \tag{18}$$

Fig. 3 shows the average of Higgs field, Chern–Simons number and Higgs winding numbers as functions of time. The average value 5/334 and the error estimate are indicated by blue dashed and dotted lines, respectively. The same picture arises as for the example configurations: Chern–Simons and winding numbers vanish until the first Higgs minimum, where winding number is generated potentially in a CP-violating way. The Chern–Simons number accommodates to the winding number subsequently.

The baryon number density, reheating temperature and photon number density are given by

$$n_B = 3n_{\text{cs}}, \qquad V_0 = \frac{\lambda v^4}{4} = \frac{\pi^3 g^*}{30} T_{\text{reh}}^4,$$
$$n_\gamma = \frac{1}{7}\frac{2\pi^2 g^*}{45} T_{\text{reh}}^3. \tag{19}$$

For the Standard Model $g^* = 86.25$, and $v = 246$ GeV and we choose $m_H = 2m_W \simeq 160$ GeV, what yields for the baryon asymmetry

$$\frac{n_B}{n_\gamma} = \frac{\kappa_{CP}}{10} \times (4.4 \pm 1.9) \times 10^{-6}, \tag{20}$$

which for the physical value $\kappa_{CP} = 9.78$ is four orders of magnitude larger than the observed asymmetry.

4. Discussion and conclusions

We presented first results of the simulation of cold electroweak baryogenesis utilizing the operator (2) that arises in the effective action of the SM from the CP violation in the CKM matrix. In order to make this operator applicable for small Higgs fields in the simulation, we introduced a cutoff. We chose a cutoff of $c\Lambda^2 = 50^2$ GeV^2 and found a result that is four orders of magnitude larger than the observed baryon asymmetry. If the result would be dominated by the infrared modes of the Higgs field in the operator one would expect according to Eqs. (10) and (14) a scaling as Λ^{-12}. Preliminary results of extensive computer simu-

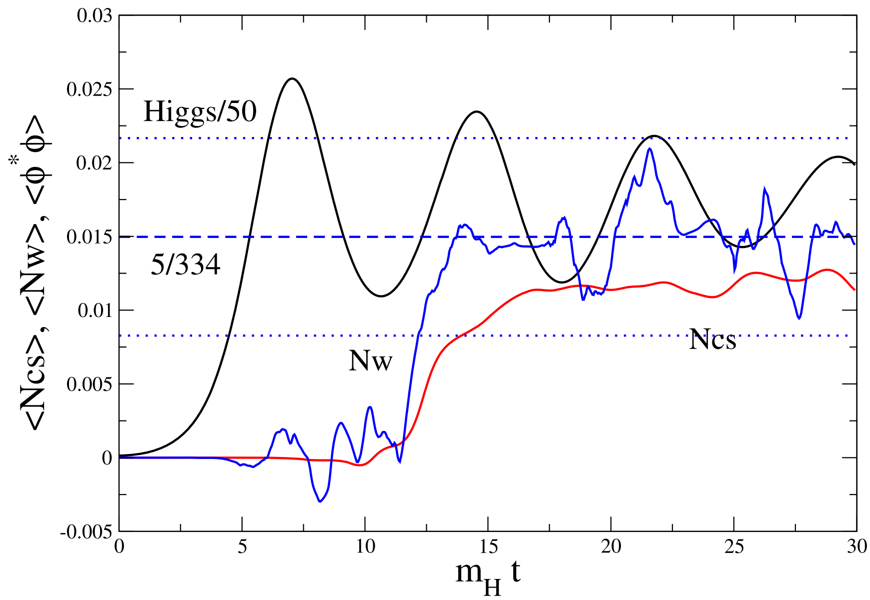

Fig. 3. The average Higgs field (scaled by a factor 1/50), Chern–Simons number and winding number as functions of time. The average value 5/334 is superposed (dashed line) as well as the estimated error band (dotted lines). (For interpretation of the references to color in this figure, the reader is referred to the web version of this Letter.).

lations confirm that the result does rather scale as Λ^{-4} which we find very encouraging. A more detailed study of the dependence of the resulting asymmetry on the cutoff and the applicability of the operator (2) in general will be topic of a subsequent work [35].

We would like to comment on why Standard Model CP violation is operative in the proposed mechanism. Main requirement is that the quark masses enter in a non-polynomial way in order to avoid the Jarlskog determinant (4) as an upper bound on CP violation. In principle, this is easily achieved as can be seen in the Kaon system. However, baryogenesis typically takes place in a hot plasma and the temperature of the plasma provides a new energy scale. The temperature effects of the plasma render most processes in the infrared finite what makes it hard to avoid (4) or a similar bound depending on temperature [23,24]. Cold electroweak baryogenesis operates at zero temperature.

In conclusion, we find it remarkable that a cosmologically viable baryon asymmetry could be created at a tachyonic electroweak transition, using only Standard Model CP violation. Although technical issues relating to the realization of low-scale inflation [36] and the gradient expansion in this context persist [20], cold electroweak baryogenesis should be considered a serious candidate scenario for explaining the baryon asymmetry and deserves further investigation.

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