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ORIGINAL ARTICLE

Optimal capacitor placement and sizing in radial electric power systems

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Abstract The use of capacitors in power systems has many well-known benefits that include improvement of the system power factor, improvement of the system voltage profile, increasing the maximum flow through cables and transformers and reduction of losses due to the compensation of the reactive component of power flow. By decreasing the flow through cables, the systems' loads can be increased without adding any new cables or overloading the existing cables. These benefits depend greatly on how capacitors are placed in the system. In this paper, the problem of how to optimally determine the locations to install capacitors and the sizes of capacitors to be installed in the buses of radial distribution systems is addressed. The proposed methodology uses loss sensitivity factors to identify the buses requiring compensation and then a discrete particle swarm optimization algorithm (PSO) is used to determine the sizes of the capacitors to be installed. The proposed algorithm deals directly with discrete nature of the design variables. The results obtained are superior to those reported in Prakash and Sydulu (2007).

1. Introduction

The application of shunt capacitor in distribution feeders has always been an important research area. It is because a portion of power loss in distribution systems could be reduced by adding shunt capacitors to supply a part of the reactive power demands. For this reason, the source of the system does not necessarily have to supply all reactive power demands and losses. Consequently, there is a possibility to decrease the losses associated with the reactive power flow through the branches in the distribution systems. The benefits of capacitor placement in distribution systems are power factor correction, bus voltage regulation, power and energy loss reduction, feeder and system capacity release as well as power quality improvement. The extent of the aforementioned advantages of capacitor placement depends on how capacitors are allocated and controlled under possible loading conditions. This means that the optimization problem, namely, capacitor placement problem should be formulated with the desired objective function (such as loss minimization) and various technical constraints (e.g. the limits of voltage levels and power flow). After that, the proper solution techniques should be applied to simultaneously determine the optimal number, location, type, size and control settings at different load levels of the capacitors to be installed [1–2].

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Because capacitor sizes and locations are discrete variables, this makes the capacitor placement problem have a combinatorial nature. The problem is a zero-one decision making problem with discrete steps of standard bank size of capacitors.

Chung and Shaoyun [3] presented a recursive linear programming based approach for minimizing line losses and finding the optimal capacitor allocation in a distribution system. Bouri et al. in [4] presented an ant colony optimization approach to shunt capacitor placement in distribution systems under certain constraints. These constraints were voltage constraints and capacitor switching constraints. The voltage constraints were taken into account by specifying upper and lower limits of voltage variations at the nodes of the distribution system. The capacitor switching constraints prevented high in-rush currents caused by the interaction between the capacitors on the distribution system.

Prakash and Sydulu in [1] presented a novel approach that determines the optimal location and size of capacitors on radial distribution systems to improve voltage profile and reduce the active power loss. Capacitor placement and sizing were done using loss sensitivity factors and PSO, respectively. The concept of loss sensitivity factors was considered as the contribution in the area of distribution systems. Loss sensitivity factors determined the candidate nodes for the placement of capacitors. The estimation of these candidate nodes basically helps in reduction of the search space for the optimization procedure. These factors are determined using a base case load flow; that is, without any compensation. PSO was used for estimation of required level of shunt capacitive compensation to improve the voltage profile of the system. The method was tested on 10, 15, 34, 69 and 85 bus distribution systems. The main advantage of that method was that it systematically decided the locations and size of capacitors to realize the optimum sizable reduction in active power loss and significant improvement in voltage profile. The method placed capacitors at a fewer number of locations with optimum sizes and offered much saving in initial investment and regular maintenance. The disadvantage of that algorithm was that the capacitor sizes were considered as continuous variables, then the capacitor sizes were rounded off to the nearest available capacitor value. In this paper, an enhancement to that algorithm is proposed.

Azim and Swarup in [5] presented a GA-based approach to determine the optimum locations and sizes of capacitors for a distribution system. The capacitor sizes were assumed as discrete known variables, which were to be placed on the buses such that they reduced the losses of the distribution system to a minimum. A genetic algorithm was used as an optimization tool, which obtained the optimal values and location of capacitors and minimized the objective function, which was the power loss in the distribution network under study. An initial base case load flow was used to calculate power loss and voltage profile of the distribution system. The problem was formulated as a constrained optimization problem. In this constrained problem, the constraint was the voltage limit; i.e. if the voltage magnitude exceeded a specified limit, this increased the value power loss function as a penalty term. Since the addition of a capacitor at any bus in the distribution system resulted in voltage magnitude increase, therefore it became imperative to model voltage magnitude as a constraint in the mathematical equation, which was to be optimized. The line flow limits were taken care of by the load flow program that calculated the losses. The encoding strategy of each individual, which forms a possible solution, is as follows: Each capacitor size value, when converted into binary form is of length 20 and if N is the number of buses in the distribution system including the slack bus, then $N-1$ locations are possible for capacitor placement, hence each individual has length equal to $N-1$. The proposed algorithm was tested on the 33-bus standard radial distribution system and a practical 29-bus radial distribution of Puth-Kalan, North Delhi, India.

In this paper, a new algorithm for solving the problem of optimal capacitor allocation and sizing in a radial power system is proposed. Loss sensitivity factors are used to determine where the capacitors are to be placed and a discrete particle swarm optimization algorithm is used to determine the sizes of the capacitors. The rest of this paper is organized as follows. In the next section, the mathematical formulation of the capacitor placement and sizing problem is provided. Loss sensitivity factors are defined in Section 3. The steps of the proposed algorithm are described in Section 4 and the obtained results are presented in Section 5. Finally, Section 6 concludes the paper.

2. Capacitor placement and sizing problem formulation

The objective of the optimal capacitor placement and sizing problem in this study is to minimize the total annual cost function of capacitor placement and power losses, which is given by

$$K^p P_{\text{loss}} + \sum_{j=1}^{J} K_j^c Q_j^c \tag{1}$$

where P_{loss} is the total power losses, K^p is the annual cost per unit of power losses (\$/KW), K_j^c is the capacitor annual cost (\$/KVAR), Q_j^c is the shunt capacitor size placed at bus j and J is the number of candidate buses for capacitor placement.

The control variables are the shunt capacitors size (Q^C), which are discrete variables. Table 1 shows the available capacitor sizes and the corresponding yearly cost [6].

The constraints that need to be satisfied are listed below.

(i) Shunt capacitors limits

$$Q_{\max}^C \le Q_{\text{total}} \tag{2}$$

where $Q_{\max}^C$ is the largest capacitor size allowed and Q_{total} is the total reactive load

(ii) Bus bar voltage limits

$$V_{\min} < V_i < V_{\max} \tag{3}$$

in radial power systems $V_{\min} = 0.9$ and $V_{\max} = 1.1$

(iii) Line flow limits

$$\text{Flow}_k < \text{Flow}_k^{\max} \tag{4}$$

where Flow_k is the power flow in kth-line and $\text{Flow}_k^{\max}$ is the maximum allowable power flow.

3. Loss sensitivity factors

Consider a distribution line connected between 'p' and 'q' buses, as shown in Fig. 1.

Table 1 Yearly cost of fixed capacitors.

Capacitor size (KVAR)	150	300	450	600	750	900	1050
Capacitor cost ($/year)	0.5	0.35	0.253	0.22	0.276	0.183	0.228
Capacitor size (KVAR)	1200	1350	1500	1650	1800	1950	2100
Capacitor cost ($/year)	0.17	0.207	0.201	0.193	0.187	0.211	0.176
Capacitor size (KVAR)	2250	2400	2550	2700	2850	3000	3150
Capacitor cost ($/year)	0.197	0.17	0.189	0.187	0.183	0.18	0.195
Capacitor size (KVAR)	3300	3450	3600	3750	3900	4050	
Capacitor cost ($/year)	0.174	0.188	0.17	0.183	0.182	0.179	

The active power loss P_{loss_k} in the kth line is given by

$$P_{\text{loss}_k} = I_{pq}^2 R_{pq} = \frac{S_{pq}^2}{V_p^2} R_{pq} = \frac{P_{pq}^2 + Q_{pq}^2}{V_p^2} R_{pq} \tag{5}$$

Now, the loss sensitivity factor is given by:

$$\frac{\partial P_{\text{loss}_k}}{\partial Q_{pq}} = \frac{2Q_{pq}}{V_p^2} R_{pq} \tag{6}$$

The loss sensitivity factors are calculated from the base case load flow (that is, without compensation) and the values are arranged in descending order for all the transmission lines of the given system. A vector that holds the respective "end buses" of the lines arranged in descending order of the values of the loss sensitivity factors is stored. The bus with inflowing power is the one considered for capacitor placement. The descending order of the loss sensitivity factors will decide the sequence in which the buses are to be considered for compensation [1].

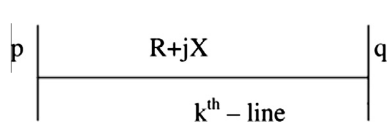

Figure 1 A distribution line between buses p and q.

4. The proposed algorithm

In this section, the details of the developed solution procedure are provided.

(1) Input data: the data to be fed as input are listed below.
  (a) Number of buses.
  (b) Load demand (active (watt) and reactive (VAR) power) at each bus.
  (c) Bus voltage limits ($V_{\min}$, $V_{\max}$).
  (d) Transmission lines' impedances (resistance and reactance).
  (e) Transmission lines' capacity (maximum allowable power flow).
  (f) Particle swarm parameters (number of particles, max number of iterations, acceleration constants C_1 and C_2, initial inertial weight).

(2) Perform the initial power flow (or load flow) analysis using Gauss Seidel or Newton Raphson algorithm without capacitor compensation to calculate the loss sensitivity factors and the values are arranged in descending order for all the lines of the given system. Then, determine the candidate buses requiring capacitor placement.

(3) Select the suitable range of capacitors from Table 1 such that

$$Q_i^c \le Q_{\max}^c \le Q_{\text{total}} \tag{7}$$

(4) Randomly initialize the position of particles. Each position is a two-dimensional matrix. Thus, the positions of the particles are represented by the following three-dimensional matrix

$$X_{J\times R\times \text{NIND}} = [Q_1^c, \ldots, Q_i^c, \ldots, Q_J^c]$$

where J is the number of candidate buses for reactive compensation, NIND is the number of particles, R is the number of allowable capacitor sizes (e.g. if the available Q^c sizes that satisfy (7) are [150, 300, 450, 600], then $R = 4$) and Q_i^c is the reactive power installed at bus i. To select the capacitor size Q_i^c to be placed at bus i, a combination of capacitor sizes is chosen from Table 1,

$$Q_i^c = b_1\, sz_1 + b_2\, sz_2 + \ldots + \ldots + b_R sz_R$$

where $b_k \in \{0,1\}$ and sz_k are capacitor sizes.

(5) Initialize the velocity of the particles.

(6) Perform the power flow (or load flow) analysis using Gauss Seidel or Newton Raphson algorithm for each particle to get the following
  (a) The active power losses (P_{loss}).
  (b) The voltage at each bus (V_{bus}).
  (c) Transmission line flows to determine the overloaded lines.

(7) Calculate the total annual cost function for each particle.

(8) Calculate the fitness function (FF) for each particle.

$$\text{FF} = \left\{ \begin{array}{l} \text{cost} + \sum_{i=1}^{N_{\text{load}}} (\text{penalty factor}) \times (V_i - V_{\max})^2 + \sum_{i=1}^{N_{\text{load}}} (\text{penalty factor}) \times (V_i - V_{\min})^2 \\ + \sum_{i=1}^{N_{\text{line}}} (\text{penalty factor}) \times (\text{Flow}_i - \text{Flow}_i^{\max})^2 \end{array} \right\} \tag{8}$$

where the penalty factor is assigned as follows for radial distribution systems

$$\text{penalty factor} = \left\{ \begin{array}{ll} 0 & \text{if constraints are not violated} \\ 500 \times \text{cost} \times \text{iteration}^2 & \text{if constraints are violated} \end{array} \right\} \tag{9}$$

(9) Compare particles' fitness functions and update the personal (individual) best (pbest_i) for each particle and the global best (gbest) through all the particles.

(10) Update the particles' velocities according to the following equation

$$v_i^{(t+1)} = w^{(t)}\, v_i^{(t)} + c_1 \times r_1 \times \left(\text{pBest}_i^{(t)} - X_i^{(t)}\right) + c_2 \times r_2 \times \left(\text{gBest}^{(t)} - X_i^{(t)}\right) \quad (10)$$

where $X_i^{(t)}$ is the position vector of the ith particle in iteration number t, $v_i^{(t)}$ is its velocity vector, r_1 and r_2 are two random numbers in the unit interval, c_1 and c_2 are two acceleration constants giving different weights to the personal best position and the global best position, and $w^{(t)}$ is the inertial weight.

(11) Update the particles' positions. For a binary discrete search space, Kennedy and Eberhart have adapted the PSO to search in binary spaces by applying a sigmoid transformation to the velocity component to squash the velocities into a range [0, 1], and then forcing the component values of the locations of particles to be 0's or 1's. The equations for updating positions are given by

$$\text{sigmoid}\left(v_i^{(t+1)}\right) = \frac{1}{1 + e^{-v_i^{(t+1)}}} \quad (11)$$

$$X_i^{(t+1)} = \begin{cases} 1 & \text{if} \quad \text{rand} < \text{sigmoid}\left(v_i^{(t+1)}\right) \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

where rand is a random number between [0, 1].

(12) Update the inertial weight according to the following equation

$$w^{(t)} = w_{\max} - \left(\frac{w_{\max} - w_{\min}}{\text{iterm}}\right) \times t \quad (13)$$

where $w^{(t)}$ is the inertial weight in iteration t, $w_{\max}$ and $w_{\min}$ are the upper and lower limit of the inertial weight respectively, iterm is the maximum number of iterations and t is the iteration counter. The inertial weight balances global and local exploration and it decreases linearly from $w_{\max}$ to $w_{\min}$.

(13) Return to step (6) until reaching the maximum number of iterations.

Table 3 Discrete PSO parameters.

C_1	C_2	$w_{\max}$	$w_{\min}$	Itermax	NIND
2	2	0.9	0.4	100	100

5. Results

The proposed algorithm has been tested on three radial distribution systems, which are the standard 10-bus, 15-bus, and 34-bus radial distribution systems, their specifications can be found in [6–8] respectively. The annual cost per unit of power losses K^p is taken as 168 \$/KW. The voltage limits are taken as $V_{\min} = 0.9$ and $V_{\max} = 1.1$.

5.1. Results of the 10-bus system

The rated line voltage of the system is 23 kV. After running an initial load flow without any capacitor compensation, the total power loss is 783.8 KW and the annual cost function of the power losses is \$131,677. The loss sensitivity factors are calculated from this initial load flow for each transmission line. The loss sensitivity factors arranged in descending order are shown in Table 2 along with the corresponding end bus identification and normalized voltage magnitudes. The buses requiring compensation are given as 5, 4, 8, 9, 7 and 6, which are stored in a 'rank bus' vector. The remaining buses are healthy buses because their normalized voltage is greater than 1.01 [1], so they do not need any compensation. Now, sizing of capacitors at buses listed in the 'rank bus' vector is done by using the discrete PSO algorithm. The parameters of the PSO algorithm are shown in Table 3.

After performing 10 independent runs, the obtained results are summarized in Table 4, which include the average, maximum and minimum active power losses and the standard deviation of these 10 results. Table 5 shows the average, maximum and minimum total cost (capacitor cost plus cost of power losses). Table 6 shows the average, maximum and minimum net saving. Table 7 shows a comparison between the optimal sizing of capacitors obtained by the proposed discrete PSO and those obtained in [1]. It is clear from Table 7 that the proposed algorithm deals explicitly with the discrete nature of the capacitor sizes, but in [1], the capacitor sizes are considered as continuous variables then the capacitor values are rounded off to the nearest available value. The total KVAR installed is 6000 KVAR. The cost of the installed capacitors is 1559 \$/

Table 2 Loss sensitivity factors arranged in descending order of a 10-bus radial distribution.

Order	$\frac{\partial P_{\text{loss}}}{\partial Q}$	Start bus (p)	End bus (q)	V_p	V_q	Norm [i] = $V_q(i)/0.95$
1	0.0119	4	5	0.948	0.9172	0.9654
2	0.0119	2	3	0.9874	0.9634	1.0141
3	0.0096	3	4	0.9634	0.948	0.9979
4	0.0092	7	8	0.889	0.8587	0.9039
5	0.0061	8	9	0.8587	0.8375	0.8816
6	0.0048	6	7	0.9072	0.889	0.9357
7	0.0027	5	6	0.9172	0.9072	0.9549
8	0.0024	10	1	1	0.9929	1.0452
9	0.0002	1	2	0.9929	0.9874	1.0393

Table 4 Statistics for the power losses for the 10-bus radial system.

Average (KW)	Minimum (KW) (best)	Maximum (KW) (worst)	Standard deviation
708.4	701.2	716.2	4.4

Table 5 Statistics for the total cost for the 10-bus radial system.

Average ($/year)	Minimum ($/year) (best)	Maximum ($/year) (worst)	Standard deviation
120,381	119,352	121,574	646

Table 6 Statistics for the net saving for the 10-bus radial system.

Average ($/year)	Minimum ($/year) (worst)	Maximum ($/year) (best)	Standard deviation
11,296	10,103	12,325	646

Table 10 Statistics for the power losses for the 15-bus radial system.

Average (KW)	Minimum (KW) (best)	Maximum (KW) (worst)	Standard deviation
30.7	30.3	31.5	0.3

Table 11 Statistics for the total cost for the 15-bus radial system.

Average ($/year)	Minimum ($/year) (best)	Maximum ($/year) (worst)	Standard deviation
5539	5491	5625	47

Table 12 Statistics for the net saving for the 15-bus radial system.

Average ($/year)	Minimum ($/year) (worst)	Maximum ($/year) (best)	Standard deviation
4841	4755	4888	47

Table 7 Optimum sizing of capacitors for the 10-bus radial system.

Bus No	4	5	6	7	8	9	10
Q^c in KVAR (proposed discrete PSO)	3000	1500	150	450	600	300	0
Q^c in KVAR (method in [1])	0	1182	1174	0	0	264	566

Table 8 Comparison between obtained results and those obtained in [1] for the 10-bus radial system.

Items	Total KVAR installed	Active power losses (KW)	Cost of power losses ($/year)	Saving due to reduction of power losses ($/year)	Cost of capacitors ($/year)	Net saving ($/year)	V_{min}	V_{max}
Proposed discrete PSO	6000	701.2	117,793	13,877	1559	12,325	0.9002	1
Method in [1]	3186	704.4703	118,351	13,324	645	12,679	0.8729	1

Table 9 Loss sensitivity factors placed in descending order of a 15-bus radial distribution system.

Order	$\frac{\partial P_{loss}}{\partial Q}$	Start Bus (p)	End bus (q)	V_p	V_q	Norm [i] = $V_q(i)/0.95$
1	0.0293	1	2	1	0.9713	1.0224
2	0.0162	2	6	0.9713	0.9582	1.0087
3	0.0153	2	3	0.9713	0.9567	1.007
4	0.0085	3	11	0.9567	0.95	1
5	0.0062	3	4	0.9567	0.9509	1.001
6	0.0052	11	12	0.95	0.9458	0.9956
7	0.0041	2	9	0.9713	0.968	1.0189
8	0.0031	4	15	0.9509	0.9484	0.9984
9	0.0029	4	14	0.9509	0.9486	0.9985
10	0.0028	6	7	0.9582	0.956	1.0063
11	0.0017	12	13	0.9458	0.9445	0.9942
12	0.0016	6	8	0.9582	0.957	1.0073
13	0.0013	9	10	0.968	0.9669	1.0178
14	0.0013	4	5	0.9509	0.9499	0.9999

year. The maximum voltage is 1 p.u. and the minimum voltage is 0. 9002 p.u. Total power loss is 701.2 KW. This means that the proposed algorithm reduces the total power losses by 10.54% compared to the base case where no capacitor compensation is used. The net saving is 12,325 $/year, which is the difference between the cost of the reduced power losses and the capacitor cost. Table 8 shows a comparison between the results obtained by the proposed discrete PSO and those obtained in [1]. From Table 8, it is clear that the obtained net saving is 12,325 $/year which is worse than 12678.8, but

Table 13 Optimum sizing of capacitors for the 15-bus radial system.

Bus no.	3	4	5	6	7	8	11	12	13	14	15
Q^c in KVAR (proposed discrete PSO)	0	450	0	450	0	0	0	0	150	0	150
Q^c in KVAR (method in [1])	871	0	0	321	0	0	0	0	0	0	0

Table 14 Comparison between obtained results and those obtained in [1] for the 15-bus radial system.

Items	Total KVAR i nstalled	Active power losses (KW)	Cost of power losses ($/year)	Saving due to reduction of power losses ($/year)	Cost of capacitors ($/year)	Net Saving ($/year)	$V_{\min}$	$V_{\max}$
Proposed discrete PSO	1200	30.4463	5114.978	5265	378	4888	0.9712	1
Method in [1]	1192	32.7	5493.6	4886.33	270	4616.33	0.9673	1

Table 15 Loss sensitivity factors placed in descending order of a 34-bus radial distribution system.

Order	$\frac{\partial P_{\text{loss}}}{\partial Q}$	Start bus (p)	End bus (q)	V_p	V_q	Norm [i] = $V_q(i)/0.95$
1	0.0073	3	4	0.989	0.9821	1.0338
2	0.0063	4	5	0.9821	0.9761	1.0274
3	0.006	5	6	0.9761	0.9704	1.0215
4	0.0057	1	2	1	0.9941	1.0465
5	0.005	2	3	0.9941	0.989	1.0411
6	0.0048	6	17	0.9704	0.966	1.0168
7	0.0046	18	19	0.9623	0.9582	1.0086
8	0.0044	6	7	0.9704	0.9666	1.0175
9	0.004	17	18	0.966	0.9623	1.0129
10	0.0038	21	22	0.952	0.9487	0.9987
11	0.0037	19	20	0.9582	0.9549	1.0051
12	0.0032	20	21	0.9549	0.952	1.0021
13	0.0032	22	23	0.9487	0.9461	0.9959
14	0.003	23	24	0.9461	0.9435	0.9932
15	0.0028	8	9	0.9645	0.962	1.0127
16	0.0024	7	8	0.9666	0.9645	1.0153
17	0.0014	24	25	0.9435	0.9423	0.9919
18	0.0014	9	10	0.962	0.9609	1.0114
19	0.0006	25	26	0.9423	0.9419	0.9914
20	0.0005	10	11	0.9609	0.9604	1.0109
21	0.0004	7	28	0.9666	0.9663	1.0171
22	0.0004	31	32	0.9605	0.9602	1.0107
23	0.0004	10	31	0.9609	0.9605	1.0111
24	0.0004	3	13	0.989	0.9887	1.0407
25	0.0003	13	14	0.9887	0.9884	1.0404
26	0.0003	28	29	0.9663	0.966	1.0169
27	0.0002	32	33	0.9602	0.96	1.0105
28	0.0002	26	27	0.9419	0.9417	0.9913
29	0.0002	11	12	0.9604	0.9603	1.0108
30	0.0001	29	30	0.966	0.9659	1.0168
31	0.0001	14	15	0.9884	0.9883	1.0403
32	0.0001	33	34	0.96	0.9599	1.0105
33	0	15	16	0.9883	0.9883	1.0403

in [1] the minimum voltage is 0.8729 and this means that the voltage constraint is violated.

5.2. Results of the 15-bus system

The rated line voltage of the system is 11 kV. After running an initial load flow, the total power losses are found to be 61.7853 KW and the annual cost function of the power losses is $10,380. The loss sensitivity factors are calculated from this initial load flow for each transmission line. The loss sensitivity factors arranged in descending order are shown in Table 9 along with its bus identification and normalized voltage magnitudes. The rank bus vector of the 15-bus radial distribution system includes the buses 6, 3, 11, 4, 12, 15, 14, 7, 13, 8 and 5. The remaining buses require no compensation since the normalized voltage is greater than 1.01 [1]. The sizing of capacitors at buses listed in the 'rank bus' vector is done by using the discrete PSO algorithm with the same parameters shown in Table 3.

The results obtained in 10 independent runs are summarized in Table 10, which include the average, maximum and minimum active power losses and the standard deviation of these 10 results. Table 11 shows the average, maximum and minimum total cost (capacitor cost plus cost of power losses). Table 12 shows the average, maximum and minimum net saving. Table 13 shows a comparison between the optimal capacitor sizing capacitor obtained by the proposed discrete PSO and those obtained in [1]. The total KVAR installed is 1200 KVAR. The cost of the installed capacitors is 378 $/year. The maximum voltage is 1 p.u. and the minimum voltage is 0.9712 p.u. Total power loss is found to be 30.4463 KW. This means that the proposed algorithm reduces the total power losses by 50.7%. The net saving is 4888 $/year. Table 14 shows a comparison between the results obtained by the proposed discrete PSO and those obtained in [1]. Table 14 shows that the obtained net saving is 4888 $/year, which is better than 4616 although the obtained capacitor cost is greater than that in [1]. This is because the saving due to reduction of power losses is better.

5.3. Results of the 34-bus system

The rated line voltage of the system is 11 kV. The total power losses without any capacitor compensation are 221.7235 KW and the annual cost function of the power losses is $37,250. The loss sensitivity factors are calculated for each transmission line from this initial load flow. The loss sensitivity factors listed in descending order are given in Table 15 along with bus identification and normalized voltage magnitudes. The rank bus vector of the 34-bus radial distribution system includes the following buses: 19, 22, 20, 21, 23, 24, 25, 26 and 27. The optimal capacitors sizes are then determined using discrete PSO with the parameters indicated in Table 3.

The average, maximum and minimum active power losses and the standard deviation of the results of 10 independent runs are listed in Table 16. Table 17 shows the average, maximum and minimum total cost. Table 18 shows the average, maximum and minimum net saving. Table 19 shows a comparison between the optimum sizing of capacitor obtained by the proposed discrete PSO and those obtained in [1]. The total KVAR installed is 1950 KVAR. The cost of the installed capacitors is 483 $/year. The maximum voltage is 1 p.u. and the minimum voltage is 0. 9501 p.u. Total power loss is 163.3 KW. This means that the proposed algorithm reduces the total power losses by 26.27%. The net saving is 9295 $/year.

Table 16 Statistics for the power losses for the 34-bus radial system.

Average (KW)	Minimum (KW) (best)	Maximum (KW) (worst)	Standard deviation
163.4	163.1	163.8	0.2

Table 17 Statistics for the total cost for the 34-bus radial system.

Average ($/year)	Minimum ($/year) (best)	Maximum ($/year) (worst)	Standard deviation
27,997	27,923	28,081	39

Table 18 Statistics for the net saving for the 34-bus radial system.

Average ($/year)	Minimum ($/year) (worst)	Maximum ($/year) (best)	Standard deviation
9221	9137	9295	39

Table 19 Optimum sizing of capacitors for the 34-bus radial bus system.

Bus No	19	20	21	22	23	24	25	26	27
Q^c in KVAR (proposed discrete PSO)	1050	0	0	300	0	0	0	600	0
Q^c in KVAR (method in [1])	781	479	0	803	0	0	0	0	0

Table 20 Comparison between obtained results and those obtained in [1] for the 34-bus radial system.

Items	Total KVAR installed	Active power losses (KW)	Cost of power losses ($/year)	Saving due to reduction of power losses ($/year)	Cost of capacitors ($/year)	Net Saving ($/year)	V_{min}	V_{max}
Proposed discrete PSO	1950	163.3	27,440	9778	483	9295	0.9501	1
Method in [1]	2063	168.8975	28,375	8875	527	8348	0.9496	1

Table 20 shows a comparison between the results obtained by the proposed discrete PSO and those obtained in [1]. It is clear from Table 20 that the obtained net saving is 9295 $/year which is better than 8348.

6. Conclusion

In this paper, the problem of optimal capacitor placement and sizing (OCPS) is solved. The PSO technique is used as the optimization tool, with its discrete version. Unlike the traditional methods, PSO is a population based search algorithm, i.e. PSO has implicit parallelism. This property ensures PSO to be less susceptible to being trapped in local minima. Moreover, it has been noticed that the solution quality of the proposed algorithm does not rely on the initial population. Starting anywhere in the search space, the algorithm converges to the optimal solution. The proposed algorithm has been tested on the standard 10-bus, 15-bus and 34-bus radial distribution systems and it gives superior results compared with those reported in literature with a relatively small number of iterations. Solving the OCPS problem gives the advantages of using capacitors, which improve the system power factor, improve the system voltage profile, decrease the flow through cables and transformers and reduce losses due to the compensation of the reactive component of power flow. By decreasing the flow through cables, the systems' loads can be increased without adding any new cables or overloading the existing cables.

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