DUAL SERIES EQUATIONS INVOLVING JACOBI AND LAGUERRE POLYNOMIALS

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ABSTRACT

An exact solution for some dual series equations involving Jacobi polynomials is obtained. Results for similar dual series equations involving Laguerre polynomials are also deduced by applying a limit process.

1. Introduction

In recent years there has been considerable interest in dual series equations involving Jacobi and Laguerre polynomials. By going through the literature, however, one feels that once a result has been obtained for Jacobi polynomials, it becomes a routine matter to work out similar results for Laguerre polynomials (see, for example, [1], [2]; [3], [4]). The source of this symmetry seems to be the fact that Laguerre polynomials are certain limiting cases of Jacobi polynomials. Indeed, if adequate care is taken in presenting the results on Jacobi polynomials it is possible to avoid the duplication of work and similar results for Laguerre polynomials can be deduced through a limit process.

In the present paper we consider certain dual series equations involving Jacobi polynomials which are generalizations of those considered by Noble [3]. We deduce results for similar dual series equations involving Laguerre polynomials by applying a limit process. In order to emphasize that in most of the cases it is unnecessary to consider dual series equations for Jacobi and Laguerre polynomials separately, the proofs have been carried out in such a way that the limit process can be applied not only to the final results but to any intermediate step and to any formula being used thereof. It turns out that the results which we deduce for dual series equations involving Laguerre polynomials are generalizations of those given by Lowndes [4] and Srivastava [10]. In the next section we give, for ready reference, some results which will be needed in the course of analysis.

2. Preliminary results

In Szegö notation the Jacobi polynomials may be defined [6, p. 254], in terms of hypergeometric functions, as

$$(2.1)\qquad P_n^{(\alpha,\beta)}\left(1-\frac{2x}{c}\right)=\frac{\Gamma(n+\alpha+1)}{n!\Gamma(\alpha+1)}\,{}_2F_1\left(-n,\, n+\alpha+\beta+1;\alpha+1,\frac{x}{c}\right).$$

We shall be working throughout this paper with the Szegö notation which is now standard in mathematics literature but to compare our results with other works we shall at times need the following relation between the Szegö notation and the one used by NOBLE [3]:

$$(2.2)\qquad P_n^{(\alpha,\delta)}\left(1-\frac{2x}{c}\right)=\frac{\Gamma(n+\alpha+1)}{n!\Gamma(\alpha+1)}J_n\left(\alpha+\delta+1,\alpha+1;\frac{x}{c}\right).$$

One of the limit formulas that will be needed is the generalized form of a result given in [5; p. 75]:

$$(2.3)\qquad \lim_{\delta\to\infty}\left(1-\frac{x}{\delta}\right)^{\delta+q}=e^{-x},$$

where q is any real number. It may be deduced from [8; p. 191 (35)] or may be shown using (3) and the generating functions for Jacobi polynomials [8; p. 172 (29)] and Laguerre polynomials [8; p. 189 (17)] that for any real number q

$$(2.4)\qquad \lim_{\delta\to\infty}P_n^{(\alpha,\delta+q)}\left(1-\frac{2x}{\delta}\right)=L_n^{(\alpha)}(x),$$

where $L_n^{(\alpha)}(x)$ are the Laguerre polynomials. A limit formula involving gamma functions which follows from [7; p. 47 (4)] is given by

$$(2.5)\qquad \lim_{\delta\to\infty}[\delta^{(q_1-q_2)}\cdot\Gamma(\delta+q_2)/\Gamma(\delta+q_1)]=1.$$

From the results (5), (7) and (17) of [6; p. 264] we obtain the following differentiation formulas for the Jacobi polynomials:

$$(2.6)\qquad \left\{\begin{aligned}&\frac{d^m}{dx^m}\left[x^\alpha P_n^{(\alpha,\delta)}\left(1-\frac{2x}{c}\right)\right]=\\&\qquad=\frac{\Gamma(\alpha+n+1)}{\Gamma(\alpha-m+n+1)}x^{\alpha-m}P_n^{(\alpha-m,\delta+m)}\left(1-\frac{2x}{c}\right),\end{aligned}\right.$$

$$(2.7)\qquad \left\{\begin{aligned}&\frac{d^m}{dx^m}\left[\left(1-\frac{x}{c}\right)^\delta P_n^{(\alpha,\delta)}\left(1-\frac{2x}{c}\right)\right]=\\&\qquad=\frac{(-c)^{-m}\,\Gamma(\delta+n+1)}{\Gamma(\delta-m+n+1)}\left(1-\frac{x}{c}\right)^{\delta-m}P_n^{(\alpha+m,\delta-m)}\left(1-\frac{2x}{c}\right).\end{aligned}\right.$$

From [9; p. 191 (43)] and [9; p. 191 (44)] we have the following formulas which are similar to the Sonine integrals of the first and second kinds:

$$(2.8)\quad \begin{cases} \int\limits_0^y \dfrac{x^\alpha P_n^{(\alpha,\delta)}(1-2x/c)}{(y-x)^{1-\mu}}\,dx= \\ \quad = \dfrac{\Gamma(\mu)\Gamma(\alpha+n+1)}{\Gamma(\alpha+\mu+n+1)}\, y^{\alpha+\mu}\, P_n^{(\alpha+\mu,\,\delta-\mu)}\left(1-\dfrac{2y}{c}\right),\ \alpha>-1,\ \mu>0, \end{cases}$$

$$(2.9)\quad \begin{cases} \int\limits_y^c \dfrac{(1-x/c)^\delta P_n^{(\alpha,\delta)}(1-2x/c)}{(x-y)^{1-\mu}}\,dx= \\ \quad = \dfrac{c^\mu\Gamma(\mu)\Gamma(\delta+n+1)}{\Gamma(\delta+\mu+n+1)}\left(1-\dfrac{y}{c}\right)^{\delta+\mu} P_n^{(\alpha-\mu,\,\delta+\mu)}\left(1-\dfrac{2y}{c}\right),\ \delta>-1,\ \mu>0. \end{cases}$$

The orthogonality relation for Jacobi polynomials [6; p. 135] may be written as

$$(2.10)\quad \begin{cases} \int\limits_0^c x^\alpha\left(1-\dfrac{x}{c}\right)^\delta P_n^{(\alpha,\delta)}\left(1-\dfrac{2x}{c}\right) P_m^{(\alpha,\delta)}\left(1-\dfrac{2x}{c}\right)dx \\ \qquad = \dfrac{c^{\alpha+1}\,\Gamma(\alpha+n+1)\,\Gamma(\delta+n+1)\,\delta_{mn}}{n!(2n+\alpha+\delta+1)\,\Gamma(n+\alpha+\delta+1)},\ \alpha>-1,\ \delta>-1, \end{cases}$$

where δ_{mn} is the Kronecker delta. It follows from the orthogonality relation (2.10) and the formula (2.8) that

$$(2.11)\quad \begin{cases} \sum\limits_{n=0}^\infty \dfrac{n!(2n+\delta+\alpha+1)\,\Gamma(n+\alpha+\delta+1)}{c^{\alpha+1}\,\Gamma(\delta+n+1)\,\Gamma(\alpha+\mu+n+1)} P_n^{(\alpha,\delta)}\left(1-\dfrac{2x}{c}\right) \\ P_n^{(\alpha+\mu,\,\delta-\mu)}\left(1-\dfrac{2y}{c}\right) = \dfrac{H(y-x)(y-x)^{\mu-1}}{(1-x/c)^\delta\, y^{\alpha+\mu}\,\Gamma(\mu)},\ \alpha>-1,\ \delta>-1,\ \mu>0, \end{cases}$$

where $H(x)$ is the Heaviside's unit function.

For $c=1$, the results (2.8)–(2.11) may be found in [3]. On the other hand if we put $c=\delta$ in (2.8)–(2.11) and let δ approach infinity then, using (2.3)–(2.5), it can be easily seen by making appropriate changes in the parameters that the resulting equations are those appearing in [4].

3. Dual series equations

In this section we give an exact solution of the dual series equations

$$(3.1)\qquad \sum_{n=0}^\infty \frac{A_n\,\Gamma(\alpha-\sigma+n+1)}{\Gamma(\alpha+n+1)} P_n^{(\alpha,\,\delta+p)}\left(1-\frac{2x}{c}\right)=f(x),\ 0<x<a,$$

$$(3.2)\qquad \sum_{n=0}^\infty \frac{A_n\,\Gamma(\delta+\sigma+p+n+1)}{c^{\sigma+p}\,\Gamma(\delta+n+1)} P_n^{(\alpha+p,\,\delta)}\left(1-\frac{2x}{c}\right)=g(x),\ a<x<c,$$

where the parameters α, p, σ and δ satisfy, for some non-negative integers m and k, the inequalities

$$(3.3)\quad \begin{cases} \text{(i) } \alpha+1>\max(0,\sigma,-p), \text{ (ii) } m-\sigma>0, \text{ (iii) } p+\delta+\sigma+1>m, \\ \text{(iv) } p+\sigma+k>0 \text{ and (v) } \delta+1>k. \end{cases}$$

The dual series equations (3.1) and (3.2) have been considered by NOBLE [3] for $p=0$, $\delta>-1$, $\alpha+1>\sigma$, $0<\sigma<1$, $c=1$, that is, when $p=0$, $m=1$, $k=0$, $c=1$. In equations (3.1) and (3.2) if we put $c=\delta$ and let $\delta\to\infty$ then, using (2.4) and (2.5), we find that they become

$$(3.4)\qquad \sum_{n=0}^{\infty} \frac{A_n\,\Gamma(\alpha-\sigma+n+1)}{\Gamma(\alpha+n+1)} L_n^{(\alpha)}(x)=f(x),\ 0<x<a,$$

$$(3.5)\qquad \sum_{n=0}^{\infty} A_n\, L_n^{(\alpha+p)}(x)=g(x),\ a<x<\infty,$$

and the conditions (3.3) on the parameters reduce to the only genuine condition

$$(3.6)\qquad \alpha+1>\max(0,\sigma,-p).$$

The equations (3.4) and (3.5) have been solved by LOWNDES [4] for $p=0$, $0<\sigma<1$. More recently SRIVASTAVA [10] has given the solution of (3.4) and (3.5) valid under the condition (3.6) and an additional condition $\sigma+p>0$.

We proceed now to give the solution of the dual equations (3.1)–(3.2) under the conditions (3.3). Multiplying (3.1) by $x^{\alpha}(y-x)^{m-\sigma-1}$ and integrating over $(0,y)$ (with $y<a$) we find, using (2.8), that

$$(3.7)\quad \begin{cases} \displaystyle\sum_{n=0}^{\infty} \frac{A_n\,\Gamma(\alpha-\sigma+n+1)y^{\alpha+m-\sigma}}{\Gamma(\alpha-\sigma+m+n+1)} P_n^{(\alpha+m-\sigma,\,\delta+p+\sigma-m)}\left(1-\frac{2y}{c}\right) \\ \displaystyle\qquad =\frac{1}{\Gamma(m-\sigma)}\int_0^y \frac{x^{\alpha}f(x)\,dx}{(y-x)^{1+\sigma-m}},\ 0<y<a. \end{cases}$$

Differentiating (3.7) m times and using (2.6) we get

$$(3.8)\qquad \sum_{n=0}^{\infty} A_n P_n^{(\alpha-\sigma,\,\delta+p+\sigma)}\left(1-\frac{2y}{c}\right)=\frac{y^{\sigma-\alpha}}{\Gamma(m-\sigma)} f_1(y),\ 0<y<a,$$

where

$$(3.9)\qquad f_1(y)=\frac{d^m}{dy^m}\int_0^y \frac{x^{\alpha}f(x)\,dx}{(y-x)^{1+\sigma-m}}.$$

In deriving (3.8) from (3.1) we have used the conditions (i), (ii) and (iii) of (3.3). If we multiply the equation (3.2) by $(1-x/c)^{\delta}$ and differentiate

k times we find using (2.7) that it becomes

$$\text{(3.10)} \quad \left\{ \begin{aligned} & \sum_{n=0}^{\infty} \frac{A_n\, \Gamma(\delta+\sigma+p+n+1)}{c^{k+\sigma+p}\; \Gamma(\delta-k+n+1)} P_n^{(\alpha+p+k,\, \delta-k)}\left(1-\frac{2x}{c}\right) \\ & \qquad = (-1)^k \left(1-\frac{x}{c}\right)^{k-\delta} \frac{d^k}{dx^k}\left[\left(1-\frac{x}{c}\right)^{\delta} g(x)\right],\ a<x<c. \end{aligned} \right.$$

Multiplying (3.10) by $(1-x/c)^{\delta-k}$ $(x-y)^{(\sigma+p+k-1)}$ and integrating with respect to x over (y, c) (with $y>a$) we get, using (2.9),

$$\text{(3.11)} \quad \sum_{n=0}^{\infty} A_n\, P_n^{(\alpha-\sigma,\, \delta+p+\sigma)}\left(1-\frac{2y}{c}\right) = \frac{(-1)^k\, (1-y/c)^{-(\delta+p+\sigma)}}{\Gamma(\sigma+p+k)}\, g_1(y),\ a<y<c,$$

where

$$\text{(3.12)} \qquad g_1(y) = \int_y^c \frac{d^k/dx^k\, [(1-x/c)^{\delta}\, g(x)]\, dx}{(x-y)^{1-\sigma-p-k}}.$$

The conditions (i), (iv) and (v) of (3.3) have been used in obtaining (3.11) from (3.2). The left hand sides of equations (3.8) and (3.11) are now identical and using the orthogonality relation (2.10) we obtain

$$\text{(3.13)} \qquad \left\{ \begin{aligned} A_n = {} & \frac{1}{\Gamma(m-\sigma)} \int_0^a \left(1-\frac{y}{c}\right)^{\delta+\sigma+p} f_1(y)\, a_n(y)\, dy + \\ & + \frac{(-1)^k}{\Gamma(\sigma+p+k)} \int_a^c y^{\alpha-\sigma}\, g_1(y)\, a_n(y)\, dy, \end{aligned} \right.$$

where

$$\text{(3.14)} \qquad \left\{ \begin{aligned} a_n(y) = {} & \frac{n!\, (2n+\alpha+\delta+p+1)\, \Gamma(n+\alpha+\delta+p+1)}{c^{\alpha-\sigma+1}\, \Gamma(\alpha-\sigma+n+1)\, \Gamma(\delta+p+\sigma+n+1)} \\ & P_n^{(\alpha-\sigma,\, \delta+p+\sigma)}\left(1-\frac{2y}{c}\right). \end{aligned} \right.$$

The coefficients A_n satisfying the dual series equations (3.1) and (3.2) under the conditions (3.3) are thus given by (3.13), (3.14), (3.9) and (3.12). For $p=0$, $m=1$, $k=0$, $c=1$ the solution is in complete agreement with the one obtained by NOBLE [3].

If we put $c=\delta$ in equations (3.13), (3.14), (3.9) and (3.12) and take the limit as $\delta \to \infty$ then, using (2.3) to (2.5), we find that they become

$$\text{(3.15)} \qquad \left\{ \begin{aligned} A_n = {} & \frac{1}{\Gamma(m-\sigma)} \int_0^a e^{-y} f_1(y)\, a_n(y)\, dy + \\ & \frac{(-1)^k}{\Gamma(\sigma+p+k)} \int_a^{\infty} y^{\alpha-\sigma}\, g_1(y)\, a_n(y)\, dy, \end{aligned} \right.$$

where

$$a_n(y) = \frac{n!}{\Gamma(\alpha-\sigma+n+1)} L_n^{(\alpha-\sigma)}(y), \tag{3.16}$$

$$f_1(y) = \frac{d^m}{dy^m} \int_0^y \frac{x^\alpha f(x)\,dx}{(y-x)^{1+\sigma-m}}, \tag{3.17}$$

$$g_1(y) = \int_y^\infty \frac{d^k/dx^k\,[e^{-x}g(x)]}{(x-y)^{1-\sigma-p-k}}\,dx. \tag{3.18}$$

The equations (3.15) to (3.18) provide us with the solution of the dual series equations (3.4) and (3.5) under the condition (3.6), where m and k are non-negative integers satisfying $m-\sigma>0$, $p+\sigma+k>0$. For $\sigma+p>0$, i.e., $k=0$ the solution is in complete agreement with the one obtained by SRIVASTAVA [10].

4. The quantities of interest in physical applications are the values of the series in (3.1) and (3.2) on the intervals where their values are not specified. We define

$$F(x) = \sum_{n=0}^\infty \frac{A_n\,\Gamma(\alpha-\sigma+n+1)}{\Gamma(\alpha+n+1)} P_n^{(\alpha,\,\delta+p)}\left(1-\frac{2x}{c}\right), \quad a<x<c. \tag{4.1}$$

$$G(x) = \sum_{n=0}^\infty \frac{A_n\,\Gamma(\delta+\sigma+p+n+1)}{c^{\sigma+p}\,\Gamma(\delta+n+1)} P_n^{(\alpha+p,\,\delta)}\left(1-\frac{2x}{c}\right), \quad 0<x<a, \tag{4.2}$$

where the parameters α, p, σ and δ satisfy the conditions (3.3) and for some non-negative integers r and s,

$$\left\{\begin{array}{l} \text{(vi) } \sigma+r>0, \text{ (vii) } \delta+p+1>r, \text{ (viii) } s-p-\sigma>0, \\ \text{(ix) } \delta+p+\sigma+1>s. \end{array}\right. \tag{4.3}$$

In view of (2.6) we can write (4.1) as

$$\left\{\begin{array}{l} F(x) = x^{-\alpha}\dfrac{\partial^r}{\partial x^r}\left[x^{\alpha+r}\displaystyle\sum_{n=0}^\infty \frac{A_n\,\Gamma(\alpha-\sigma+n+1)}{\Gamma(\alpha+r+n+1)}\right. \\ \left. P_n^{(\alpha+r,\,\delta+p-r)}\left(1-\dfrac{2x}{c}\right)\right], \quad a<x<c. \end{array}\right. \tag{4.4}$$

Substituting the value of A_n from (3.13) in (4.4), interchanging the order of integration and summation and evaluating the series with the help of formula (2.11) we obtain

$$
(4.5)\quad \begin{cases} F(x)=\dfrac{x^{-\alpha}}{\Gamma(\sigma+r)}\dfrac{\partial^r}{\partial x^r}\left[\dfrac{1}{\Gamma(m-\sigma)}\displaystyle\int_0^a (x-y)^{\sigma+r-1} f_1(y)\,dy\right. \\ \qquad\left.+\dfrac{1}{\Gamma(\sigma+p+k)}\displaystyle\int_a^x \dfrac{y^{\alpha-\sigma}(x-y)^{\sigma+r-1}}{(1-y/c)^{\delta+p+\sigma}}\,g_1(y)\,dy\right],\ a<x<c, \end{cases}
$$

where $f_1(y)$ and $g_1(y)$ are given by (3.9) and (3.12) and m, r, k are non-negative integers satisfying (3.3) and (4.3).

To evaluate $G(x)$ we first multiply equations (3.8) and (3.11) by $(1-y/c)^{\delta+p+\sigma}$ and differentiate them s times, then simplify the expressions using (2.7) and secure with the help of the orthogonality relation (2.10) an alternative expression for the coefficients A_n given by

$$
(4.6)\quad \begin{cases} A_n=\dfrac{(-1)^s}{\Gamma(m-\sigma)}\displaystyle\int_0^a y^{\alpha-\sigma+s}\,b_n(y)\dfrac{d^s}{dy^s}\left[y^{\sigma-\alpha}\left(1-\dfrac{y}{c}\right)^{\delta+p+\sigma} f_1(y)\right]dy \\ \qquad+\dfrac{(-1)^{s+k}}{\Gamma(\sigma+p+k)}\displaystyle\int_a^c y^{\alpha-\sigma+s}\,b_n(y)\dfrac{d^s g_1(y)}{dy^s}\,dy, \end{cases}
$$

where

$$
(4.7)\quad \begin{cases} b_n(y)=\dfrac{n!\,(2n+\alpha+\delta+p+1)\,(n+\alpha+\delta+p+1)}{c^{\alpha-\sigma+1}\,\Gamma(\alpha-\sigma+s+n+1)\,\Gamma(\delta+p+\sigma+n+1)} \\ \qquad P_n^{(\alpha-\sigma+s,\,\delta+p+\sigma-s)}\left(1-\dfrac{2y}{c}\right). \end{cases}
$$

Substituting A_n from (4.6) in (4.2), interchanging the order of integration and summation and evaluating the series with the help of formula (2.11), we get

$$
(4{\cdot}8)\quad \begin{cases} G(x)=\dfrac{(1-x/c)^{-\delta}}{\Gamma(s-\sigma-p)}\left[\dfrac{(-1)^s}{\Gamma(m-\sigma)}\displaystyle\int_x^a \dfrac{d^s/dy^s\left[y^{\sigma-\alpha}(1-y/c)^{\delta+p+\sigma} f_1(y)\right]dy}{(y-x)^{1+p+\sigma-s}}\right. \\ \qquad\left.+\dfrac{(-1)^{s+k}}{\Gamma(\sigma+p+k)}\displaystyle\int_a^c \dfrac{d^s/dy^s\left[g_1(y)\right]dy}{(y-x)^{1+p+\sigma-s}}\right],\ 0<x<a, \end{cases}
$$

where $f_1(y)$ and $g_1(y)$ are given by (3.9) and (3.12) and m, s, k are non-negative integers satisfying (3.3) and (4.3). For $p=0$, $m=s=1$, $r=k=0$, $c=1$ the results (4.5) and (4.8) are in complete agreement with those obtained by NOBLE [3]. Of course, in comparing (4.8) one has to make a trivial simplification in equation (3.12) of [3; p. 368].

To obtain the values of the series in (3.4) and (3.5) on the intervals where their values are not specified, we put $c=\delta$ in (4.5), (4.8), (3.9) and (3.12) and let $\delta \to \infty$. Using (2.3) to (2.5) we find

$$
(4.9)\quad \begin{cases} F(x) = \dfrac{x^{-\alpha}}{\Gamma(\sigma+r)} \dfrac{\partial^r}{\partial x^r} \left[\dfrac{1}{\Gamma(m-\sigma)} \displaystyle\int_0^a (x-y)^{\sigma+r-1} f_1(y)\, dy \right. \\ \qquad \left. + \dfrac{1}{\Gamma(\sigma+p+k)} \displaystyle\int_a^x y^{\alpha-\sigma} (x-y)^{\sigma+r-1} e^y g_1(y)\, dy \right], \quad a<x<\infty; \end{cases}
$$

$$
(4.10)\quad \begin{cases} G(x) = \dfrac{e^x}{\Gamma(s-\sigma-p)} \left[\dfrac{(-1)^s}{\Gamma(m-\sigma)} \displaystyle\int_x^a \dfrac{d^s/dy^s\,[y^{-\alpha} e^{-y} f_1(y)]}{(y-x)^{1+p+\sigma-s}}\, dy + \right. \\ \qquad \left. + \dfrac{(-1)^{s+k}}{\Gamma(\sigma+p+k)} \displaystyle\int_a^\infty \dfrac{d^s/dy^s\,[g_1(y)]\, dy}{(y-x)^{1+p+\sigma-s}} \right], \quad 0<x<a, \end{cases}
$$

where $f_1(y)$ and $g_1(y)$ are given by (3.17) and (3.18) and m, k, r and s are non-negative integers satisfying $m-\sigma>0$, $p+\sigma+k>0$, $\sigma+r>0$, $s-p-\sigma>0$. For $p=0$, $m=s=1$, $r=k=0$, the results (4.9) and (4.10) are in complete agreement with those obtained by LOWNDES in [4, p. 126].

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