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Possible theoretical limits on holographic quintessence from weak gravity conjecture

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Abstract

The holographic dark energy model is one of the important ways for dealing with the dark energy problems in the quantum gravity framework. In this model, the dimensionless parameter c plays an essential role in determining the evolution of the holographic dark energy. In particular, the holographic dark energy with $c \geqslant 1$ can be effectively described by a quintessence scalar-field. However, according to the requirement of the weak gravity conjecture the variation of the quintessence scalar-field should be less than the Planck mass, which would give theoretic constraints on the parameters c and Ω_{m0}. Therefore, we get the possible theoretical limits on the parameter c for the holographic quintessence model.

It has been realized firmly that our universe is experiencing an accelerated expansion at the present time, through the astronomical observations, such as observations of large scale structure (LSS) [1], searches for type Ia supernovae (SNIa) [2], and measurements of the cosmic microwave background (CMB) anisotropy [3]. The acceleration of the universe strongly indicates the existence of a mysterious exotic matter, namely the dark energy, which has large enough negative pressure and has been a dominative power of the universe (for reviews see e.g. [4]). The combined analysis of observational data suggests that the universe is spatially flat, and consists of approximately 70% dark energy, 30% dust matter (cold dark matter plus baryons), and negligible radiation. Although it can be affirmed that the ultimate destiny of our universe is determined by the feature of dark energy, we still know little about the nature of dark energy. However, one still can propose some candidates to interpret or describe its various properties. The most simple yet indispensable theoretical candidate for dark energy is the Einstein's cosmological constant λ (vacuum energy) [5] which has the equation of state $w_\lambda = -1$. However, as is well known, the cosmological constant scenario is always plagued with the two famous cosmological constant problems regarding why ρ_λ is much smaller than many known contributions to it and why it is comparable to the energy density of matter today.

Another candidate for dark energy is the energy density associated with dynamical scalar-field, a slowly varying, spatially homogeneous component. An example of scalar-field dark energy is the so-called quintessence [6], a scalar field ϕ slowly evolving down its potential $V(\phi)$. Provided that the evolution of the field is slow enough, the kinetic energy density is less than the potential energy density, giving rise to the negative pressure responsible to the cosmic acceleration. So far a wide variety of scalar-field dark energy models have been proposed. Besides quintessence, these also include phantom [7], K-essence [8], tachyon [9], ghost condensate [10] and quintom [11] amongst many. However, we should note that the mainstream viewpoint regards the scalar-field dark energy models as an low-energy effective description of the underlying theory of dark energy. In addition, other proposals on dark energy include interact-

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ing dark energy models [12], variable cosmological constant models [13], braneworld models [14], and Chaplygin gas models [15], etc.

Theoretical physicists have made lots of efforts trying to resolve the cosmological constant problems, but all these efforts seem to be unsuccessful. Of course the theoretical considerations have made some progress and are still in process. In recent years, many string theorists have devoted to shedding light on the cosmological constant or dark energy problems within the string theory framework. The famous Kachru–Kallosh–Linde–Trivedi (KKLT) model [16] is a typical example, which tries to construct metastable de Sitter vacua in the light of type IIB string theory. Furthermore, string landscape idea [17] has been proposed for shedding light on the cosmological constant problems based upon the anthropic principle and multiverse speculation. Another way of endeavoring to probe the nature of dark energy within the fundamental theory framework originates from some considerations of the features of the quantum gravity theory. It is generally believed by theorists that we cannot entirely understand the nature of dark energy before a complete theory of quantum gravity is available. However, although we are lacking a quantum gravity theory today, we still can make some attempts to probe the nature of dark energy according to some principles of quantum gravity. The holographic dark energy model [18] is just an appropriate example, which is constructed in the light of the holographic principle [19] of quantum gravity theory. That is to say, the holographic dark energy model possesses some significant features of the underlying theory of dark energy.

According to the holographic principle, the number of degrees of freedom for a system within a finite region should be finite and bounded roughly by the area of its boundary. In the cosmological context, the holographic principle will set an upper bound on the entropy of the universe. Motivated by the Bekenstein entropy bound, it seems plausible to require that for an effective quantum field theory in a box of size L with UV cutoff Λ, the total entropy should satisfy $S = L^3\Lambda^3 \leqslant S_{\rm BH} \equiv \pi M_{\rm pl}^2 L^2$, where $S_{\rm BH}$ is the entropy of a black hole with the same size L. However, Cohen et al. [20] pointed out that to saturate this inequality some states with Schwartzschild radius much larger than the box size have to be counted in. As a result, a more restrictive bound, the energy bound, has been proposed to constrain the degrees of freedom of the system, requiring that the total energy of a system with size L should not exceed the mass of a black hole with the same size, namely, $L^3\Lambda^4 = L^3\rho_\Lambda \leqslant L M_{\rm pl}^2$. This means that the maximum entropy is in order of $S_{\rm BH}^{3/4}$. When we take the whole universe into account, the vacuum energy related to this holographic principle is viewed as dark energy, usually dubbed "holographic dark energy". The largest IR cut-off L is chosen by saturating the inequality so that we get the holographic dark energy density

$$\rho_\Lambda = 3c^2 M_{\rm pl}^2 L^{-2}, \tag{1}$$

where c is a numerical constant (note that $c > 0$ is assumed), and as usual $M_{\rm pl}$ is the reduced Planck mass. It has been conjectured by Li [18] that the IR cutoff L should be given by the future event horizon of the universe

$$R_{\rm eh}(a) = a\int_t^\infty \frac{dt'}{a(t')} = a\int_a^\infty \frac{da'}{Ha'^2}. \tag{2}$$

Such a holographic dark energy looks reasonable, since it may simultaneously provide natural solutions to both dark energy problems as demonstrated in Ref. [18]. The holographic dark energy model has been tested and constrained by various astronomical observations [21–26]. For other extensive studies on the holographic dark energy, see e.g. Refs. [27,28].

The holographic dark energy scenario reveals the dynamical nature of the vacuum energy. When taking the holographic principle into account, the vacuum energy density will evolve dynamically. Though the underlying theory of dark energy is unavailable presently, we can, nevertheless, speculate on the underlying theory of dark energy by taking some principles of quantum gravity into account. The holographic dark energy model is no doubt a tentative in this way. Now, we are interested in that if we assume the holographic vacuum energy scenario as the underlying theory of dark energy, how the low-energy effective scalar-field model can be used to describe it. In this direction, some work has been done. The holographic versions of scalar-field models, such as quintessence, tachyon, and quintom, have been constructed [29–31]. In this Letter, we focus on the canonical scalar-field description of the holographic dark energy, namely the "holographic quintessence" [29].

It is generally believed that string theory is the most promising consistent theory of quantum gravity. By means of the KKLT mechanism [16] (see also [32]), a vast number of metastable de Sitter vacua can be constructed through the flux compactification on a Calabi–Yau manifold. These string vacua can be described by the low-energy effective theories. However, recently, it was realized that the vast series of semiclassically consistent field theories are actually inconsistent. These actually inconsistent effective field theories are viewed as located in the so-called "swampland" [33]. The self-consistent landscape is surrounded by the swampland.

Undoubtedly, it is an important mission to distinguish the landscape and the swampland. Vafa has proposed some criterion to the consistent effective field theories [33]. Furthermore, recently, it was conjectured by Arkani-Hamed et al. [34] that the gravity is the weakest force, which helps to rule out those effective field theories in the swampland. This conjecture is supported by string theory and some evidence involving black holes and symmetries [34] (for the other arguments in string theory to support this conjecture see also [35]). Arkani-Hamed et al. pointed out [34] that when considering the quantum gravity, the gravity and other gauge forces should not be treated separately. For example, in four dimensions a new intrinsic UV cutoff for the U(1) gauge theory, $\Lambda = gM_{\rm pl}$, is suggested, where g is the gauge coupling [34]. This conjecture was generalized to asymptotic dS/AdS background [36]. In [36], the weak gravity conjecture together with the requirement that the IR cutoff should be smaller than the UV cutoff leads to an upper bound for the cosmological constant. In addition, for the inflationary cosmology, the application of the weak gravity conjecture shows that the chaotic inflation model is in the swampland [37]. This conjecture even implies that the eternal inflation may not be achieved [38]. Furthermore, Huang conjectured [39] that the variation of the inflaton should be smaller than the Planck scale $M_{\rm pl}$, and this can make stringent constraint on the spectral index.

Naturally, the weak gravity conjecture can also be applied to the dark energy problem. This suggests that the variation of the quintessence field value ϕ should be less than $M_{\rm pl}$ [40]. This criterion may give important theoretical constraints on the equation-of-state parameter of quintessence models, and some of these constraints are even stringent than those of the present experiments [40]. In this Letter we shall investigate the possible theoretical constraints on the parameters of the holographic quintessence from the weak gravity conjecture.

First, we briefly review the holographic dark energy model. Since the spatial flatness is motivated by theoretical considerations (such as the inflationary theory) and astronomical observations, we consider a spatially flat universe filled with matter component $\rho_{\rm m}$ (including both baryon matter and cold dark matter) and holographic dark energy component ρ_Λ, thus the Friedmann equation reads

$$3M_{\rm pl}^2H^2=\rho_{\rm m}+\rho_\Lambda, \tag{3}$$

or equivalently,

$$E(z)\equiv\frac{H(z)}{H_0}=\left(\frac{\Omega_{\rm m0}(1+z)^3}{1-\Omega_\Lambda}\right)^{1/2}, \tag{4}$$

where $z=(1/a)-1$ is the redshift of the universe. Combining the definition of the holographic dark energy (1) and the definition of the future event horizon (2), we derive

$$\int_a^\infty\frac{d\ln a'}{Ha'}=\frac{c}{Ha\sqrt{\Omega_\Lambda}}. \tag{5}$$

The Friedmann equation (4) implies

$$\frac{1}{Ha}=\sqrt{a(1-\Omega_\Lambda)}\frac{1}{H_0\sqrt{\Omega_{\rm m0}}}. \tag{6}$$

Substituting (6) into (5), one obtains the following equation

$$\int_x^\infty e^{x'/2}\sqrt{1-\Omega_\Lambda}\,dx'=ce^{x/2}\sqrt{\frac{1}{\Omega_\Lambda}-1}, \tag{7}$$

where $x=\ln a$. Then taking derivative with respect to x in both sides of the above relation, we easily get the dynamics satisfied by the dark energy, i.e. the differential equation about the fractional density of dark energy,

$$\Omega'_\Lambda=-(1+z)^{-1}\Omega_\Lambda(1-\Omega_\Lambda)\left(1+\frac{2}{c}\sqrt{\Omega_\Lambda}\right), \tag{8}$$

where the prime denotes the derivative with respect to the redshift z. This equation describes the behavior of the holographic dark energy completely, and it can be solved exactly [18,21]. From the energy conservation equation of the dark energy, the equation of state of the dark energy can be given by [21]

$$w_\Lambda=-1-\frac{1}{3}\frac{d\ln\rho_\Lambda}{d\ln a}=-\frac{1}{3}\left(1+\frac{2}{c}\sqrt{\Omega_\Lambda}\right). \tag{9}$$

Note that the formula $\rho_\Lambda=\frac{\Omega_\Lambda}{1-\Omega_\Lambda}\rho_{\rm m0}a^{-3}$ and the differential equation of Ω_Λ (8) are used in the second equal sign.

The property of the holographic dark energy is mainly governed by the numerical parameter c. From Eq. (9), it can be easily found that the evolution of the equation of state satisfies $-(1+2/c)/3\leqslant w_\Lambda\leqslant -1/3$ due to $0\leqslant\Omega_\Lambda\leqslant 1$. Thus, the parameter c plays a significant role in the holographic evolution of the universe. When $c<1$, the holographic evolution will make the equation of state cross $w=-1$ (from $w>-1$ evolves to $w<-1$); when $c\geqslant 1$, the equation of state will evolve in the region of $-1\leqslant w\leqslant -1/3$.

Next, let us consider the quintessence scalar-field model. The quintessence scalar field ϕ evolves in its potential $V(\phi)$ and rolls towards its minimum of the potential, according to the Klein–Gordon equation $\ddot\phi+3H\dot\phi=-dV/d\phi$. The slope of the potential drives the rate of evolution while the cosmic expansion damps this evolution through the Hubble parameter H. The energy density and pressure are $\rho_\phi=\dot\phi^2/2+V$, $p_\phi=\dot\phi^2/2-V$, so that the equation of state of quintessence $w_\phi=p_\phi/\rho_\phi$ evolves in a region of $-1<w_\phi<1$. Usually, in order to make the universe's expansion accelerate, w_ϕ should be required less than $-1/3$. Nevertheless, it can be seen clearly that the quintessence scalar field cannot realize the equation of state crossing -1. Therefore, only the holographic dark energy in cases of $c\geqslant 1$ can be described by the quintessence [29].[1]

In fact, in the holographic scenario, the value of c should be determined by cosmological observations. However, current observational data are not precise enough to determine the value of c very accurately. An analysis of the latest observational data, including the gold sample of 182 SNIa, the CMB shift parameter given by the 3-year WMAP observations, and the baryon acoustic oscillation (BAO) measurement from the Sloan Digital Sky Survey (SDSS), shows that the possibilities of $c>1$ and $c<1$ both exist and their likelihoods are almost equal within 3 sigma error range [25]. Therefore, neither quintessence feature nor quintom one can be ruled out by observational data presently available. In [25], the fit values for the model parameters with 1-σ errors are $c=0.91^{+0.26}_{-0.18}$ and $\Omega_{\rm m0}=0.29\pm 0.03$ with $\chi_{\rm min}=158.97$. Clearly, the range of c in the 1-σ error, $0.73<c<1.17$, is not capable of ruling out the probability of $c>1$; this conclusion is somewhat different from those derived from previous investigations using earlier data. In previous work, for instance [23,24], the 1-σ range of c obtained can basically exclude the probability of $c>1$ giving

[1] Apparently, the quintessence model is consistent with the second law of thermodynamics. In the holographic dark energy model, the entropy of the whole system is described by $S=\pi M_{\rm pl}^2R_{\rm eh}^2$. To satisfy the second law of thermodynamics, one requires that $\dot R_{\rm eh}\geqslant 0$, which leads to $c\geqslant\sqrt{\Omega_\Lambda}$ (for the general case in non-flat space, see [22]). For the quintessence model, $w\geqslant -1$, this together with Eq. (9) also leads to $c\geqslant\sqrt{\Omega_\Lambda}$. Furthermore, since the maximum of Ω_Λ is 1, we thus obtain the condition $c\geqslant 1$ for the quintessence-like behavior realization of the holographic dark energy.

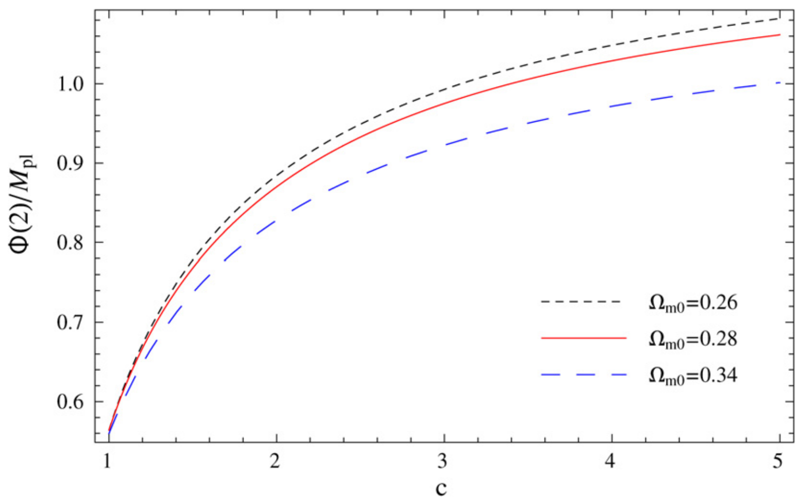

Fig. 1. The relationship between $\phi(z)$ (at $z = 2$) and c. Selected curves correspond to $\Omega_{m0} = 0.26$, 0.28 and 0.34, respectively.

rise to the quintessence-like behavior, supporting the quintom-like behavior evidently.[2] Though the present result (in 1-σ error range) from the analysis of the up-to-date observational data does not support the quintom-like feature as strongly as before, the best-fit value ($c = 0.91$) still exhibits the holographic quintom characteristic. However, the cases of $c < 1$ will bring theoretical problems: (i) This will lead to dark energy behaving as a phantom eventually, which violates the weak energy condition of general relativity, and the Gibbons–Hawking entropy will thus decrease since the event horizon shrinks, which violates the second law of thermodynamics as well. (ii) The quantum instability may often be encountered in quintom models when the $w = -1$ crossing happens. (iii) When the future event horizon as the IR cut-off becomes shorter than the UV cut-off within a finite time in the future, the definition of the holographic dark energy will break down. Consequently, from the theoretical perspective, the holographic dark energy with $c \geqslant 1$ is more reasonable. On the whole, since the data analysis cannot rule out the possibility of $c \geqslant 1$ completely, the cases of $c \geqslant 1$ are worth investigating in detail. In order to describe the holographic dark energy with the quintessence scalar-field (the low-energy effective theory), we in this Letter restrict $c \geqslant 1$ for the holographic dark energy model.

According to the forms of quintessence energy density and pressure, one can easily derive the scalar potential and kinetic energy term as

$$\frac{V(\phi)}{\rho_{c0}} = \frac{1}{2}(1 - w_\phi)\Omega_\phi E^2, \tag{10}$$

$$\frac{\dot{\phi}^2}{\rho_{c0}} = (1 + w_\phi)\Omega_\phi E^2, \tag{11}$$

where $\rho_{c0} = 3M_{pl}^2 H_0^2$ is today's critical density of the universe. Imposing the holographic nature (with $c \geqslant 1$) to the quintessence, the energy density of quintessence is needed to satisfy the requirement of holographic principle, i.e., we should identify ρ_ϕ with ρ_Λ. Then, the quintessence field acquires the holographic nature, namely, E, Ω_ϕ and w_ϕ are given by Eqs. (4), (8) and (9). Without loss of generality, we assume $V' > 0$ and $\dot{\phi} < 0$ in this Letter. Then, the derivative of the scalar field ϕ with respect to the redshift z can be given by

$$\frac{\phi'}{M_{pl}} = \frac{\sqrt{3(1 + w_\phi)\Omega_\phi}}{1 + z}. \tag{12}$$

Consequently, we can easily obtain the evolutionary form of the field by integrating the above equation

$$\phi(z) = \int_0^z \phi'\, dz, \tag{13}$$

where the field amplitude at the present epoch ($z = 0$) is fixed to be zero, namely $\phi(0) = 0$. In what follows, we use the criterion $\phi(z)/M_{pl} \leqslant 1$ to give the possible theoretical constraints on the values of c and Ω_{m0}.

First, one can solve Eq. (8) numerically and plot $w_\Lambda(z)$ (see Fig. 1 in Ref. [29]). From that figure, one can see that larger value of c makes the value of w_Λ relatively larger. This makes the amplitude of field $\phi(z)$ larger if the value of c becomes larger (see Fig. 3 in Ref. [29]). For making this point more clear, we plot $\phi(z = 2)$ versus c in Fig. 1, where selected curves correspond to $\Omega_{m0} = 0.26$, 0.28 and 0.34, respectively. Thus, if c becomes large, the value of ϕ will become large (see Fig. 1) and in some cases, it may disobey the criterion that the variation of quintessence scalar-field should be less than M_{pl}. Therefore, the criterion $|\Delta\phi(z)| = \phi(z) \leqslant M_{pl}$ is able to give important theoretical constraints on the values of c and Ω_{m0}.

Generically, the dark energy component is negligible in early times of the universe. Hence, one should confirm when the dark energy starts to operate in the universe. In general, the redshift at $z \sim 2$ can be viewed as the onset of dark energy evolution, since at which dark energy begins to take over the mantle of

[2] In [23], the joint fitting of SNIa + CMB + LSS for the holographic dark energy model gives the parameter constraints in 1-σ: $c = 0.81^{+0.23}_{-0.16}$, $\Omega_{m0} = 0.28 \pm 0.03$, with $\chi^2_{min} = 176.67$. In [24], using the f_{gas} values provided by *Chandra* observational data (the X-ray gas mass fraction of 26 rich clusters), the 1-σ fit values for c and Ω_{m0} are given: $c = 0.61^{+0.45}_{-0.21}$ and $\Omega_{m0} = 0.24^{+0.06}_{-0.05}$, with the best-fit chi-square $\chi^2_{min} = 25.00$.

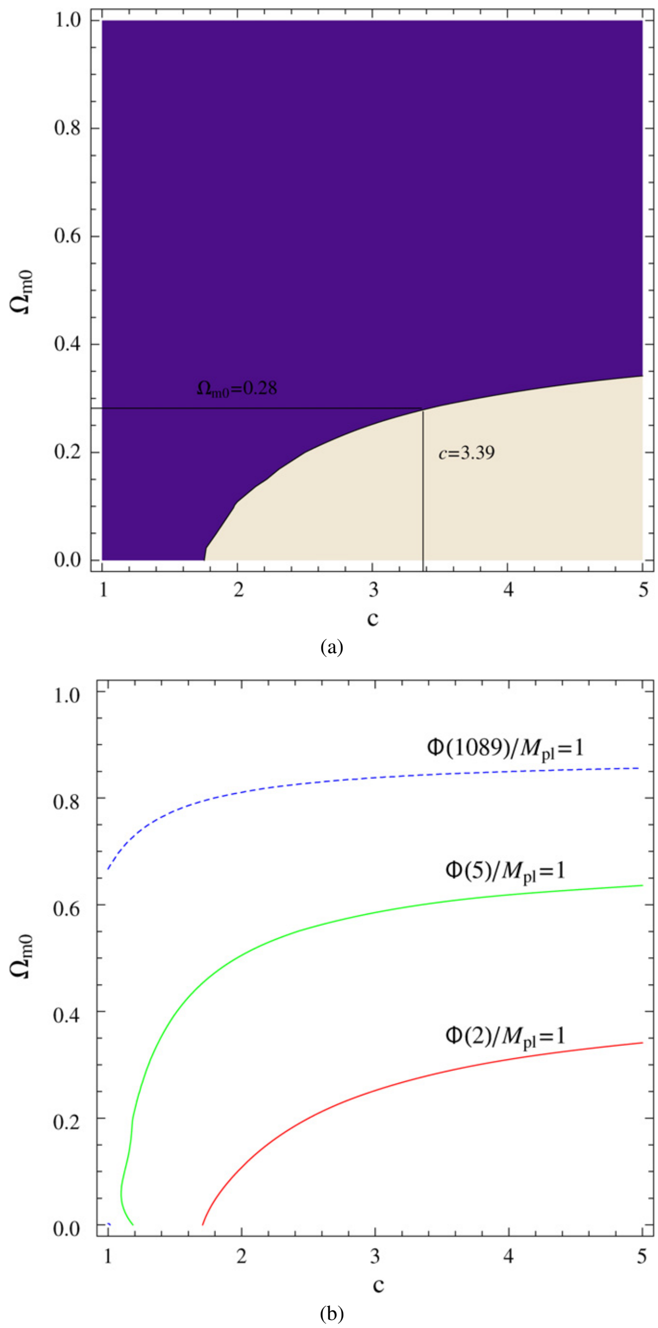

Fig. 2. Panel (a): Constraint on the parameter-space of the holographic quintessence from the theoretical criterion $\phi(z=2)/M_{\rm pl} \leqslant 1$. Upper shaded area represents the allowed region. Panel (b): Curves corresponding to $\phi(z)/M_{\rm pl} = 1$ in the c–$\Omega_{\rm m0}$ parameter-plane, where z is taken to be 2, 5, and 1089, respectively.

matter component (albeit at that time matter component still dominates the evolution of the universe). Therefore, we can set $z = 2$ as the onset of dark energy evolution. Of course, for the sake of safety, we can also take, say, $z = 5$ as the onset of dark energy evolution. An example is shown in the panel (a) of Fig. 2, where $z = 2$ is taken. This figure shows the constraints for the c–$\Omega_{\rm m0}$ parameter-space of the holographic quintessence from the theoretical criterion $\phi(z)/M_{\rm pl} \leqslant 1$, where the borderline is set by $\phi(z=2)/M_{\rm pl} = 1$, and the allowed region of the parameter-space is represented by the shaded area. For comparison, we also plot the curves corresponding to $\phi(z=5) = M_{\rm pl}$ and $\phi(z=1089) = M_{\rm pl}$ in the c–$\Omega_{\rm m0}$ plane in the panel (b) of Fig. 2, which shows that the borderline will get an upper shift when enlarging the redshift, leading to the allowed region shrinks. Note that the case of $z = 1089$ is not an appropriate example because at the time of CMB formation the universe is dominated by non-relativistic matter and the dark energy is totally negligible.

The theoretical limit of c is shown explicitly in the panel (a) of Fig. 2. When fixing the value of $\Omega_{\rm m0}$, the upper bound of c can be read from this figure directly. For example, choosing $\Omega_{\rm m0} = 0.28$ which is favored by the current observations, the criterion $\phi(2)/M_{\rm pl} \leqslant 1$ directly leads to $c_{\rm max} = 3.39$ which is the theoretical limit of the parameter c. However, since the relationship between $c_{\rm max}$ and $\Omega_{\rm m0}$ is derived from Eq. (13) which is an integral formula making the relation $c_{\rm max}(\Omega_{\rm m0})$ difficult to identify, we should furthermore find an empirical relation between $c_{\rm max}$ and $\Omega_{\rm m0}$. Thus, we output the data along the curve $\phi(z=2)/M_{\rm pl} = 1$ and fit them with the elementary functions, then we obtain

$$c_{\rm max}(\Omega_{\rm m0}) = 15341.8 e^{\Omega_{\rm m0}} - 15340.1 - 15342.9\Omega_{\rm m0} - 7609.9\Omega_{\rm m0}^2 - 2875.4\Omega_{\rm m0}^3,$$

which is very easy for us to operate. For instance, when substituting $\Omega_{\rm m0} = 0.24$ in it, it gives $c_{\rm max} = 2.77$; when substituting $\Omega_{\rm m0} = 0.28$, it gives $c_{\rm max} = 3.34$. Therefore, this empirical function $c_{\rm max}(\Omega_{\rm m0})$ is very convenient for us to get the theoretical limit of the parameter c. Likewise, we can also get an empirical relation $c_{\rm max}(\Omega_{\rm m0})$ for the $z = 5$ case. We do not exhibit this case explicitly here, but gives an output point as example: the input $\Omega_{\rm m0} = 0.28$ gives $c_{\rm max} = 1.27$. The numerical fitting curves with data points are shown in Fig. 3.

To summarize, in this Letter we investigate the possible theoretical limits on the parameter c of the holographic quintessence. We adopt the perspective that the scalar-field model is an effective description for the underlying theory of dark energy. In the holographic dark energy model, the equation of state with $c \geqslant 1$ evolves within the range $-1 \leqslant w \leqslant -1/3$, so it looks like a quintessence. Quintessence scalar-field can thus be used to effectively describe the holographic dark energy with $c \geqslant 1$. For quintessence scalar-field, the requirement (from the weak gravity conjecture) that the variation of the field should be less than $M_{\rm pl}$ will set a theoretical bound on the model. So, in this Letter, we tested the holographic quintessence model using this criterion and obtained the theoretical limits on the parameter c for the model. Anyway, the theoretical limits discussed in this Letter is only a possibility. The requirement that the variation of the canonical quintessence field minimally coupled to gravity is less than the Planck scale may arise from the consistent theory of quantum gravity. In this sense, the results derived in this Letter can, to some extent, be viewed as the prediction of quantum gravity. Though the constraints on the parameter c are rather loose, the possible theoretical limits of the holographic quintessence model are worth investigating.

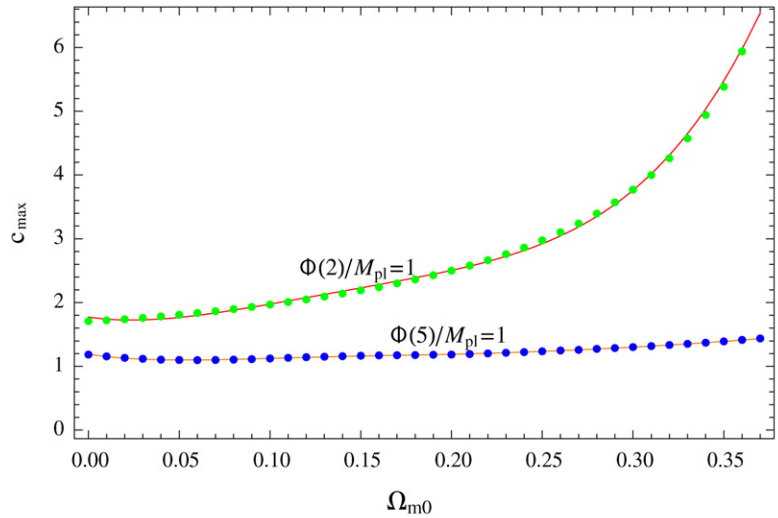

Fig. 3. The empirical relations between $c_{\max}$ and Ω_{m0}. Points are generated from the equation $\phi(z)/M_{pl} = 1$, where z is taken to be 2 (green points) and 5 (blue points), respectively, and curves are the numerical fitting results. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)

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