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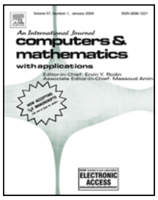

Modeling cooperative and competitive behaviors in emergency evacuation: A game-theoretical approach

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ABSTRACT

A game-theoretical model to study evacuees' cooperative and competitive behaviors during an emergency evacuation is proposed. The model integrated with evacuation dynamics model determines the density of cooperative and competitive evacuees and their related evacuation times. Computer simulation results show that (1) as urgency of evacuation increases, cooperation among evacuees' decreases; (2) in an emergency situation, individual hyper-rationality among evacuees diminishes evacuation efficiency; (3) the imitation effect enhances cooperation among evacuees, yet reduces evacuation efficiency. This study provides a methodological pattern to research crowd behaviors in emergency evacuation.

1. Introduction

Of increasing importance in the field of emergency management is the study of evacuation dynamics. Particularly, researchers are looking for methods for a better understanding of the crowd dynamics of evacuees. As public facilities have grown over time in terms of their max capacities, the challenge of keeping evacuation times as low as possible has likewise increased. The key to this problem is a growing body of knowledge in regard to crowd dynamics [1].

Regarding the research of crowd dynamics, there are two fundamentally distinct approaches [2]. The first approach views the crowd as a whole, i.e., interpreting crowd flow as fluid flow; this is better suited for large-scale crowds with relatively high density [3]. While such an approach is promising for applications [4], it cannot represent the dynamics of individual heterogeneous behaviors. The second approach treats pedestrians as discrete individuals, mainly utilizing cellular automata (CA) models [5–8] and social force models [9,10]. While these models can account for individual factors to reproduce an evacuation situation, as a particle-based model in physics [9,10], they are usually used to focus on the movement behaviors of the particle-like individuals.

During emergency evacuations evacuees at bottleneck often exhibit competitive and cooperative behaviors [8]. These behaviors, varying in different individuals, have great impact upon evacuation times. Rational evacuees will choose either competitive or cooperative behaviors in response to other evacuees' choices. It is straightforward to describe such a process by using a form of mathematics known as the game theory. Game theory has been widely used in the research of various scientific disciplines, from biological systems to economic and social systems [11,12]. With the help of game theory, researchers can conduct extensive studies on the pedestrian and evacuation dynamics [1,13–17], and therefore, discover means to enhance evacuation efficiency and ultimately save more lives. However, game-theoretical models are still limited in the study of the crowd's behaviors in evacuation process.

The game-theoretical approaches have been employed to study the routine choice problems of pedestrians and crowds [14,15] and was briefly introduced in a study on evacuation crowd flow rates [16]. Recently, Dogbé [17] proposed

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a macroscopic preliminary methodological pattern of crowd behavior dynamics in terms of differential game theory and mean-field limit approaches. And Zheng et al. [13] established microscopic framework to research crowd dynamic during evacuation processes based on evolutionary game theory. Yet, to our knowledge, it has not been found that any researches on the individual cooperative and competitive behaviors in the process of evacuation by game-theoretical approach are performed. Therefore, the present study is an attempt to establish a framework on modeling the competitive and cooperative behaviors among evacuees by game theory. A unified approach is proposed, integrating a game-theoretical model with a cellular automation model of evacuation dynamics, simulating the motions of crowds based on their evacuees' competitive and cooperative strategies.

This paper is organized as follows: Section 2 introduces the notion of evacuees' games characterized by CA evacuation dynamics and evacuees' strategy. In Section 3, results of computer simulations of the evacuees' game are presented, the evacuation dynamics are considered, and its implications for evacuation are discussed. Section 4 presents comparisons with the extended CA model with friction effect [7,8] and discussions on why the present model is able to go beyond those obtained with the model including friction. Lastly, research results are summarized regarding their implications for improving crowd evacuations.

2. Model

2.1. Evacuees' game

Suppose two evacuees can chose to either compete (D) or cooperate (C) when attempting to occupy a desired position. Such conflict between the evacuees leads to a 2×2 game, as shown in Fig. 1. The payoff of the game is as follows: (1) if both evacuees are willing to cooperate, they will each reach their desired position, and subsequently, each will obtain utility R; (2) if both evacuees choose to be in a state of competition, each will be locked at the initial position, and each will, in turn, obtain utility P; (3) When one evacuee chooses to be cooperative and the other, competitive, the cooperative evacuee remains motionless, obtaining utility S, while the competitive evacuee attains his or her desired position, utility T. If the reward for attaining the desired position is 1, the cost of competition will be c. Thus, the payoff of the game is such that $R = 1/2, S = 0, T = 1 - c$, and $P = -c$.

If $0 < c < 1/2$, then $T > R > S > P$ and $T + S < 2R$, and the game is therefore deemed a *Chicken Game*. The corresponding pure Nash equilibrium of the above-mentioned game is (C, D) or (D, C), and the mixed Nash equilibrium for the densities of C and D are $2c$ and $1-2c$, respectively. As c approaches 0, the density of cooperation is approximately 0. Being that the *Chicken game* represents a kind of social dilemma [18], in this study, it is referred to as *Evacuee's Dilemma*, which is a game in the case that c is relatively small, indicating that the evacuation crowd is trapped in competitive conditions.

If $1/2 < c < 1$, then $R > T > S > P$, and the game is deemed a *Harmony Game*. The pure NE is (C, C), indicating the corresponding mixed NE for the density of cooperation is 1. The same Nash solutions can be obtained in the case that $c > 1$. In these kinds of games evacuees behave orderly.

When evacuation is not the result of imminent danger, evacuees will maintain their distance from others [9], indicating the evacuees consider the cost of competition too large to compete with others, which leads to a *harmony game*. However, when imminent danger is present, evacuees will neglect the cost of competition after comparing it with the cost of their life [19]. In this situation, an *evacuees' dilemma* game will occur.

For this research, the cost of competition, c, as the evacuees' subjective utility, reflects the degree of emergency for a given evacuation. In the case where $c = 0$, the degree of emergency is at its highest. This is discussed in more detail in the following section.

During an evacuation, it often occurs that more than two persons strive for the same desired position, thus the game should be extended to a N-players game ($N > 2$). The configuration of the game is the same as in the case of the two-player. If every evacuee in the game is cooperative, the evacuees can obtain the desired position, and in turn, each will obtain payoff $1/n$. Should one evacuee be willing to compete, the benefit for this person is $1 - c$, and 0 for others. If there are at least two evacuees in a competitive state, they will receive $-c$, while others obtain 0. Thus the unified expression of the utility function can be defined as follows:

$$u_i(a_i, a_{-i}) = \begin{cases} 1/n, & \text{if } h(a) = 0, \\ (1-c) \cdot I(a_i), & \text{if } h(a) = 1, \\ -c \cdot I(a_i), & \text{if } h(a) > 1 \end{cases} \tag{1}$$

where $h(a)$ denotes the number of competitive evacuees; $I(x)$ is an indicating function, where in if strategy x is competition, $I(x) = 1$, otherwise, 0.

2.2. Game dynamic

A cellular automata (CA) model is used to describe the evacuation dynamics. In the model, the evacuation space is discretized into small cells, which can either be vacant or occupied by exactly one evacuee. Each of these evacuees can move to one of their unoccupied neighboring cells (4 directions: up, down, left, or right). Suppose that in the CA model the

	C	D
C	R	S
D	T	P

Fig. 1. Payoff matrix of evacuee's game with two players.

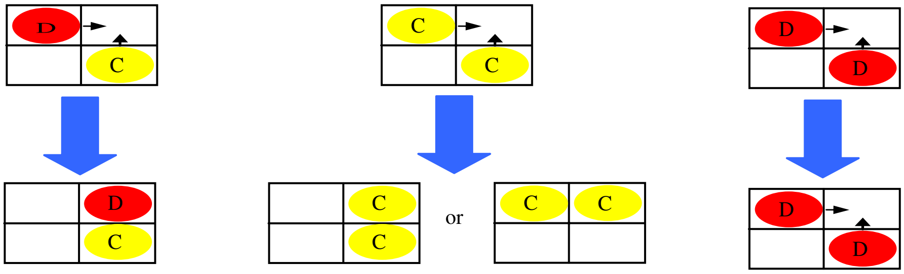

Fig. 2. Movements of evacuees due to different strategies (the red circle represents the competitive evacuee, and yellow, the cooperative). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

evacuees can determine their next move according to transition probability $p_{i\to j}$, determined by the static floor field (FF) [6],

$$p_{i\to j} = \frac{\exp(-k_s(S_j - S_i))}{\sum\limits_{j'\in\Omega^*} \exp(-k_s(S_{j'} - S_i))} \tag{2}$$

where the static FF Si of cell i can be calculated by its nearest Manhattan distance. The dynamic floor field can be neglected in the case of a single exit situation [8]. The parameter, k_s in the calculation of floor field is considered a speed control parameter [20], measuring the average efficient speed. In this research, the parameter is fixed at 10, which ensures the evacuees are utilizing their greatest efforts to reach the exit during a given emergency evacuation.

While synchronously updating the position of each evacuee in the CA model, a situation will occur in which several evacuees wish to occupy one cell at one time step, as shown in Fig. 2. It is assumed that attaining the desired position depends on each evacuee's strategy. Results correspond to the *Evacuees' game* discussed above.

Suppose that the evacuees have inductive rationality rather than perfect, logical, deductive rationality [21]. Evacuees will follow others' strategies or use their own judgment and the updating rules of the players' strategies are based on Fermi-type distribution [22].

As evacuees choose to follow others during a game, herding behavior will occur in the evacuation process. Similarly, evacuees are likely to *follow others' strategies* according to the current payoffs of a given player in a game. During the current game, the probability that evacuee, i chooses another evacuee, j, at random and adopts j's strategy is as follows

$$W[s_j \to s_i] = \frac{1}{1 + \exp[r(U_i - U_j)]} \tag{3}$$

where U_i represent the payoff of person i at the current game, and r as a parameter represent evacuee's rationality when judging the outcome of a strategy.

Evacuees may also choose a strategy by *using their own judgments*, i.e. according to their perception of the success or failure of previous strategies from what they have experienced in the past. The transition probability of strategy x can be written as the following equation

$$W_i[y \to x] = \frac{1}{1 + \exp[r(F_i(x) - F_i(y))]} \tag{4}$$

where r denotes an evacuee's rationality, as it does in Eq. (2), and $F_i(x)$ is the average accumulated payoff over time for an evacuee's strategy x. Although only one strategy can obtain the real payoff in a game, the other strategy can be counted in terms of virtual payoff. The accumulated payoff for $F_i(x)$ is the sum of real payoff and virtual payoff.

Parameter r in (3) and (4) represents the rationality of evacuees. The parameter r is viewed as a rationality parameter [13,23], which means that the larger this parameter, the higher the evacuees' rationality. In this study, rationality parameter can be used as an index to demonstrate the influence of rationality on the evacuation efficiency.

To determine how evacuees choose strategies, (3) and (4) are combined, and a new parameter α is introduced as follows

$$p_i(y \to x) = \begin{cases} W[s_j \to s_i], & \text{with probability } \alpha \\ W_i[y \to x], & \text{with probability } 1-\alpha \end{cases} \tag{5}$$

where α serves as a copying parameter to indicate the likelihood to which one evacuee follows another's strategy and $1-\alpha$ is the probability to evacuees using ones' own judgments. Therefore, α is a measurement of evacuees' imitation behaviors.

3. Computer simulation and results

The simulation of evacuation process is implemented based on evacuation dynamics given by Eq. (2) and game dynamics from Eq. (5). Movement rules of evacuees are determined by the CA model, and the rules that evacuees transfer their behaviors between cooperative and competitive are determined by the game-theoretical model. The proposed principles "following others' strategies" and "using ones' own judgments" lead to Eqs. (3) and (4), respectively, and (3) and (4) compose Eq. (5), providing the final rule for evacuees to choose strategies. The simulation procedure for the evacuation process is as follows

Step 1: Evacuees' initial positions and strategies are determined. At this time, all evacuees are distributed randomly in a space with $L \times W$ sites, with density of p_c. Furthermore, initial strategies of the all evacuees are set.

Step 2: If all evacuees have escaped the space, the simulation is terminated; otherwise, the subsequent position for each remaining evacuee is determined in accordance with the mobility rule in Eq. (2).

Step 3: Positions of evacuees not in conflict are updated. If there are several evacuees compete to move into one position, a conflict occurs; otherwise step 2 is repeated.

Step 4: After a conflict has occurred the players' behaviors are determined by their strategies determined in the previous conflict, and their new strategies are updated in accordance strategy rule in Eq. (5).

Step 5: New positions of evacuees, according to *evacuees' game* and their strategies, are updated in each game, and step 2 is repeated.

The process of establishing simulations settings for the evacuation of a large room with a single exit are presented in the following. First, a space is divided into a grid size of 63 × 63 sites, 0.40 m × 0.40 m, with one site in the middle of one wall representing the exit as shown in Fig. 3. Evacuee speed is assumed to be 1.33 m/s and the time step size, 0.3 s [13]. The evacuees are then distributed randomly, each occupying just one site, before they attempt to exit the room. At this time, the initial density of evacuees is 0.03, therefore there are 112 evacuees. And the initial states of the evacuees are all set as cooperative. Results in this study using various parameters are obtained by averaging over 100 different initial evacuation distributions at random.

The evacuation process can be interpreted as a series of evacuees' games. Each evacuee updates his or her strategy according to Eq. (5). Upon beginning the evacuation, each evacuee is assumed as cooperative, and the final chosen strategies of crowds upon exiting are the results of such process. As for different rationality parameters ($r = 0.2$, 1 and 10) and copying parameters ($\alpha = 0.1$, 0.5 and 0.9), the evacuees' cooperation frequency and evacuation time varies with the cost of competition as shown in Fig. 4.

From Fig. 4(a), (c), (e), it can be seen that cooperation frequency increases with that of cost. Even as the cost approaches zero, there will be a 25%–80% cooperation frequency in the evacuation crowd. This reflects that in situations where the degree of emergency is high, evacuees will still exhibit cooperation to some extent and high rationality inhibits cooperative behaviors. As cost increases, the crowd with high rationality will become cooperative completely. As shown in Fig. 4(b), (d), (f), evacuation times will reduce significantly as cost increases, reflecting that evacuations times in response to imminent danger are less than those where the degree of emergency is low.

Fig. 5 is a plot of the relationship between evacuation time and cooperation frequency from simulations with different degree of emergency. It shows that for different evacuation crowds, higher cooperation frequency will lead to shorter evacuation times, reflecting that an orderly evacuation process will ensure highly efficient evacuations. Only for the situation that the cooperation frequency approaches 1, the evacuation time of different scenarios will converge to 73 s in the simulations. The different points in Fig. 5 correspond to various cooperation frequencies and evacuation times. Taking the points *A* and *B* in Fig. 5 as an example, one can see that point *A* (a crowd with $r = 10$ and $\alpha = 0.9$), as compared with point *B* (a crowd with $r = 10$ and $\alpha = 0.1$), has the same cooperation frequency, 44%. However, there is a large gap between point *A* and *B* in terms of their evacuation times, i.e. 211.5 s for point *A*, 83.2 s for point *B*. The above results demonstrate cooperation frequency cannot fully determine evacuation time, which is contrary to the previous study by the FF model with friction effects [7,8]. Fig. 5 illustrates the emergence of complex evacuation behaviors in a real evacuation process, and it also shows that the previous FF model including friction effects has its limitation on studying cooperative and competitive behaviors of evacuees without considering the dynamics of individuals' the strategies or behaviors.

For evacuations with the degree of emergency is at its highest, i.e. $c = 0$, the copying parameters are set at three different values, 0.1, 0.5 and 0.9. From Fig. 6, one can see that as the rationality parameter, r approaches 0, the cooperation frequencies approaches 55.5%, 61.0% and 83.0%, respectively, and evacuation time, 91.0, 91.7 and 89.3 s under these three copying parameters. It can also be found that as the rationality parameter, r approaches 10 (enough large value), the frequency of cooperation reduces to 26.6%, 29.6% and 45.9%, respectively and corresponding evacuation times increase to 129.1, 137.7

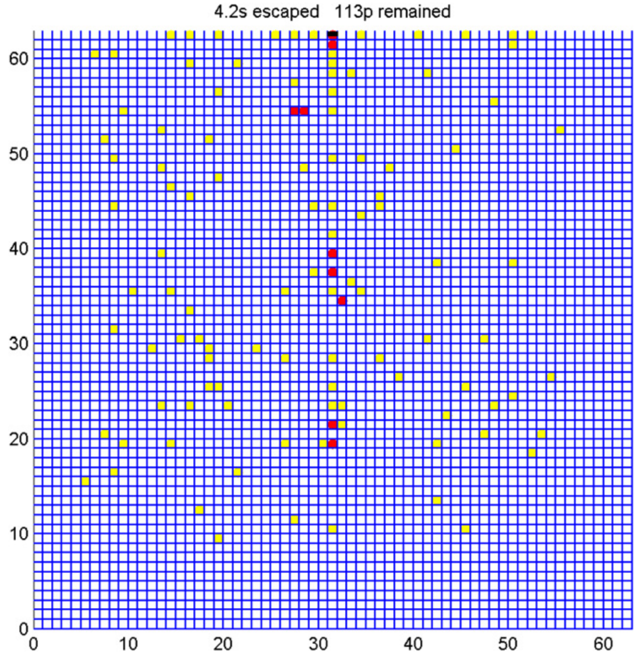

Fig. 3. Snapshot of evacuees' distribution in an evacuation simulation at 4.2 s with $c = 0$, $r = 1$, and $\alpha = 0.5$.

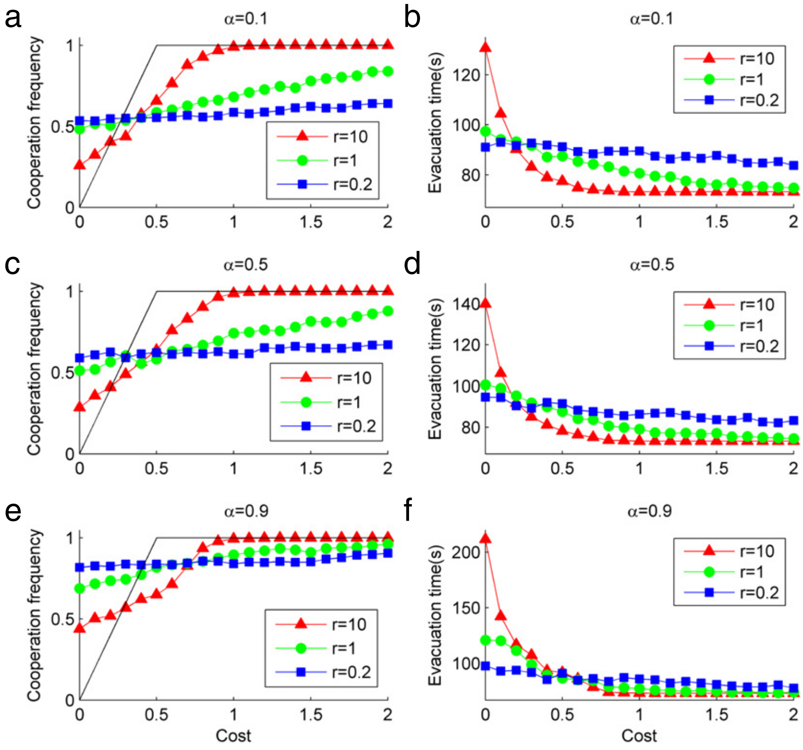

Fig. 4. Cooperation frequency (a), (c), (e) and corresponding evacuation time (b), (d), (f) as functions of the cost of competition.

and 212.9 s. Therefore, during times of imminent danger, hyper-rationality will inhibit cooperation among evacuees and delay evacuation time. This kind of local and individual hyper-rationality is the likely cause of crowd's congestion at the bottleneck. In addition, the fluctuation of cooperation frequency in Fig. 7(a) is relatively stable while the fluctuation of the corresponding evacuation time becomes larger as rationality parameter increases, which shows that in the evacuation crowd with higher rationality appears higher uncertainty in the evacuation process.

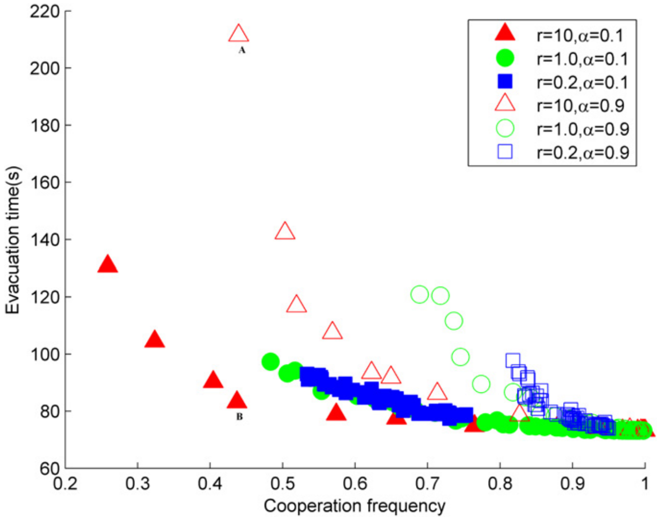

Fig. 5. Plots of evacuation time against cooperation frequency.

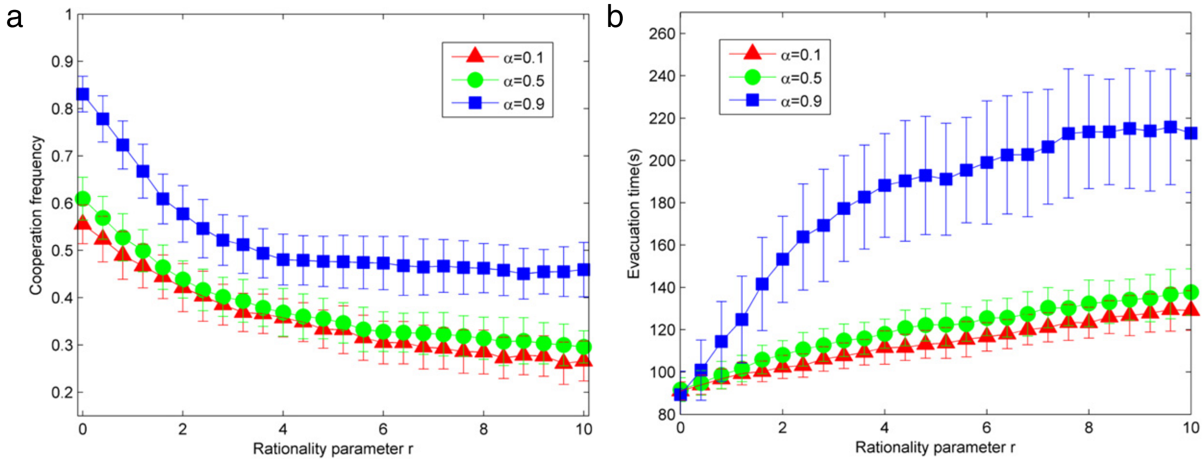

Fig. 6. (a) Cooperation frequency and (b) evacuation time as functions of rationality parameter r with copying parameter $\alpha = 0.1$, 0.5 and 0.9. Error bars correspond to one standard error.

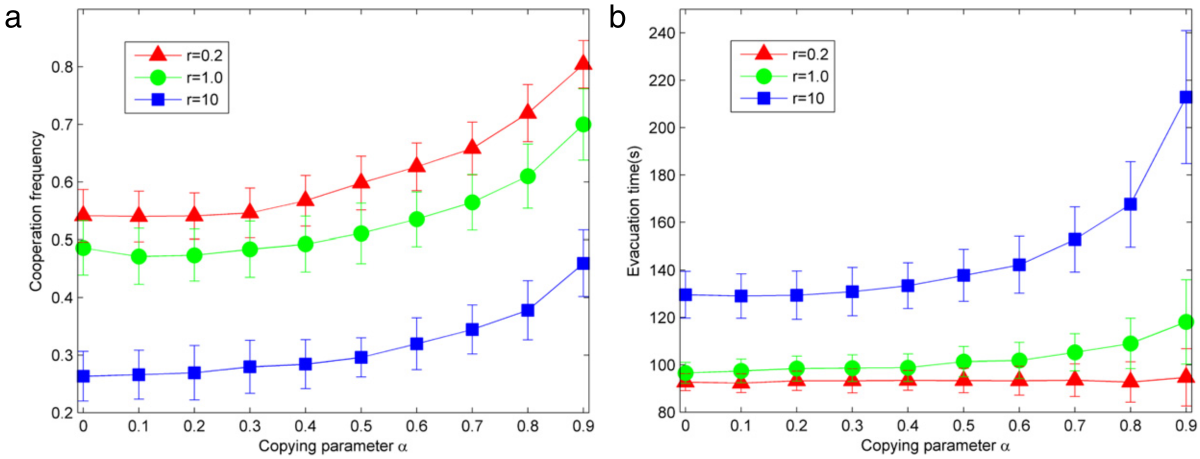

Fig. 7. (a) Cooperation Frequency and (b) evacuation time as functions of copying parameter with rationality parameter $r = 10$, 1.0, and 0.2. Error bars correspond to one standard error.

Setting rationality parameter r at 10, 1, and 0.2, Fig. 7 shows the relationship between cooperation frequency and copying parameter. From Fig. 7(a) one can see that in crowds with different rationality parameter, cooperation density will increase with the increase of copying parameter, which indicates that imitation can lead to cooperation during the evacuation process. It can be seen in Fig. 7(b) that evacuation time and its fluctuation will increase with the copying parameter as well, especially for crowds with high rationality. It shows that higher cooperation frequency and longer evacuation time occur at the same time as the imitation effect increases. Such results are contrary to common intuition that higher level of cooperation leads to shorter evacuation time. The results show that evacuation crowds with more cooperation are readily influenced by minor competitive evacuees in terms of their evacuation efficiency under high imitation effect.

4. Comparisons and discussions

In this research the FF model is used to simulate movements of evacuees. Applying game-theoretical approach is a mindset more than an extended model. And it is straightforward to applying game dynamics to other evacuation dynamics model such as continuous dynamic system like social force model [9,10]. This work is aimed at providing a methodological framework to research crowd behaviors in emergency evacuation. For situations with high degree of emergency it is difficult to collect the related experimental data, and it also has ethical issues in performing such experiments. Therefore, it is important and necessary to study evacuation process by mathematical models and computer simulations. However, comparisons with other existing evacuation model and discussions on the present model's advantage still need to be presented.

The floor field model, as one of the standard models of evacuation dynamics has been extended by taking into account evacuees' cooperative and competitive behavior through friction effect [7,8]. Although the extended model with friction effect seems to behave similar to this model, it is essentially different from the game-theoretical approach. The focus of this work is not the evacuation time like the FF models with friction constant, but the change of cooperative or competitive states which is influenced by the factors that evacuees' rationality, the degree of emergency and degree of evacuees' imitation. The dynamics of the behaviors cannot be considered in the FF model with friction constant. Indeed an emergency evacuation is a dynamic process and the choice of cooperative or competitive behavior is determined by varying mental states of evacuees and external environments. In the present model, the mental state is determined by rules given in Eqs. (3)–(5), and the external environment is characterized by the degree of emergency in terms of cost. These features have not been contained in the extended FF model.

In the model including friction effect, by a fiction parameter μ the basic model is extended to describe clogging effects between crowds [7]. As two or more individuals attempt to move to the same target cell, one of the evacuees can move to the desired cell with probability $1 - \mu$. From a perspective of mean-field approximation, this friction parameter μ seems equal to the probability that two competitive evacuees meet in the case of 2-players game, and therefore the "cooperation frequency" in the FF model can be defined as $1 - \sqrt{\mu}$. Nevertheless this mean-field approximation method cannot be used unless the state of evacuation behaviors reaches equilibrium. Since in a real evacuation processes an equilibrium state cannot be reached in a short time, the mean-field approximation method cannot capture the dynamic feature only by a fiction constant. Indeed, the cooperation frequency and its corresponding friction parameter will vary with time. As is shown in Fig. 5, the different points that possess the same cooperation frequency have significantly different evacuation time. Such results cannot be derived from the FF model with friction parameter. Furthermore in the FF model, for conflict situations without friction, the evacuee who can move is picked up at random, while in the present model the people who are able to move to the desired position are determined by choice of strategy according to the rules given by game-theoretical approach. In fact, for a real evacuation process, an individual choosing the appropriate behaviors should depend on others' behaviors and self-judgment rather than behave in a random way. In Figs. 4, 6 and 7, the present model shows that how the degree of emergency, evacuee's rationality and degree of evacuee's imitation impact cooperation frequency respectively. These discussions on the cooperative behavior itself cannot be obtained from the FF model but only through other methodology like game theory.

In summary, the FF model with friction effect can describe conflicts and fiction from an indirect population perspective, different from the present model from a direct individual perspective. As for the calculation of evacuation time, the FF model with friction effect seems more straightforward than the present model. However, as for individual cooperative or competitive behavior itself, setting a friction constant cannot provide any understanding that how those behaviors occur and evolve under the influence of other factors. So in this sense it is necessary to resort to other methods. In this work through a game-theoretical approach, the evacuation model is endowed with more advantages to exhibit dynamics of evacuees' cooperative and competitive behaviors besides simple movements.

5. Conclusions

Emergency evacuations are characterized by complexity and non-equilibrium. Evacuees update their behaviors during evacuations, with games occurring at various times and positions, leading to cooperation and competition among evacuees. The game-theoretical approach in the present model is essentially different from FF model which accounts for friction effects. This work is not concentrated on the evacuation time like the FF models, but on the cooperative and competitive behaviors as the following results.

(1) Even when the degree of emergency is high for an evacuation, cooperation is still present. However, the frequency of cooperation will reduce with the increase of the degree of emergency during an evacuation. And higher degrees of emergency during an evacuation lead to longer evacuation times.
(2) During the evacuation process, hyper-rationality in crowd behaviors will inhibit cooperation and delay evacuation times, which is the likely cause of crowd disaster.
(3) The frequency of cooperation will increase with the increasing degree of imitation among evacuees, indicating the imitation effect promotes evacuees' cooperation. Meanwhile, evacuation times will increase with the increasing degree of imitation, particularly in the case of crowds with high rationality, which shows cooperation is not an absolute promise for highly efficient evacuation in the crowd with large imitation effect.

This work provides a methodological pattern to research the crowd behaviors in emergency evacuation by game theory. The game-theoretical approach is different from the method in the FF model and its extension, and based on this approach, the problems related to the dynamics of evacuees' behaviors can be studied further.

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