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On $Sp(0)$ factors and orientifolds

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Abstract

We discuss the geometric engineering of SO/Sp gauge theories with symmetric or antisymmetric tensor matter and show that the 'mysterious' rank zero gauge group factors observed by a few authors can be traced back to the effects of an orientifold which survives the geometric transition. By mapping the Konishi constraints of such models to those of the $U(N)$ theory with adjoint matter, we show that the required shifts in the ranks of the unbroken gauge group components is due to the flux contribution of the orientifold after the transition.

1. Introduction

It was recently pointed out [1] that the Konishi constraints [2] of the $Sp(N)$ theory with antisymmetric matter and a tree-level superpotential of degree $d+1$ can be mapped to those of the $U(N+2d)$ theory with adjoint matter. By noticing that this relation involves a shift in the rank of the components of the unbroken gauge groups, it was shown [1,3] that an apparent discrepancy found in [4] and further explored in [5] can be removed by relaxing an unwarranted assumption. Moreover, it was speculated that the somewhat mysterious $Sp(0)$ factors involved in this relation originate in the IIB realization of such field theories.

In the present Letter, we show that the observations of [1] have a simple interpretation in the geometric engineering of such models, and they admit an obvious generalization. By considering the four SO/Sp theories with symmetric or antisymmetric matter, we show that their IIB realization involves a $\mathbb{Z}_2$ orientifold of an A_1 fibration. As in [6,7], we find that the orientifold 5-plane involved in this construction survives the geometric transition of [8–11]. This allows us to show that the phenomena observed in [1] are due to the flux contribution of this orientifold after the transition. Moreover, we show that the Konishi constraints of all four models can be mapped to those of a theory with unitary gauge group and adjoint matter, and that this map amounts to replacing the orientifold by its flux contribution. This gives an elementary explanation of the relation found in [1].

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We shall be interested in $\mathcal{N}=1$ gauge theories with gauge group $G=SO(N)$ or $Sp(N)$[1] and a single chiral superfield X with $X^T=\epsilon X$ and $\epsilon=\pm 1$ for the symmetric or antisymmetric representation. The gauge transformation is:

$$X \to UXU^T, \tag{1}$$

with U valued in G. Consider the tree-level superpotential

$$W_{\mathrm{tree}} = \mathrm{tr}\big[W(\Phi)\big], \tag{2}$$

where $\Phi = X$ for $G=SO(N)$ and $\Phi = XJ$ for $G=Sp(N)$, where

$$J = \begin{bmatrix} 0 & 1_{N/2} \\ -1_{N/2} & 0 \end{bmatrix}.$$

Here

$$W(z) = \sum_{j=1}^{d+1} \frac{t_j}{j} z^j \tag{3}$$

is a complex polynomial of degree $d+1$. Throughout this Letter, we assume that $W'(z)$ has simple zeroes. Since $U^{-T}JU^{-1}=J$ for $U\in Sp(N)$ and $U^T=U^{-1}$ for $U\in SO(N)$, the field Φ always transforms as $\Phi \to U\Phi U^{-1}$. In particular, Φ is in the adjoint representation for the antisymmetric representation of $SO(N)$ and the symmetric representation of $Sp(N)$. In these cases, we can assume that W is an even polynomial since only even powers of Φ contribute to (2).

2. Geometric engineering

To find the IIB realization of our models, we distinguish the cases:

(A) $SO(N)$ with symmetric matter or $Sp(N)$ with antisymmetric matter;
(B) $SO(N)$ with antisymmetric matter or $Sp(N)$ with symmetric matter.

The engineering of (A) was given in [7] and that of (B) was discussed in [6].[2] In both cases, we start with the singular A_1 fibration given by:

$$X_0:\quad xy = \big(u - W'(z)\big)\big(u + W'(z)\big), \tag{4}$$

which admits the two-section:

$$\Sigma_0:\quad x=y=0,\quad \big(u-W'(z)\big)\big(u+W'(z)\big)=0. \tag{5}$$

This is a union of two rational curves which intersect at the critical points z_j of W. Since W is even in case (B), we let $d=2n+1$ and take $j=-n,\ldots,n$. For case (A) we take $j=1,\ldots,d$.

The resolution $\hat{X}$ can be described as the complete intersection:

$$\begin{aligned}
&\beta\big(u-W'(z)\big) = \alpha x,\\
&\alpha\big(u+W'(z)\big) = \beta y,\\
&\big(u-W'(z)\big)\big(u+W'(z)\big) = xy,
\end{aligned} \tag{6}$$

in the ambient space $\mathbb{P}^1[\alpha,\beta]\times\mathbb{C}^4[z,u,x,y]$. The exceptional $\mathbb{P}^1$'s are denoted by D_j and sit above the singular points of X_0, which are determined by $x=y=u=0$ and $z=z_j$. The resolved space admits the $U(1)$ action:

$$\begin{aligned}
&\big([\alpha,\beta],z,u,x,y\big)\\
&\quad\to \big(\big[e^{-i\theta}\alpha,\beta\big],z,u,e^{i\theta}x,e^{-i\theta}y\big).
\end{aligned} \tag{7}$$

For the two cases, consider the holomorphic $\mathbb{Z}_2$ actions:[3]

$$\begin{aligned}
&\hat{k}_{\mathrm{A}}:\quad \big([\alpha,\beta],z,u,x,y\big) \to \big([-\beta,\alpha],z,-u,y,x\big),\\
&\hat{k}_{\mathrm{B}}:\quad \big([\alpha,\beta],z,u,x,y\big)\\
&\qquad\qquad \to \big([-\beta,\alpha],-z,u,-y,-x\big),
\end{aligned} \tag{8}$$

which obviously preserve (6) (remember that $W'(-z)=-W'(z)$ in case (B)). The first symmetry preserves each exceptional curve, while the second preserves D_0 while exchanging D_j with D_{-j}.

These symmetries project to the following involutions of X_0:

$$\begin{aligned}
&\kappa_{\mathrm{A0}}:\quad (z,u,x,y)\to(z,-u,y,x),\\
&\kappa_{\mathrm{B0}}:\quad (z,u,x,y)\to(-z,u,-y,-x),
\end{aligned} \tag{9}$$

[1] We use conventions in which N is even for $Sp(N)$.

[2] Case (B) had already been engineered in [12,13], but in a framework different from the one we shall find useful here. In the approach of [12,13], the IIA T-dual involves an orientifold 4-plane. In this Letter, we use the construction of [6], whose IIA dual involves an orientifold 6-plane. The relation between the two realizations is discussed in [6].

[3] These square to the identity since $[-\alpha,-\beta]=[\alpha,\beta]$ in $\mathbb{P}^1$.

whose fixed point sets are given by:

$$O_{A0}:\quad x=y,\quad u=0,\quad x^2+W'(z)^2=0,$$
$$O_{B0}:\quad x=-y,\quad z=0,\quad x^2+u^2=0.$$

The fixed point loci of (8) are:

$$\hat{O}_A:\quad x-y=u=x^2+W'(z)^2=0,\quad \frac{\alpha}{\beta}=\pm i, \tag{10}$$
$$\hat{O}_B:\quad x+y=z=x^2+u^2=0,\quad \frac{\alpha}{\beta}=\pm i. \tag{11}$$

We shall use the geometric symmetries (8) to define orientifolds of our IIB background upon combing them with worldsheet parity reversal. More precisely, we choose the orientifold projections such that $\hat{O}_A$ corresponds to an $O_5^{-\epsilon}$ plane and $\hat{O}_B$ corresponds to an $O_5^{+\epsilon}$ plane.

It is not hard to check that this construction engineers our theories. The matter content can be recovered geometrically or by a fractional brane construction. More directly, one can follow the approach of [6,7,14] by using T-duality to map our background to the Hanany–Witten realizations of these models.

3. Dual configurations

To extract the T-dual Hanany–Witten systems, we use a local description valid on a subset $\tilde{X}\subset\hat{X}$. This is given by two copies U_0 and U_1 of $\mathbb{C}^3$ with coordinates (x_i,u_i,z_i) $(i=0,1)$ which are glued according to:

$$(x_1,u_1,z_1)=\left(\frac{1}{u_0},x_0u_0^2-2W'(z_0)u_0,z_0\right). \tag{12}$$

The resolution map is given by:

$$\begin{aligned}(z,u,x,y)&=\big(z_0,x_0u_0-W'(z_0),x_0,u_0\big(x_0u_0-2W'(z_0)\big)\big)\\&=\big(z_1,x_1u_1+W'(z_1),x_1\big(x_1u_1+2W'(z_1)\big),u_1\big),\end{aligned} \tag{13}$$

while the $U(1)$ action (7) takes the form:

$$(z_i,u_i,x_i)\to\big(z_i,e^{-i\theta}u_i,e^{i\theta}x_i\big). \tag{14}$$

Its fixed point set is the union of rational curves $x_0=u_0=0$ and $x_1=u_1=0$. This action stabilizes the exceptional curves D_j: $x_0=u_1=z-z_j=0$.

The Hanany–Witten construction results by T-duality with respect to the circle orbits of this action. Following [6], we use the following ansatz for the T-dual coordinates:

$$\begin{aligned}w&:=x^4+ix^5=x_0u_0-W'(z_0)\\&=x_1u_1+W'(z_1),\\x^6&=\frac{1}{2}\big(|x_1|^2-|u_0|^2\big),\\z&=x^8+ix^9,\end{aligned} \tag{15}$$

together with the periodic coordinate x^7 along the orbits of (14).

Expressing the fixed point set of (14) in these coordinates, we find that the dual background contains two NS5-branes $\mathcal{N}_0$ and $\mathcal{N}_1$ sitting at:

$$\mathcal{N}_0:\quad w=-W'(z),\quad x^6=+\infty,$$
$$\mathcal{N}_1:\quad w=+W'(z),\quad x^6=-\infty.$$

We also have D4-branes $\mathcal{D}_j$ stretching between the NS5-branes at $z=z_j$.

The orientifolds (8) act as:

$$\begin{aligned}(z_0,x_0,u_0)&\overset{\hat{\kappa}_A}{\longleftrightarrow}(z_1,u_1,-x_1),\\(z_0,x_0,u_0)&\overset{\hat{\kappa}_B}{\longleftrightarrow}(-z_1,-u_1,-x_1).\end{aligned} \tag{16}$$

In the first case, the fixed point set is $\hat{O}_A$: $u_0^2+1=x_0+W'(z)u_0=0$. In the second case, it is $\hat{O}_B$: $u_0^2+1=z=0$. Both of these are unions of two disjoint rational curves. The IIA orientifold action is:

$$\text{(A)}:\quad x^6\to -x^6,\quad z\to z,\quad w\to -w,$$
$$\text{(B)}:\quad x^6\to -x^6,\quad z\to -z,\quad w\to w. \tag{17}$$

Using (15) we find that under T-duality these loci map to O6-planes sitting at $x^4=x^5=x^6=0$ and $x^6=x^8=x^9=0$, respectively (Fig. 1). This recovers the Hanany–Witten realization of our models.[4]

4. Description after the geometric transition

After the geometric transition of Refs. [8–11], the Calabi–Yau space (4) is deformed to:

$$X:\quad xy=u^2-W'(z)^2-f(z), \tag{18}$$

[4] For a detailed discussion of these constructions and further references see [15].

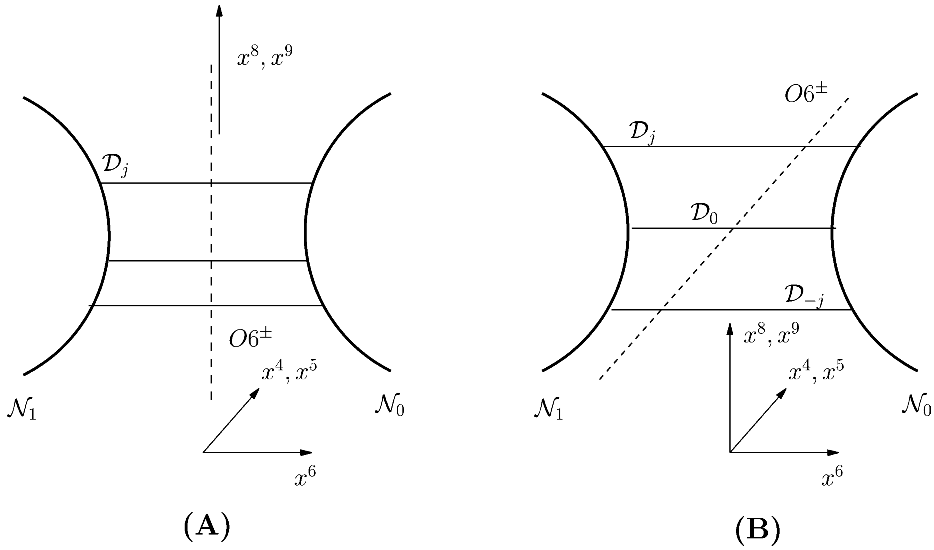

Fig. 1. Brane configuration for the $SO(N)/Sp(N)$ theories with symmetric or antisymmetric matter. The outer NS5-branes are bent in the directions x^4 and x^5, which cannot be shown properly in this two-dimensional figure. The orientifold plane has charge -4ϵ in case (A) and $+4\epsilon$ in case (B).

where $f(z)$ is a polynomial of degree at most $d-1$. This fibration admits the two-section:

$$\Sigma:\quad x=y=0,\quad u^2-W'(z)^2-f(z)=0. \tag{19}$$

The D5-branes wrapping the exceptional divisors are replaced by fluxes. Writing

$$W'(z)^2+f(z)=\prod_j(z-a_j)(z-b_j),$$

we can choose the cuts I_j of (19) to connect a_j and b_j. We also choose a symplectic basis of cycles A_j, B_j with A_j associated with the cut I_j. In case (B) we can choose these such that $I_{-j}=-I_j$. In particular, we have the cut I_0 which passes through the origin.

In case (A), the deformed space (18) is still invariant under the $\mathbb{Z}_2$ action (9) so the orientifold 5-plane survives the transition. Its internal part is deformed to the irreducible curve:

$$\begin{aligned} O_{\mathrm{A}}:\quad & x=y,\quad u=0,\\ & x^2+W'(z)^2+f(z)=0. \end{aligned} \tag{20}$$

In case (B), the polynomial $f(z)$ must be even in order to preserve the orientifold symmetry. Again the orientifold survives the transition, after which its internal part becomes:

$$\begin{aligned} O_{\mathrm{B}}:\quad & x=-y,\quad z=0,\\ & x^2+u^2-f(0)=0. \end{aligned} \tag{21}$$

The Riemann surface (19) arises naturally in the confining phase of the $SO(N)/Sp(N)$ theories with (anti)symmetric matter [16,17]. This curve can be extracted by analyzing the generalized Konishi anomalies of such theories.

5. Relation to generalized Konishi constraints

Consider the field theory quantities

$$T(z)=\left\langle \mathrm{tr}\,\frac{1}{z-\Phi}\right\rangle,\qquad R(z)=\left\langle \mathrm{tr}\,\frac{\mathcal{W}^2}{z-\Phi}\right\rangle,$$

where $\mathcal{W}_\alpha$ is the superfield strength.

5.1. Case (A)

Using the method of generalized Konishi anomalies, it was shown in [16,17] that $R(z)$ and $T(z)$ satisfy:

$$\begin{aligned} W'R &= \frac{1}{2}R^2-\frac{f}{2},\\ W'T &= TR-2\epsilon R'+c, \end{aligned} \tag{22}$$

where f and c are polynomials of degree at most $d-1$. The solution is:

$$R = W' - u,$$
$$T = \frac{c}{u} - 2\epsilon\frac{W'' - u'}{u} = \tilde{T} - \Psi, \tag{23}$$

where $\tilde{T} = \tilde{c}/u$ with $\tilde{c} = c - 2\epsilon W''$ a polynomial of degree at most $d-1$, $\Psi = -2\epsilon u'/u$ and $u = \sqrt{(W')^2 + f}$ is the appropriate branch of the spectral curve (19). The pair $(R, \tilde{T})$ satisfies the relations:

$$W'R = \frac{1}{2}R^2 - \frac{f}{2},$$
$$W'\tilde{T} = \tilde{T}R + \tilde{c}, \tag{24}$$

which are also obeyed by the quantities

$$r = \left\langle \mathrm{tr}\,\frac{\mathcal{W}^2}{z-\phi}\right\rangle, \qquad t = \left\langle \mathrm{tr}\,\frac{1}{z-\phi}\right\rangle$$

of a theory with unitary gauge group and an adjoint chiral multiplet ϕ. It is clear that $R\,dz$ and $\tilde{T}\,dz$ have no poles at finite z on the spectral curve (19), while $\Psi\,dz$ has simple poles at the branching points of Σ.

At the branching points, Ψ behaves like $-\frac{\epsilon}{z-a_j}$ or $-\frac{\epsilon}{z-b_j}$. The quantity $\mathcal{A} = \Psi\,dz$ satisfies:[5]

$$\bar{\partial}\mathcal{A} = -\epsilon\pi\left[\sum_{j=1}^{d}\delta(z-a_j) + \sum_{j=1}^{d}\delta(z-b_j)\right]d\bar{z}\,dz. \tag{25}$$

Thus $\mathcal{A}$ can be viewed as the potential produced by charges equal to $-\epsilon$ placed at branching points of Σ. The 'vacuum' term $\tilde{T}\,dz$ in $\mathcal{B} := T\,dz = \tilde{T}\,dz - \mathcal{A}$ contributes fluxes through the A-cycles of Σ:

$$N_j := \oint_{A_j}\frac{dz}{2\pi i}T = \tilde{N}_j + 2\epsilon, \tag{26}$$

where

$$\tilde{N}_j = \oint_{A_j}\frac{dz}{2\pi i}\tilde{T}$$

with $+2\epsilon$ the contribution from $-\mathcal{A}$. In view of the above, relation (26) maps a vacuum of our theory with unbroken gauge group $\prod_{j=1}^{d} SO(N_j)$ ($\epsilon = +1$) or $\prod_{j=1}^{d} Sp(N_j)$ ($\epsilon = -1$) to a $\prod_{j=1}^{d} U(N_j - 2\epsilon)$ vacuum of the $U(N - 2\epsilon d)$ theory with adjoint matter.

It is easy to find the IIB interpretation of this map. Recall that the orientifold survives the geometric transition, giving an $O_5^{-\epsilon}$ plane whose internal directions wrap the curve (20). This curve intersects the Riemann surface (19) precisely at its branching points $(z, y) = (a_j, 0)$ or $(b_j, 0)$, and contributes to the flux through the 3-cycles S_j associated with the cuts I_j.[6] This accounts for the shift by 2ϵ in relation (26). More precisely, N_j is the number of D-branes wrapping the exceptional curves D_j before the transition, while $\tilde{N}_j = N_j - 2\epsilon$ is the total RR flux through the associated 3-cycle produced after the transition. The flux contribution N_j is due to the D-brane wrapping D_j, which is replaced by a RR flux during the transition, while -2ϵ is the flux contribution of the $O_5^{-\epsilon}$ plane (20).[7]

Thus the shift observed in [1] is explained by the presence of an O5 plane after the geometric transition. Moreover, it is clear that the map $(R, T) \to (R, \tilde{T})$ to the $U(N - 2\epsilon d)$ theory amounts to replacing the orientifold by its flux contribution, i.e., considering the IIB theory with the same total RR fluxes and on the same geometry (18), but without the orientifold plane (20). The latter IIB background is well known to engineer the $U(N - 2\epsilon d)$ theory with adjoint matter. Hence, the map of [1] has an elementary interpretation in geometric engineering.[8]

[5] Remember that $\bar{\partial}_z\frac{1}{z-a} = \pi\delta(z-a)$.

[6] As in [9,18], the 3-cycles of X can be constructed by fibering two-spheres over the cuts.

[7] In our case, the orientifold 5-plane intersects the 3-cycle S_j along a circle. The RR 3-form H is not closed due to the presence of the orientifold (H has a source supported along the curve (20)). One can construct an S^2 fibration $\mathcal{S}$ of X over the z-plane whose S^2 fibers are themselves obtained by fibering circles over the intervals $I_z = [u_-(z), u_+(z)]$ in the u-plane, where

$$u_\pm(z) := \pm\sqrt{W'(z)^2 + f(z)}.$$

The fibers of $\mathcal{S}$ collapse to points for $z = a_j$ or $z = b_j$. Then the integral of H over S_j equals the integral of $\tilde{\mathcal{A}}$ over A_j, where $\tilde{\mathcal{A}}$ is a (non-meromorphic) one-form on Σ obtained from $\frac{1}{2}H$ by 'push-forward' along the S^2 fibration $\mathcal{S}$. As in [19] $\tilde{\mathcal{A}}$ has integral periods but differs from $\mathcal{A}$ by a one-form whose periods vanish on-shell.

[8] Other relations of this type were considered in [6], where they were shown to have similarly straightforward interpretations.

Of course, this map only refers to matching of the associated Konishi constraints, and should not be taken at face value regarding other quantities of physical interest. For case of an $SO(N)$ group with symmetric matter (i.e., $\epsilon = +1$) we can have $\tilde{N}_j < 0$ for some j. This simply means that the total flux through the associated 3-cycle is allowed to become negative. This is of course purely formal in the context of the $U(N - 2\epsilon d)$ theory, and only receives its proper physical interpretation once one considers the orientifold, thereby recovering the SO/Sp model.

For $N_j = 2$ one finds that an $SO(2)$ factor group is mapped to a $U(0)$ factor. In the engineering of the $U(N - 2\epsilon d)$ model, this means that there are no branes wrapping the corresponding $\mathbb{P}^1$ before the transition, and no RR flux through the associated 3-cycle after the transition. In particular, one can keep this cycle collapsed, in which case the associated cut of the spectral curve (19) is reduced to a double point. Nevertheless, it is clear that the period of T does not vanish in this limit because of the flux contribution of the orientifold, which passes through this double point. This behavior of the SO theory with symmetric matter was conjectured in [1]. We note that similar effects were already found in [20] for the more complicated case of $U(N)$ theories with adjoint and symmetric or antisymmetric matter, and explained in [6] in terms of an orientifold which survives the geometric transition.

5.2. Case (B)

It was shown in [16,17] that $R(z)$ and $T(z)$ satisfy:

$$W'R = \frac{1}{2}R^2 - \frac{f}{2},$$
$$W'T = TR + \frac{2\epsilon}{z}R + c, \tag{27}$$

where f and c are polynomials of degree at most $d-1$. The solution is:

$$R = W' - u,$$
$$T = \frac{c}{u} + \frac{2\epsilon}{z}\left[\frac{W'}{u} - 1\right] = \tilde{T} - \Psi, \tag{28}$$

where $\tilde{T} = \tilde{c}/u$ with $\tilde{c} = c + 2\epsilon W'/u$ a polynomial of degree at most $d - 1 = 2n$ (remember that W' is odd!) and $\Psi = +\frac{2\epsilon}{z}$. The pair $(R, \tilde{T})$ satisfies the relations (24) of a theory with unitary gauge group and an adjoint chiral multiplet. We have

$$\bar{\partial}\Psi = 2\pi\epsilon\delta(z)\,d\bar{z} \tag{29}$$

and

$$N_j := \oint_{A_j} \frac{dz}{2\pi i} T = \tilde{N}_j \quad (\text{for } j \neq 0),$$
$$N_0 := \oint_{A_0} \frac{dz}{2\pi i} T = \tilde{N}_0 - 2\epsilon, \tag{30}$$

with $\tilde{N}_j$ the contributions from $\tilde{T}$. We have $N_{-j} = N_j$ for all j.

The IIB interpretation is as before. After the geometric transition, the $O_5^{+\epsilon}$ plane (21) pierces the spectral curve (19) in the two points $u = \pm\sqrt{f(0)}$ sitting above $z = 0$. It contributes $+2\epsilon$ to the RR flux $\tilde{N}_0$ through the associated S^3 cycle in X, leading to the relation $\tilde{N}_0 = N_0 + 2\epsilon$. This allows us to identify a vacuum of our theory with unbroken gauge group

$$SO(N_0) \times \prod_{j=1}^{n} SU(N_j) \quad (\epsilon = -1)$$

or

$$Sp(N_0) \times \prod_{j=1}^{n} SU(N_j) \quad (\epsilon = +1)$$

with an

$$SU(N_0 + 2\epsilon) \times \prod_{j=1}^{n} \bigl(U(N_j) \times U(N_j)\bigr)$$

vacuum of the $U(N + 2\epsilon)$ theory with adjoint matter. Again this identification is only formal in the case $\epsilon = -1$ (i.e., $SO(N)$ with antisymmetric matter) and $N_0 = 0$.

6. Conclusions

We considered the geometric engineering and T-dual Hanany–Witten realizations of four field theories, namely, $SO(N)$ with symmetric or antisymmetric matter and $Sp(N)$ with symmetric or antisymmetric matter. As in [6,7], we found that the IIB realization of such models involves a $\mathbb{Z}_2$ orientifold which survives the geometric transition of [8–11] and therefore contributes to the effective superpotential and fluxes. Following [1], we extracted a relation between the Konishi constraints of such theories and those of the $U(\tilde{N})$ field theory with adjoint matter, where $\tilde{N} = N - 2\epsilon d$ for SO/Sp with symmetric ($\epsilon = 1$)/antisymmetric ($\epsilon = -1$) matter and $\tilde{N} = N + 2\epsilon$ for SO/Sp with antisymmetric/symmetric matter. Its interpretation in geometric engineering amounts to the trivial operation of replacing the orientifold 5-plane by its flux contribution.

The fact that the orientifold contributes to the flux through various 3-cycles after the transition is responsible for the phenomena discussed in [1] and formalized in [3]. In particular, it gives an elementary explanation of the rank shifts required by the relation with the $U(\tilde{N})$ theory. It also recovers and generalizes the role of $Sp(0)$ factors in the $Sp(N)$ theory with antisymmetric matter. For the particular case of the $SO(N)$ theory with symmetric matter, we confirmed the conjecture of [1] that $T(z)$ can have non-vanishing period even if the associated branch cut on the Riemann surface is collapsed to a double point. As in [6], we find that simple operations in geometric engineering account for non-obvious relations between strongly coupled field theories.

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