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Subleading light-cone distribution functions in the decays $\Lambda_b \to X_s\gamma$ and $\Lambda_b \to X_u\ell\bar{\nu}_\ell$

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Abstract

In this Letter we investigate the subleading twist corrections to the photon energy spectrum in the decay $\Lambda_b \to X_s\gamma$ and to the lepton energy spectrum in $\Lambda_b \to X_u\ell\bar{\nu}_\ell$. As a first step we rederive the matching coefficients of the subleading twist using a much simpler method. Parametrizing the matrix elements for Λ_b and using reparametrization invariance, we show that the energy spectra close to the endpoints are given in terms of only a single universal function.

1. Introduction

The existence of baryons with bottom quarks is well established by now, and at future facilities a detailed study of the decay modes of such states will become feasible. However, the B factories are running below the threshold for the production of these modes, which means that the measurements we are going to discuss will be—at least in the foreseeable future—the domain of hadron colliders.

From the theoretical point of view, the ground state bottom baryon Λ_b is the simplest heavy state, although it contains three quarks. The reason is that the spin of the heavy b quark decouples in the limit $m_b \to \infty$ and thus the light degrees of freedom have to be in a spin-zero state [1]. In other words, the spin of a Λ_b is the spin of the heavy quark, which in principle allows interesting polarization studies. Some of these studies require polarized Λ_b's which may become available at GIGA-Z [2], a high-luminosity linear collider running at the Z_0 resonance.

In the present Letter we investigate energy spectra of inclusive heavy-to-light decays of the Λ_b, in particular the photon spectrum of the rare decay $\Lambda_b \to X_s\gamma$ and the lepton energy spectrum of the semileptonic decay $\Lambda_b \to X_u\ell\bar{\nu}_\ell$. The situation here is very similar to the one in the decays $B \to X_s\gamma$ and $B \to X_u\ell\bar{\nu}_\ell$: The photon spectrum in $B \to X_s\gamma$ is mainly concentrated in the endpoint of small hadronic mass of X_s, where the strict $1/m$ expansion breaks down. This forces us to switch to an expansion in twists similar to deep inelastic scattering. The leading twist contribution is given in terms of a light-cone-distribution function which can be formally written as a forward matrix element of a non-local operator [3]. This function describes all inclusive heavy to light transitions and

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is universal. Applying this to $b \to u\ell\bar{\nu}_\ell$ and $b \to s\gamma$ one may obtain a model independent determination of $|V_{ub}/V_{ts}|$ [4].

Recently subleading terms have been included into the analysis of B meson decays [5], and the purpose of the present Letter is to include subleading terms for the inclusive decays of heavy baryons. The main result of the Letter is that only a single distribution function is needed to parametrize the differential decay rates to subleading order; in other words, the endpoints of the photon energy spectrum and the charged lepton energy spectrum in $\Lambda_b \to X_s\gamma$ and $\Lambda_b \to X_u\ell\bar{\nu}_\ell$ are given in terms of one universal function up to terms of order $1/m_b^2$. This has to be compared to the case of the corresponding B decays, where three universal functions are needed [5].

In Section 2 we consider the matching including the terms of subleading twist. We reproduce the results of [5] using a much simpler method based on a background field, in which the light quark propagates. In Section 3 we parametrize the forward matrix elements for the non-local operators between Λ_b states. In Section 4 we discuss our result and conclude.

2. Matching on non-local operators to order $1/m$

The tree-level matching of full QCD to the non-local operators to order $1/m_b$ has been performed for both $b \to s\gamma$ [5] and $b \to u\ell\bar{\nu}_\ell$ [6,7] and the results of these papers can be used also for the case of the Λ_b. However, we find it useful to rederive the results of [5] in a different way, which is very transparent and is manifestly gauge invariant. The method resembles the one used in [8] for the shape function for mesons.

The starting point is the usual decomposition of the heavy quark momentum according to

$$p_b = m_b v + k, \tag{1}$$

where the components of the residual momentum k are small compared to m_b. At tree level, the inclusive decay $b \to s\gamma$ requires to consider the propagator of the (massless) s quark, which is

$$S = \frac{i}{\not{p}_b - \not{q}} = \frac{i}{m_b\not{v} + \not{k} - \not{q}}, \tag{2}$$

where q is the photon momentum.

In position space and in terms of the static heavy-quark field $h_v(x)$ the residual momentum k corresponds to a covariant derivative, since the large part $m_b v$ has been removed by a field redefinition. This leads us to consider the following (non-local) operator

$$T = \bar{h}_v(0)\Gamma^\dagger\left(\frac{i}{m_b\not{v} + i\not{D} - \not{q}}\right)\Gamma h_v(x), \tag{3}$$

where D is the (QCD) covariant derivative and Γ is the Dirac matrix of the corresponding heavy-to-light current $\bar{s}\Gamma b$.

It has been shown in [8] that (3) yields the correct tree level matching for the shape function and we shall use the same arguments to rederive the subleading terms. In order to define the power counting properly we introduce light-cone vectors as

$$v = \frac{1}{2}(n + \bar{n}), \qquad n^2 = 0 = \bar{n}^2,$$
$$n \cdot \bar{n} = 2, \qquad q = (v \cdot q)\bar{n}. \tag{4}$$

In this way we can decompose any vector into its components according to

$$g_{\mu\nu} = \frac{1}{2}n_\mu\bar{n}_\nu + \frac{1}{2}\bar{n}_\mu n_\nu + g^\perp_{\mu\nu}. \tag{5}$$

We want to expand (3) in the kinematical region where

$$(m_b v - q) \cdot n = m - n \cdot q \sim \Lambda_{\rm QCD} \tag{6}$$

which is the kinematical region for the shape function. In order to perform this expansion we write

$$\frac{1}{m_b\not{v} + i\not{D} - \not{q}} = \frac{m_b\not{v} + i\not{D} - \not{q}}{(m_b\not{v} + i\not{D} - \not{q})^2} \tag{7}$$

and consider the denominator, which can be written in the form

$$\begin{aligned}(m_b\not{v} &+ i\not{D} - \not{q})^2 \\ &= m_b Q_+ + \frac{1}{2}\{iD_-, Q_+\} + \left(iD^\perp\right)^2 \\ &\quad - i\sigma^{\mu\nu} iD_\mu iD_\nu,\end{aligned} \tag{8}$$

where we have defined

$$Q_+ = m_b - n \cdot q + iD_+, \qquad iD_+ = i(n \cdot D),$$
$$iD_- = i(\bar{n} \cdot D), \qquad D^\perp_\mu = g^\perp_{\mu\nu}D^\nu. \tag{9}$$

Since $m_b - n \cdot q \sim \Lambda_{\rm QCD}$ we have $Q_+ \sim \Lambda_{\rm QCD}$ and thus the term $m_B Q_+$ is the leading term (of order

$m_b \Lambda_{\mathrm{QCD}}$) in the denominator. All other terms are of order Λ_{QCD}^2 and are expanded as

$$\frac{1}{(m\not{v} - \not{q} + i\not{D})^2} = \frac{1}{mQ_+} - \frac{1}{mQ_+}\frac{1}{2}\{iD_-, Q_+\}\frac{1}{mQ_+} - \frac{1}{mQ_+}\left(\left(iD^\perp\right)^2 - i\sigma^{\mu\nu} iD_\mu iD_\nu\right)\frac{1}{mQ_+} + \cdots. \tag{10}$$

Next we reinsert the numerator, which we do in a symmetrized fashion. After some simple algebra we get

$$\frac{m\not{v} - \not{q} + i\not{D}}{(m\not{v} - \not{q} + i\not{D})^2} = \frac{\not{n}}{2Q_+} + \frac{\not{\bar{n}}}{2m} + \frac{1}{2m}\left\{i\not{D}^\perp, \frac{1}{Q_+}\right\} - \frac{1}{2m}\frac{1}{Q_+}\left(\not{n}\left(iD^\perp\right)^2 - \frac{1}{2}\left\{\not{n}, i\sigma^{\mu\nu}\right\} iD_\mu iD_\nu\right)\frac{1}{Q_+}. \tag{11}$$

The last term can be rewritten in terms of the perpendicular components of the covariant derivative. We have

$$\left\{\not{n}, \sigma^{\mu\nu}\right\} n_\mu n_\nu = \frac{i}{2}\left(\not{n}(\not{n}\not{\bar{n}} - \not{\bar{n}}\not{n}) + (\not{n}\not{\bar{n}} - \not{\bar{n}}\not{n})\not{n}\right) = 0, \tag{12}$$

$$\left\{\not{n}, \sigma^{\mu\nu}\right\} n_\mu g^\perp_{\nu\alpha} = \frac{i}{2}\not{n}\left(\gamma^\perp_\alpha \not{n} - \not{n}\gamma^\perp_\alpha\right) = 0 \tag{13}$$

from which we derive

$$\left\{\not{n}, i\sigma^{\mu\nu}\right\} iD_\mu iD_\nu = i\sigma^{\alpha\beta} n_\alpha \bar{n}_\beta \gamma^\mu_\perp \left[iD^\perp_\mu, Q_+\right] + 2\not{n} i\sigma^{\mu\nu}_\perp iD^\perp_\mu iD^\perp_\nu. \tag{14}$$

Combining this with the other terms we get

$$\frac{m\not{v} - \not{q} + i\not{D}}{(m\not{v} - \not{q} + i\not{D})^2} = \frac{\not{n}}{2Q_+} + \frac{\not{\bar{n}}}{2m} + \frac{\gamma^\mu_\perp}{2m}\left\{iD_\mu, \frac{1}{Q_+}\right\} + \frac{i\sigma^{\alpha\beta} n_\alpha \bar{n}_\beta \gamma^\mu_\perp}{4m}\left[iD_\mu, \frac{1}{Q_+}\right] - \frac{1}{2m}\frac{\not{n}}{Q_+}\left(\frac{1}{2}g^{\mu\nu}_\perp\left\{iD^\perp_\mu, iD^\perp_\nu\right\} - \frac{1}{2}i\sigma^{\mu\nu}_\perp\left[iD^\perp_\mu, iD^\perp_\nu\right]\right)\frac{1}{Q_+}. \tag{15}$$

This result can now be used to perform the matching on the non-local operators as they have been defined in [5]. These operators are

$$O_0(\omega) = \bar{h}_v \delta(\omega + i\widehat{D}_+) h_v, \tag{16}$$

$$O_1^\mu(\omega) = \bar{h}_v \left\{iD_\mu, \delta(\omega + i\widehat{D}_+)\right\} h_v, \tag{17}$$

$$O_2^\mu(\omega) = \bar{h}_v \left[iD_\mu, \delta(\omega + i\widehat{D}_+)\right] h_v, \tag{18}$$

$$O_3^{\mu\nu}(\omega_1, \omega_2) = \bar{h}_v \delta(\omega_2 + i\widehat{D}_+)\left\{iD^\mu_\perp, iD^\nu_\perp\right\}\delta(\omega_1 + i\widehat{D}_+) h_v, \tag{19}$$

$$O_4^{\mu\nu}(\omega_1, \omega_2) = \bar{h}_v \delta(\omega_2 + i\widehat{D}_+)\left[iD^\mu_\perp, iD^\nu_\perp\right]\delta(\omega_1 + i\widehat{D}_+) h_v \tag{20}$$

and

$$P_{0,\alpha}(\omega) = \bar{h}_v \delta(w + i\widehat{D}_+)\gamma_\alpha\gamma_5 h_v, \tag{21}$$

$$P_{1,\alpha}^\mu(\omega) = \bar{h}_v \left\{iD_\mu, \delta(\omega + i\widehat{D}_+)\right\}\gamma_\alpha\gamma_5 h_v, \tag{22}$$

$$P_{2,\alpha}^\mu(\omega) = \bar{h}_v \left[iD_\mu, \delta(\omega + i\widehat{D}_+)\right]\gamma_\alpha\gamma_5 h_v, \tag{23}$$

$$P_{3,\alpha}^{\mu\nu}(\omega_1, \omega_2) = \bar{h}_v \delta(\omega_2 + i\widehat{D}_+)\left\{iD^\mu_\perp, iD^\nu_\perp\right\} \times \delta(\omega_1 + i\widehat{D}_+)\gamma_\alpha\gamma_5 h_v, \tag{24}$$

$$P_{4,\alpha}^{\mu\nu}(\omega_1, \omega_2) = \bar{h}_v \delta(\omega_2 + i\widehat{D}_+)\left[iD^\mu_\perp, iD^\nu_\perp\right] \times \delta(\omega_1 + i\widehat{D}_+)\gamma_\alpha\gamma_5 h_v. \tag{25}$$

The matching onto these operators is performed for the decay rate. In order to obtain the rate we have to take the imaginary part of a forward matrix element with the Λ_b. Inserting (15) back into (3) and taking the imaginary part we obtain for the leading term

$$\begin{aligned}
\mathrm{Im}\,\bar{h}_v \frac{\overline{\Gamma}\not{n}\Gamma}{2Q_+} h_v
&= -\frac{1}{2m}\int d\omega \left(\frac{\pi}{2}\,\mathrm{Tr}[P_+\Gamma\not{n}\Gamma]\delta(1 - x - \omega)\bar{h}_v \times \delta(w + i\widehat{D}_+) h_v - \frac{\pi}{2}\,\mathrm{Tr}\left[s^\alpha\Gamma\not{n}\Gamma\right]\delta(1 - x - \omega)\bar{h}_v \times \delta(w + i\widehat{D}_+)\gamma_\alpha\gamma_5 h_v\right) \\
&= -\frac{1}{2m}\int d\omega \left[C_0(\omega) O_0(\omega) + C^\alpha_{5,0}(\omega) P_{0,\alpha}(\omega)\right]
\end{aligned} \tag{26}$$

from which we can read off the matching coefficients C_0 and $C^{\alpha}_{5,0}$ as they have been derived already in [5].[1]

In the same way we obtain for the subleading contributions

$$\begin{aligned}
&\operatorname{Im}\bar{h}_v \frac{\bar{\Gamma}\gamma_\perp^\mu \Gamma}{2m}\left\{iD_\mu, \frac{1}{Q_+}\right\}h_v \\
&\quad = \operatorname{Im}\int d\omega\,\bar{h}_v \frac{\bar{\Gamma}\gamma_\perp^\mu \Gamma}{2m^2}\left\{iD_\mu, \delta(\omega + i\widehat{D}_+)\right\} \\
&\qquad \times \frac{1}{1-x-\omega}h_v \\
&\quad = -\frac{1}{2m^2}\int d\omega\left(\frac{\pi}{2}\operatorname{Tr}\left[P_+\bar{\Gamma}\gamma_\perp^\mu \Gamma\right]\delta(1-x-\omega)\bar{h}_v \right.\\
&\qquad\qquad \times \left\{iD_\mu, \delta(\omega + i\widehat{D}_+)\right\}h_v \\
&\qquad\qquad - \frac{\pi}{2}\operatorname{Tr}\left[s^\alpha \bar{\Gamma}\gamma_\perp^\mu \Gamma\right] \\
&\qquad\qquad \times \delta(1-x-\omega)\bar{h}_v \\
&\qquad\qquad \left. \times \left\{iD_\mu, \delta(\omega + i\widehat{D}_+)\right\}\gamma_\alpha\gamma_5 h_v\right),
\end{aligned} \tag{27}$$

from which we obtain the matching coefficients C_1^μ and $C_{5,1}^{\alpha,\mu}$.

A further restriction of the structure of the matching coefficients, which should hold even beyond tree level, is due to reparametrization invariance [9]. It turns out that reparametrization invariance [10,11] relates the coefficients of O_0 and $O_3^{\mu\nu}$ in such a way that only one function is needed to parametrize the effect of the two operators O_0 and $O_3^{\mu\nu}$. We shall use the relations from [10] in the next section when we will count the number of unknown functions needed to describe the spectrum of $\Lambda_b \to X_s\gamma$ including subleading contributions.

In addition to these operators we have also to take into account the subleading terms of the Lagrangian. They are given as

$$\begin{aligned}
O_T(\omega) = i\int d^4x \int \frac{dt}{2\pi}e^{-i\omega t} \\
\times T\left[\bar{h}_v(0)h_v(t)\mathcal{L}_{1/m}(x)\right],
\end{aligned} \tag{28}$$

[1] Here we made us of the decomposition formula

$$P_+\Gamma P_+ = \frac{1}{2}P_+\operatorname{Tr}[P_+\Gamma] - \frac{1}{2}s^\mu\operatorname{Tr}[s_\mu\Gamma],$$

$$s_\mu = P_+\gamma_\mu\gamma_5 P_+, \qquad P_+ = \frac{1}{2}(1+\slashed{v}).$$

$$\begin{aligned}
P_{T,\alpha}(\omega) = i\int d^4x \int \frac{dt}{2\pi}e^{-i\omega t} \\
\times T\left[\bar{h}_v(0)\gamma_\alpha\gamma_5 h_v(t)\mathcal{L}_{1/m}(x)\right],
\end{aligned} \tag{29}$$

where $\mathcal{L}_{1/m}$ is the subleading contribution to the Lagrangian

$$\mathcal{L}_{1/m} = \frac{1}{2m}\bar{h}_v(i\slashed{D})^2 h_v. \tag{30}$$

Once matrix elements are taken, these pieces can be interpreted as the corrections to the states, which means that they will appear always in a specific combination with the leading terms, i.e., these will not introduce any new unknown functions.

3. Matrix elements for Λ_b decays

After having computed the matching we have to consider the forward matrix elements with a Λ_b. These matrix elements have to be evaluated in the static limit in which the Λ_b becomes a very simple object. We obtain for the matrix elements:

$$\begin{aligned}
&\left\langle \Lambda_b(v,s)\middle| O_0(\omega)\middle| \Lambda_b(v,s)\right\rangle \\
&\quad = \bar{u}(v,s)u(v,s)f_\Lambda(\omega),
\end{aligned} \tag{31}$$

$$\begin{aligned}
&\left\langle \Lambda_b(v,s)\middle| P_{0,\alpha}(\omega)\middle| \Lambda_b(v,s)\right\rangle \\
&\quad = \bar{u}(v,s)\gamma_\alpha\gamma_5 u(v,s)f_\Lambda(\omega),
\end{aligned} \tag{32}$$

where $u(v,s)$ describes the static Λ_b with velocity v and spin s.

For the subleading operators the non-vanishing matrix elements are parametrized as

$$\begin{aligned}
&\left\langle \Lambda_b(v,s)\middle| O_1^\mu(\omega)\middle| \Lambda_b(v,s)\right\rangle \\
&\quad = -2\bar{u}(v,s)u(v,s)\left(v^\mu - n^\mu\right)\omega f_\Lambda(\omega),
\end{aligned} \tag{33}$$

$$\begin{aligned}
&\left\langle \Lambda_b(v,s)\middle| P_{1,\alpha}^\mu(\omega)\middle| \Lambda_b(v,s)\right\rangle \\
&\quad = -2\bar{u}(v,s)\gamma_\alpha\gamma_5 u(v,s)\left(v^\mu - n^\mu\right)\omega f_\Lambda(\omega),
\end{aligned} \tag{34}$$

$$\begin{aligned}
&\left\langle \Lambda_b(v,s)\middle| O_3^{\mu\nu}(\omega_1,\omega_2)\middle| \Lambda_b(v,s)\right\rangle \\
&\quad = \bar{u}(v,s)u(v,s)g_\perp^{\mu\nu}g_\Lambda(\omega_1,\omega_2),
\end{aligned} \tag{35}$$

$$\begin{aligned}
&\left\langle \Lambda_b(v,s)\middle| P_{3,\alpha}^{\mu\nu}(\omega_1,\omega_2)\middle| \Lambda_b(v,s)\right\rangle \\
&\quad = \bar{u}(v,s)\gamma_\alpha\gamma_5 u(v,s)g_\perp^{\mu\nu}g_\Lambda(\omega_1,\omega_2),
\end{aligned} \tag{36}$$

$$\begin{aligned}
&\left\langle \Lambda_b(v,s)\middle| O_T(\omega)\middle| \Lambda_b(v,s)\right\rangle \\
&\quad = \bar{u}(v,s)u(v,s)t_\Lambda(\omega),
\end{aligned} \tag{37}$$

$$\begin{aligned}
&\left\langle \Lambda_b(v,s)\middle| P_{T,\alpha}(\omega)\middle| \Lambda_b(v,s)\right\rangle \\
&\quad = \bar{u}(v,s)\gamma_\alpha\gamma_5 u(v,s)t_\Lambda(\omega).
\end{aligned} \tag{38}$$

Consequently, only two additional functions appear at order $1/m_b$. Furthermore, these subleading function are tied to the leading order function f_Λ by reparametrization invariance, which means that it appears always the combination

$$F_\Lambda(\omega) = f_\Lambda(\omega) + t_\Lambda(\omega) - \frac{1}{m_b^2}\int d\omega_1\, d\omega_1\, g_\Lambda(\omega_1\omega_2) \times \left(\frac{\delta(\omega-\omega_1) - \delta(\omega-\omega_2)}{\omega_1 - \omega_2}\right) \tag{39}$$

with the leading order function f_Λ. In other words, due to heavy quark spin symmetry we can express $\Lambda_b \to X_s\gamma$ and $\Lambda_b \to X_u\ell\bar{\nu}_\ell$ in terms of a single universal function, up to terms of order $1/m_b^2$.

Finally we discuss the moment expansion of the resulting functions. For the leading term we have

$$f_\Lambda(\omega) = \delta(\omega) - \frac{\lambda_\Lambda}{6m_b^2}\delta''(\omega) + \cdots, \tag{40}$$

where

$$\lambda_\Lambda \bar{u}(v,s)u(v,s) = \left\langle \Lambda_b(v,s)\middle|\bar{h}_v(iD)^2 h_v\middle|\Lambda_b(v,s)\right\rangle \tag{41}$$

is the kinetic energy of the b quark inside the Λ_b baryon. For the subleading contributions we obtain

$$g_\Lambda(\omega_1,\omega_2) = \frac{2\lambda_\Lambda}{3}\delta(\omega_1)\delta(\omega_2) + \cdots, \tag{42}$$

$$t_\Lambda(\omega) = -\frac{\lambda_\Lambda}{2m_b^2}\delta'(\omega) + \cdots, \tag{43}$$

which reproduces the known result from [12] with the replacements $\lambda_1 \to \lambda_\Lambda$ and $\lambda_2 \to 0$:

$$\frac{d\Gamma}{dx}(\Lambda_b \to X_s\gamma) = \Gamma_0\left[\delta(1-x) - \frac{\lambda_\Lambda}{2m_b^2}\delta'(1-x) - \frac{\lambda_\Lambda}{6m_b^2}\delta''(1-x) + \cdots\right] \tag{44}$$

with

$$\Gamma_0 = \frac{G_F^2\alpha|V_{tb}^* V_{ts}||C_7|^2}{32\pi^4}m_b^5. \tag{45}$$

The moment expansion of the new universal function reads

$$F_\Lambda(\omega) = \delta(\omega) + \frac{\lambda_\Lambda}{6m_b^2}\delta'(\omega) + \cdots. \tag{46}$$

4. Discussion and conclusions

The main result of this Letter is that the semileptonic and radiative decays of a Λ_b baryon into light hadrons are described by a universal function, where the corrections to this statement are of order $1/m_b^2$. In this way we find the two differential decay rates:

$$\frac{d\Gamma}{dx}(\Lambda_b \to X_s\gamma) = \frac{G_F^2\alpha|V_{tb}^* V_{ts}||C_7|^2}{32\pi^4}m_b^5(2x-1)F_\Lambda(1-x), \tag{47}$$

$$\frac{d\Gamma}{dy}(\Lambda_b \to X_u\ell\bar{\nu}_\ell) = \frac{G_F^2|V_{ub}|^2}{96\pi^3}m_b^5 \times \int d\omega\,\Theta(1-y-\omega)(1-\omega)F_\Lambda(\omega), \tag{48}$$

where $x = 2E_\gamma/m_b$ is the rescaled photon energy and $y = 2E_\ell/m_b$ is the rescaled lepton energy. Note that the first moment of F still vanishes up to terms of order $1/m_b^2$, so we still get no contribution to the total rates. This serves a check, since the contributions of order $1/m$ may not contribute to the rate. As for $\Lambda_b \to X_s\gamma$, the spectrum of the semileptonic decay can be expanded in terms of moments. Inserting (46) into the spectrum, one arrives at

$$\frac{d\Gamma}{dy}(\Lambda_b \to X_u\ell\bar{\nu}_\ell) = \Gamma_0\left[\theta(1-y) - \frac{\lambda_\Lambda}{6m_b^2}\delta(1-y) - \frac{\lambda_\Lambda}{6m_b^2}\delta'(1-y) + \cdots\right], \tag{49}$$

where

$$\Gamma_0 = \frac{G_F^2|V_{ub}|^2}{96\pi^3}m_b^5. \tag{50}$$

With the replacements already mentioned above, this agrees with the result obtained in [12].

Although this may be difficult from the experimental point of view, we may investigate the subleading corrections to a determination of V_{ub} from a comparison of $\Lambda_b \to X_s\gamma$ with $\Lambda_b \to X_u\ell\bar{\nu}_\ell$. Like in the case of B mesons, an energy cut E_c on the charged lepton will be unavoidable to discriminate the large charm background, i.e., $E_c >$

$(M^2[\Lambda_b] - M^2[\Lambda_c])/(2M[\Lambda_b])$. One may define observables (similar as for the decays of B mesons, see [13]) involving partially integrated rates with suitable weight functions. Since the $1/m_b$ terms appear only a kinematic factors (the pre-factors $(2x-1)$ in (47) and $(1-\omega)$ in (48)) we have more complicated weight functions as in [13]. We define

$$\Gamma_u(E_c) \equiv \int_{E_c}^{m_\Lambda/2} dE_\ell \left(\frac{4E_\ell{}^2 - E_c m_b}{2E_\ell{}^2}\right) \frac{d\Gamma_u^{\Lambda_b \to X_u \ell \bar{\nu}_\ell}}{dE_\ell}, \tag{51}$$

$$\Gamma_s(E_c) \equiv \frac{2}{m_b} \int_{E_c}^{m_\Lambda/2} dE_\gamma\, (E_\gamma - E_c) \frac{d\Gamma_s^{\Lambda_b \to X_s \gamma}}{dE_\gamma} \tag{52}$$

from which V_{ub} can be determined as

$$\left|\frac{V_{ub}}{V_{tb}V_{ts}^*}\right| = \frac{3\alpha}{\pi} \left|C_7^{\mathrm{eff}}\right|^2 \frac{\Gamma_u(E_c)}{\Gamma_s(E_c)} + \mathcal{O}\big(1/m_b^2\big). \tag{53}$$

Comparing our result (53) for the V_{ub} determination with the one obtained in [6,7] one expects substantially smaller corrections in the case of Λ_b baryons, since the corrections are only of order $1/m_b^2$. However, this requires to measure the inclusive semileptonic and radiative rare Λ_b decays, which is more difficult experimentally.

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