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The covariant, time-dependent Aharonov–Bohm effect

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ABSTRACT

We discuss two possible covariant generalizations of the Aharonov–Bohm effect – one expression in terms of the space–time line integral of the four-vector potential and the other expression in terms of the space–time "area" integral of the electric and magnetic fields written in terms of the Faraday 2-form. These expressions allow one to calculate the Aharonov–Bohm effect for time-dependent situations. In particular, we use these expressions to study the case of an infinite solenoid with a time varying flux and find that the phase shift is zero due to a cancellation of the Aharonov–Bohm phase shift with a phase shift coming from the Lorentz force associated with the electric field, $\mathbf{E} = -\partial_t \mathbf{A}$, outside the solenoid. This result may already have been confirmed experimentally.

1. Introduction

The Aharonov–Bohm (AB) effect [1,2] lies at the interface of gauge theories and quantum mechanics. In its best known form, the AB effect predicts a shift in the interference pattern of the quantum mechanical double-slit experiment which has a magnetic flux carrying solenoid placed between the slits. If a solenoid with a magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$ (where $\mathbf{A}$ is the electromagnetic vector potential) is placed between the two slits of a double-slit experiment the phase, α, of the wave-function of the electrons going through the slits and following some *path* to the screen will be shifted by an amount

$$\alpha_B = -\frac{e}{\hbar} \int\limits_{path} \mathbf{A} \cdot d\mathbf{x} \tag{1}$$

where e is the charge of the electron. If one considers two electrons arriving at the screen via two separate paths, namely $path_1$ and $path_2$, one can reverse one of the paths and find that the phase difference between the two electrons at the screen is given by

$$\delta\alpha_B = \alpha_{B_1} - \alpha_{B_2} = \frac{e}{\hbar} \oint\limits_{2-1} \mathbf{A} \cdot d\mathbf{x} = \frac{e}{\hbar} \int \mathbf{B} \cdot d\mathbf{S} = \frac{e}{\hbar} \Phi_0 \tag{2}$$

where the subscript $2-1$ means the path going from the slits to the screen along $path_2$ and returning along $path_1$. We used Stokes' theorem on the closed line integral and $\nabla \times \mathbf{A} = \mathbf{B}$. Finally, Φ_0 is the magnetic flux through the cross sectional area, $\mathbf{S}$, of the solenoid. This shift in the phase leads to a shift in the position, x, of the interference pattern maxima and minima on the screen by $\Delta x = \frac{L\lambda}{2\pi d}\delta\alpha_B$ where L is the distance to the screen, d is the distance between the slits and λ is the wavelength of the wave-function. This shift due to the magnetic AB effect has been measured [3,4]. Note, there is some unavoidable arbitrariness in the sign $\delta\alpha_B$ depending on the rotational sense of the closed loop going around the solenoid – going along $path_1$ to the screen and returning along $path_2$ versus going along $path_2$ to the screen and returning along $path_1$. However, the shift of the interference pattern is independent of this arbitrariness.

The importance of the magnetic AB effect of (2) is that it shows (to some degree) the physical nature of the vector potential, $\mathbf{A}$, since the electrons move in a region, outside the solenoid, where $\mathbf{B} = 0$ but $\mathbf{A} \neq 0$. However, although $\mathbf{A}$ is gauge *variant* under the gauge transformation $\mathbf{A} \to \mathbf{A} - \nabla\Lambda$, where $\Lambda(\mathbf{x}, t)$ is some arbitrary function, the final result in the phase difference, $\delta\alpha_B$, is gauge independent since it can be turned into a surface integral of the magnetic field, which is gauge invariant.

The electric version of the AB effect has been less discussed and investigated. It was experimentally observed relatively recently in [5]. Similar to the magnetic case above, one can show [1] that for an electron moving through some region of space with an electric scalar potential ϕ, it will have its phase shifted by an amount

$$\alpha_E = \frac{e}{\hbar} \int\limits_{t_1}^{t_2} \phi \, dt \tag{3}$$

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where $\Delta t = t_2 - t_1$ is the time the electron spends in the potential. If one considers electrons moving along two different paths, $path_1$ and $path_2$, with different values of the potential, ϕ_1 and ϕ_2, along the different paths, then the electrons will acquire a phase difference due to traveling in different potentials given by

$$\delta\alpha_E = \frac{e}{\hbar}\int_{t_1}^{t_2} \Delta\phi\, dt = \frac{e}{\hbar}\int_{t_1}^{t_2}\int \mathbf{E}\cdot d\mathbf{x}\, dt \tag{4}$$

where $\Delta\phi = \phi_2 - \phi_1 = -\int_2^1 \nabla\phi \cdot d\mathbf{x} = \int \mathbf{E}\cdot d\mathbf{x}$ is the potential difference between the two paths through which the electrons move. The last form of the electric phase shift in (4), i.e. $\frac{e}{\hbar}\int_{t_1}^{t_2}\int \mathbf{E}\cdot d\mathbf{x}\, dt$, appears similar to the last form of the magnetic phase shift in (2), i.e. $\frac{e}{\hbar}\int \mathbf{B}\cdot d\mathbf{S}$, in that both have the form $\delta(Phase) \propto (Field) \times (Area)$ although for the electric phase shift the "area" has one space side and one time side while the magnetic phase shift has a conventional area having two space sides. One can flesh out this connection via the following heuristic argument: For a small distance, $\Delta\mathbf{x}$, between the two different potentials, ϕ_2 and ϕ_1, one can write $\Delta\phi = \mathbf{E}\cdot\Delta\mathbf{x}$. Using this in the first expression in (4), one can write $\delta\alpha_E = \frac{e}{\hbar}(\mathbf{E}\cdot\Delta\mathbf{x})\Delta t$, where again Δt is the time that the two electrons spend in their respective potentials. Now $\Delta t \propto L$ where L is the length of the region through which the electrons move where the potentials are ϕ_2 and ϕ_1 – more precisely $v_e \Delta t = L$ where v_e is the speed of the electrons as they move through these regions of constant scalar potential. Combining these results, we find $\delta\alpha_E \propto (\mathbf{E}\cdot\Delta\mathbf{x})L$, and $(\Delta\mathbf{x})L$ is the area between the two tubes of length L separated by a distance $\Delta\mathbf{x}$, i.e. $d(Area) = (\Delta\mathbf{x})L$. Thus, both magnetic and electric AB phase differences from (2) and (4) can be written in the form $\delta(Phase) \propto (Field) \times (Area)$. Pictorially, one can see this $(Area)$ as the area swept out by an imaginary string which connects the two electrons – the length of the string is $\Delta\mathbf{x}$ and the length swept out is L. Note that the phase difference in (4) is in addition to any phase difference due to the path length difference between $path_1$ and $path_2$. Also, as in the magnetic case (2), there is an unavoidable sign ambiguity in (4) depending whether one considers $\Delta\mathbf{x}$ as coming from a path going from ϕ_1 to ϕ_2 or, alternatively, a reversed path going from ϕ_2 to ϕ_1.

The expressions (2) and (4) are written in three-vector form so they are not obviously covariant. In the next section, we examine two possible covariant generalizations of the AB phase differences (2) and (4) which allow one to examine time-dependent Aharonov–Bohm experiments.

2. Covariant expressions for the AB phase shift

The first covariant version of the AB phase differences generalizes the potential form of the phase difference given by the first expressions on the right hand side of (2) and (4)

$$\delta\alpha_{EB} = \frac{e}{\hbar}\oint A_\mu\, dx^\mu = \frac{e}{\hbar}\left[\int_{t_1}^{t_2} \Delta\phi\, dt - \oint \mathbf{A}\cdot d\mathbf{x}\right]. \tag{5}$$

This covariant expression for the AB phase shift was used in [1]. The closed loop integral in the four-vector expression, $\oint A_\mu\, dx^\mu$, is not a closed time loop but is to be taken in the sense that the two electrons both start at the space–time point $(t_i, \mathbf{x}_i)$, travel along two different paths, $path_1$ and $path_2$, and end up at the same space–time point $(t_f, \mathbf{x}_f)$ with $t_f > t_i$. One reverses the direction of one of the paths and in this way gets $\Delta\phi = \phi_2 - \phi_1$ in the time integral and one gets a closed loop for the spatial integral, i.e. $\oint \mathbf{A}\cdot d\mathbf{x}$.

The second covariant version of the AB phase difference generalizes the $(Field) \times (Area)$ form of the phase difference, i.e. the last two expressions for the magnetic and electric phase differences given in (2) and (4). This second covariant version of the AB phase is best expressed in the notation of differential forms and the wedge product.[1] This second proposed expression for the covariant AB phase is

$$\delta\alpha_{EB} = -\frac{e}{2\hbar}\int F_{\mu\nu}\, dx^\mu \wedge dx^\nu = \frac{e}{\hbar}\int F \tag{6}$$

where $F = -\frac{1}{2}F_{\mu\nu}\, dx^\mu \wedge dx^\nu$ is the Faraday 2-form, dx^μ and dx^ν are differential four-vectors, and $\wedge$ is the anti-symmetric wedge product [6]. The factor of $\frac{1}{2}$ accounts for the anti-symmetry of $F_{\mu\nu}$ and $dx^\mu \wedge dx^\nu$.

We now expand the Faraday 2-form out, and show that it reproduces the standard, static AB phase results (2) and (4),

$$\begin{aligned} F &= -\frac{1}{2}F_{\mu\nu}\, dx^\mu \wedge dx^\nu \\ &= (E_x\, dx + E_y\, dy + E_z\, dz) \wedge dt + B_x\, dy \wedge dz + B_y\, dz \wedge dx \\ &\quad + B_z\, dx \wedge dy. \end{aligned} \tag{7}$$

If the electric field is zero, i.e. $\mathbf{E} = 0$, then one has $F = B_x\, dy \wedge dz + B_y\, dz \wedge dx + B_z\, dx \wedge dy = \mathbf{B}\cdot d\mathbf{S}$ where the differential forms expression has been converted back to three-vector notation and $d\mathbf{S}$ is the differential area. Thus, the expression in (6) reduces to $\delta\alpha_{EB} = \frac{e}{\hbar}\int F = \frac{e}{\hbar}\int \mathbf{B}\cdot d\mathbf{S}$ which is equivalent to the three-vector expression (2).

If, on the other hand, the magnetic field is zero, i.e. $\mathbf{B} = 0$, *and* one has a time independent system (so that $\partial_t \mathbf{A} = 0$ and $\mathbf{E} = -\nabla\phi$), then the non-zero terms of the Faraday 2-form are $F = -\partial_x\phi\, dx \wedge dt - \partial_y\phi\, dy \wedge dt - \partial_z\phi\, dz \wedge dt$. Doing the spatial integral of this expression for the Faraday 2-form yields $-\int_2^1 \nabla\phi \cdot d\mathbf{x} = \phi_2 - \phi_1 = \Delta\phi$. Thus, under these conditions, the expression in (6) reduces to $\delta\alpha_{EB} = \frac{e}{\hbar}\int F = \frac{e}{\hbar}\int \Delta\phi\, dt$ which is equivalent to the first three-vector expression on the right hand side of (4).

In summary, in this section, we have constructed two covariant versions of the AB phase difference, (5) and (6). In the next section, we will discuss how one can experimentally test these covariant expressions for the AB phase difference, (5) or (6), in the time-dependent situation of an infinite solenoid with a time varying magnetic flux.

3. Solenoid with time varying flux

For static situations, both (5) and (6) reproduce the results for the magnetic and electric AB phase differences (2) and (4). However, for certain time-dependent situations, the two expressions both lead to the conclusion that there is an exact cancellation of the magnetic and electric AB phase shifts so that one finds no net phase shift differences coming from the time-dependent electromagnetic field. In particular, we have in mind the usual magnetic AB set-up of an infinite solenoid but with a time-dependent magnetic field and vector potential, i.e. $\mathbf{B}(t)$, and $\mathbf{A}(t)$. Note that for this situation the scalar potential is still zero, $\phi = 0$. At first, one might think that for this set-up the AB phase would simply be obtained by inserting $\mathbf{A}(t)$ into the first expression on the right hand side of (2), or inserting $\mathbf{B}(t)$ into the second expression on the right hand side of (2), giving the usual magnetic AB phase shift (2) but with the time dependence of the vector potential, i.e. $\delta\alpha_B \propto \Phi_0(t)$. This is in fact what previous work [7–9] on the time-dependent AB

[1] For our purposes the elementary and excellent introduction to differential forms given in [6] is all we will need.

effect has suggested – that there would be a time-dependent AB phase. However, for this time-dependent set-up one can see there are complications since unlike the static solenoid set-up, there is now a non-vanishing electric field outside the solenoid coming from $\mathbf{E} = -\partial_t \mathbf{A}$. This induces an additional phase shift as we will show below.

We will first calculate the AB phase difference predicted by (6). The part of the AB phase difference from the three magnetic field terms of (7) is

$$\frac{e}{\hbar}\int [B_x\,dy\wedge dz + B_y\,dz\wedge dx + B_z\,dx\wedge dy] = \frac{e}{\hbar}\int \mathbf{B}(\mathbf{x},t)\cdot d\mathbf{S} \tag{8}$$

where in the last expression we have converted back to three-vector notation. The contribution to the covariant AB phase difference from the three electric field terms of (7) is

$$\frac{e}{\hbar}\int \big[(E_x\,dx + E_y\,dy + E_z\,dz)\wedge dt\big] = -\frac{e}{\hbar}\oint \mathbf{A}\cdot d\mathbf{x} = -\frac{e}{\hbar}\int \mathbf{B}(\mathbf{x},t)\cdot d\mathbf{S} \tag{9}$$

where we have taken into account that $\mathbf{E} = -\partial_t \mathbf{A}$, performed the dt integration, and in the last expression we have used Stokes' theorem. The dt integration in (9) has turned $\mathbf{E} = -\partial_t \mathbf{A}$ into $-\mathbf{A}$. The magnetic contribution from (8) is the negative of the electric contribution from (9) and the two parts cancel exactly.

We now calculate the phase shift for the time-dependent, infinite solenoid using (5) for an infinitesimal arc. First, the vector potential outside an infinite solenoid which has a time-dependent magnetic field and, therefore, a time-dependent current, $I(t)$, is

$$\mathbf{A} = \frac{kI(t)}{r}\hat{\theta} \tag{10}$$

where k is a constant whose exact form is not important for the present and $\hat{\theta}$ is a unit vector in the angular direction. Without loss of generality, we take the infinitesimal path of the particle of charge e and mass m to be along a circular arc, i.e. $d\mathbf{x} \propto \hat{\theta}$. Since $\mathbf{A} \propto \hat{\theta}$, the product $\mathbf{A}\cdot d\mathbf{x}$ will pick out the angular direction of $d\mathbf{x}$. The relationship between the angular displacement of the particle, $\Delta\theta$, the radius of the arc, r, and the velocity of the particle, v, is

$$r\Delta\theta = v\Delta t. \tag{11}$$

Actually since the particle is accelerated by the electric field outside the solenoid one should use $v \to (v_f + v_i)/2$, i.e. the average velocity using the mid-point (this assumes that the acceleration due to the electric field is a constant during this infinitesimal interval). Evaluating $\frac{e}{\hbar}\int \mathbf{A}\cdot d\mathbf{x}$ for this infinitesimal path, $d\mathbf{x} = r\Delta\theta\hat{\theta}$, gives

$$\frac{e}{\hbar}\frac{kI(t)}{r}(r\Delta\theta) = \frac{ekI(t)\Delta\theta}{\hbar}. \tag{12}$$

The question that arises now is "Where does one evaluate $I(t)$?"; "At the initial time t_i, or final time t_f?". Based on the fact that we use $v \to (v_f + v_i)/2$ for the velocity, we evaluate $I(t)$ at the midpoint time $t = t_i + \Delta t/2$. Inserting this into $I(t)$ and expanding to first order gives $I(t_i + \Delta t/2) \approx I(t_i) + I'(t_i)\frac{\Delta t}{2} + \cdots$, with the prime indicating a time derivative. The first term, $I(t_i)$, is a constant and represents the initial, static AB phase contribution. We can, without loss of generality, take the initial current to be zero, $I(t_i) = 0$, so that there will be no initial phase shift. If there were a non-zero initial current, one would instead have a constant phase shift of $\delta\alpha_B = \frac{ekI(t_i)\Delta\theta}{\hbar}$. Next, inserting the second term in the expansion into (12) gives,

$$\delta\alpha_{A(t)} = \frac{ekI'(t)\Delta t\Delta\theta}{2\hbar} \tag{13}$$

which is the phase shift due to the time-dependence of the vector potential. However, (13) is not the total phase shift in this case since there will be an electric field, $\mathbf{E} = -\partial_t \mathbf{A}$, outside the solenoid which will also contribute to the phase shift. We now calculate this shift. The electric field outside the solenoid is

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} = -\frac{kI'(t)}{r}\hat{\theta}. \tag{14}$$

The acceleration associated with this electric field for the particle is

$$\mathbf{a} = \frac{e\mathbf{E}}{m} = -\frac{ekI'(t)}{mr}\hat{\theta}. \tag{15}$$

The change in distance, Δd, due to the acceleration in (15) is

$$\Delta d = \frac{1}{2}a\Delta t^2 = -\frac{ekI'(t)}{2mr}\frac{r\Delta\theta}{v}\Delta t = -\frac{ekI'(t)\Delta\theta\Delta t}{2mv}, \tag{16}$$

where we have taken the odd (but perfectly legal) step of writing one of the Δt factors as $r\Delta\theta/v$ – see Eq. (11). The change in phase, $\delta\alpha_{E-field}$, due to this change in distance, Δd, coming from the acceleration due to the electric field in (14) is just Δd divided by $\frac{\lambda}{2\pi}$ where λ is the de Broglie wavelength of the particle, i.e. $\lambda = \frac{h}{mv}$. Putting all these together gives the phase shift due to the electric field as

$$\delta\alpha_{E-field} = \frac{\Delta d}{\lambda/(2\pi)} = -\frac{ekI'(t)\Delta t\Delta\theta}{2\hbar}. \tag{17}$$

One can see that the AB phase shift due the time variation of the potential given in (13) is canceled exactly by the phase shift due to the effect of the electric field given in (17), i.e. $\delta\alpha_{A(t)} + \delta\alpha_{E-field} = 0$. This leaves only the phase shift due to any initial, static current and magnetic flux.

Thus, both versions of the covariant AB phase, (5) and (6), predict that there will be no time-dependent AB phase shift for the solenoid with a time-dependent current and magnetic flux. For the covariant phase shift expression in terms of the four-potentials (5) this result comes from a cancellation between an AB type phase shift due to the time variation of the vector potential (13) and a phase shift due to the electric field (17). For the covariant phase shift expression (6), this result comes from an equivalent cancellation between the electric contribution (the first three terms of the Faraday 2-form) and the magnetic contribution (the last three terms of the Faraday 2-form).

Previous works on this problem of the time-dependent AB effect [7–9] predicted that one should see a time-dependent phase shift for a time-dependent vector potential and magnetic field. This is in contrast to the prediction from the covariant expressions (5) or (6) that there will be no time-dependent phase shift due to a cancellation between the magnetic and electric parts of these expressions. The suggested [7,8] and performed [9] experiments considered the time-dependent vector potential associated with a laser beam (i.e. a coherent, focused electromagnetic wave). Although the laser system considered in [7–9] is different from the time-dependent solenoid considered here, it is easy to see from the proposed covariant phase shift in terms of the Faraday 2-form as given in (6) that the magnetic field part coming from $\mathbf{B} = \nabla\times\mathbf{A}$ will always cancel the electric field part coming from $\mathbf{E} = -\partial_t\mathbf{A}$. There could still be phase shift coming $\mathbf{E} = -\nabla\phi$ if there is a non-zero scalar potential in addition to the time varying vector potentials (for both the time varying solenoid and the laser system considered in [7–9] $\phi = 0$). Now, the one experiment we did find to actually test the time-dependent AB effect did find *no time varying phase shift* [9] which would then favor our proposed expressions for the covariant AB phase shift (5) or (6). It should be stressed that some of the authors of the experiment [9] argued that their null result was due to inadequacies of the experiment [8]. Further experiments are needed to confirm which prediction is correct.

From the above explicit calculation from Eqs. (10)–(17) of the cancellation of the standard AB phase (13) with the phase contribution coming from the electric field (17) one can surmise that in order to see this effect (or rather non-effect since the two contributions are predicted to cancel) there needs to be some conditions or relationship between the time scale of the variation of the magnetic field, t_B, with respect to the time-scale of the electron to travel from the source to the screen, $t_{electron}$. If one assumes that the magnetic field varies sinusoidally as in [7–9] with a frequency f_B then $t_B = \frac{1}{f_B}$. Assuming that the electrons move with velocity v_e and if the distance between the screen and the electron source is L the time scale of the electrons is $t_{electron} = \frac{L}{v_e}$. Thus, to see this non-effect one needs $t_{electron} \sim t_B = \frac{1}{f_B}$. If one has $t_{electron} \ll t_B = \frac{1}{f_B}$ – the time scale of the electrons is much less that the time scale of the magnetic field variation – then one will get the phase shift of the static situation since the electrons move through the field much faster than it changes so that the field is effectively static. If, on the other hand, $t_{electron} \gg t_B = \frac{1}{f_B}$ – the time scale of the electrons is much greater than the time scale of the magnetic field variation – then the effect of the magnetic field on the phase shift of the electron will time average to zero. For the set-up in [8,9] and the proposed experiment in [7] the speed of the electrons was $v_e \sim 10^7 \frac{m}{s}$. Assuming $L \sim 0.1$ m, one gets $t_e \sim 10^{-8}$ s. Thus, to see this cancellation of the standard AB phase with the phase coming from the electric field $\mathbf{E} = -\partial_t \mathbf{A}$, one needs the magnetic field to vary on a time scale of $t_B \sim 10^{-8}$ s or change with a frequency of $f_B \sim 10^8$ Hz. If one wanted to have a higher/lower frequency, one should adjust the velocity of the electrons to be higher/lower according to the relationship $v_e \sim f_B L$, e.g. for $f_B \sim 10^3$ Hz and with $L \sim 0.1$ m, one should take $v_e \sim 10^2 \frac{m}{s}$.

4. Conclusions

One of the most important phenomena which lies at the interface of gauge theories and quantum mechanics is the Aharonov–Bohm effect [1,2] – an extra phase shift in the interference pattern of the quantum mechanical double-slit experiment due to the presence of electromagnetic vector, $\mathbf{A}$, and scalar, ϕ, potentials. The expressions for the magnetic and electric AB phase differences are given by (2) and (4), respectively. These expressions are non-covariant and thus one can ask for a covariant expression which should combine/unify (2) and (4). In this Letter, we have examined two possible covariant generalizations, namely Eqs. (5) and (6), of the non-covariant electric and magnetic phase differences. Expression (5) was in terms of the space–time line integral of the four-vector potential, and expression (6) was in terms of a space–time surface area integral of the Faraday 2-form. Both expressions reduce to the non-covariant AB phase differences (2) and (4) in the static limit. Additionally, for the time-dependent case of an infinite solenoid both (5) and (6) gave the same, somewhat surprising result that there would be no time-dependent AB phase shift. One would only have whatever static AB-phase shift existed before the start of any time variation of the magnetic flux. For the expression (5) in terms of the space–time line integral of the four-vector potential this null result was the result of the cancellation between a true AB phase shift, i.e. expression (13), and a non-AB type phase shift (17) due to the electric field that exists outside the solenoid in this case. Since the electron for the time-dependent, infinite solenoid did not move in a field-free region (and in addition the force on the electron was not zero) this is some generalized, or hybrid Aharonov–Bohm effect with part of the shift coming from the potential and the other part coming from the fields.

The expression for the AB phase difference given in terms of the Faraday 2-form, (6), shows that the cancellation of the magnetic part of the AB phase coming from $\nabla \times \mathbf{A}$ will generally be canceled by the $-\partial_t \mathbf{A}$ part of the electric part of the AB phase. For the time-dependent case, this then leaves only the part of the electric AB phase coming from the scalar potential ϕ. For the case of a time-varying magnetic field inside an infinite solenoid (or for the laser set-up considered in [7–9]), $\phi = 0$, and thus one gets no time varying AB phase difference.

As a final comment, we note that the expression for the AB phase difference in (5) is essentially a Wilson loop [10] which is used to study the issue of confining versus non-confining phases, i.e. the "area" law versus "perimeter" law, of Yang–Mills gauge theories like QCD. In this work, we are making the suggestion of the equivalence of the "perimeter" integral in (5) with the "area" integral in (6). Thus, it may be of interest to study non-Abelian gauge fields using the proposed AB phase difference given in (6).

Note added

After this paper was accepted for publication, we learned of the related work of [11] and [12] dealing with similar issues and having some of the same conclusions.

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References

[1] Y. Aharonov, D. Bohm, Phys. Rev. 115 (1959) 485.
[2] W. Ehrenberg, R.E. Siday, Proc. Phys. Soc. B 62 (1949) 8.
[3] R.G. Chambers, Phys. Rev. Lett. 5 (1960) 3.
[4] A. Tonomura, et al., Phys. Rev. Lett. 56 (1986) 792.
[5] A. van Oudenaarden, M.H. Devoret, Yu.V. Nazarov, J.E. Mooij, Nature 391 (1998) 768.
[6] L. Ryder, Quantum Field Theory, 2nd edition, Cambridge University Press, Cambridge, 1996 (Section 2.9).
[7] B. Lee, E. Yin, T.K. Gustafson, R. Chiao, Phys. Rev. A 45 (1992) 4319.
[8] A.N. Ageev, S.Yu. Davydov, A.G. Chirkov, Tech. Phys. Lett. 26 (2000) 392.
[9] Yu.V. Chentsov, Yu.M. Voronin, I.P. Demenchonok, A.N. Ageev, Opt. Zh. 8 (1996) 55.
[10] K. Wilson, Phys. Rev. D 10 (1974) 2445.
[11] G. Rousseaux, R. Kofman, O. Minazzoli, Eur. Phys. J. D 49 (2008) 249.
[12] K. Moulopoulos, J. Phys. A 43 (2010) 354019;
K. Moulopoulos, J. Mod. Phys. 2 (2011) 1250.