Does the holographic principle determine
the gravitational interaction?

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Abstract

It is likely that the holographic principle will be a consequence of the would be theory of quantum gravity. Thus, it is interesting to try to go in the opposite direction: can the holographic principle fix the gravitational interaction? It is shown that the classical gravitational interaction is well inside the set of potentials allowed by the holographic principle. Computations clarify which role such a principle could have in lowering the value of the cosmological constant computed in QFT to the observed one.
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1. Introduction

One of the most promising route towards a deeper understanding of quantum gravity is holography (two detailed reviews are [1,2]). The pioneering ideas of Bekenstein [3], 't Hooft [4] and Susskind [5] shed light on a very peculiar characteristic of gravitational field which is very likely to survive in the final theory of quantum gravity. While in quantum field theory (henceforth QFT), the number of degrees of freedom of a given space-like region is proportional to the volume of the region itself, if the gravitational effects are taken into account such a number will appear to be proportional to the surface of the region. In the elegant framework of [6,7] which refined the works [3,4] and [5], the above statement on the degrees of freedom is translated in a covariant entropy bounds; also a formulation of a causal entropy bound [8] is possible which discloses other important aspects of the above ideas.

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An explicit and highly non-trivial realization in (super)string theory of the holographic principle is the AdS/CFT correspondence, first introduced by Maldacena [9], in which the role of (super)gravity in decreasing the number of degrees of freedom, which one would naively expect on QFT grounds, is manifest. Such a decreasing of the number of degrees of freedom could also have important consequences, as far as the cosmological constant is concerned, since the striking disagreement between the cosmological constant computed in QFT and the observed one of about 120 orders of magnitude (the situation will be slightly better if supersymmetry is introduced but it remains extremely unpleasant) is likely to be related to an overcounting of degrees of freedom in QFT. It is now commonly believed that the property of the number of degrees of freedom to be proportional to the area of a (suitable defined) surface enclosing them will be a distinguishing feature of quantum gravity. The problem is, however, that a theory of quantum gravity is not available yet. A fruitful point of view which could give even stronger physical basis to the holographic principle is the following: since the quantum gravity is still lacking and we cannot deduce from it the holographic principle, can we try to go in the opposite direction? That means, from the holographic principle is it possible to deduce (at least) the classical gravitational interaction?

In this Letter we move some preliminary steps in this direction. We consider a closed volume containing a finite but large number of particles interacting with an a priori unknown potential. The question we try to address is: which form the potential has to have in order for the entropy to be proportional to the area of the surface which encloses the particles? The computations are almost entirely classical, nevertheless quantum mechanics has a very important role in specifying some hypothesis. The results are very encouraging: the gravitational potential is very likely to be the unique answer to the above question. This analysis also clarifies the role of the holographic principle in decreasing the QFT-computed cosmological constant to the observed value.

2. The method

Let us consider a spherical[1] three-dimensional region M with diameter 2ρ. Let N denote the number of particles and u be the interaction potential. Since we will work at a constant density of particles the number of particles N is proportional to the volume $V(M)$ of M:

$$N = c_1 V(M), \tag{1}$$

c_1 being a constant with the dimension of an inverse volume proportional to the density of the gas.[2] The classical partition function can be written as follows:

$$Z_\beta = \int \prod_{i=1}^{N} d^3 p_{(i)} \, d^3 q_{(i)} \exp\left[-\beta \sum H_{(i)}\right] \tag{2}$$

$$= \left(\frac{2\pi}{\beta}\right)^{\frac{3N}{2}} \int_{M^N} \prod^{i=1,\dots,N} d^3 q_{(i)} \exp\left[-\beta \sum_{i,j|i\neq j} u\big(\vec{q}_{(i)}, \vec{q}_{(j)}\big)\right], \tag{3}$$

$$H_{(i)} = \frac{1}{2}\big(\vec{p}_{(i)}\big)^2 + \sum_{j|j\neq i} u\big(\vec{q}_{(i)}, \vec{q}_{(j)}\big), \tag{4}$$

where β is a positive real number which can be interpreted as the inverse temperature, $\vec{q}_{(i)}$ and $\vec{p}_{(i)}$ are the position (which is assumed to vary in $M \subset R^3$) and momenta of the ith particle and u is the potential which, as usual, is assumed to be a binary interaction. The mass of the particles have been set to one. It is worth to note that, in this

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[1] The hypothesis of sphericity is not necessary, it only simplifies computations but the same conclusions can be reached by dropping it.

[2] In this section we will not pay much attention to the physical dimensions of the parameters of the model. A concrete example of the numerical estimates that one can get will be given in the last section.

"almost classical" model in which quantum mechanics will only enter through some constraints on the potential and on the parameters of the model, one can trust computations only above some temperature below which quantum effects cannot be neglected anymore. We will set the critical value of β equal to one (in suitable units): $\beta_c = 1$ and we will assume that

$$\beta < \beta_c = 1.$$

To be more precise, we could leave β unspecified, in this case computations themselves would tell that $\beta < 1$.

The first restriction is to assume u to be a non-decreasing function of the Euclidean distance between the ith and the jth particles. The point is that if u does not increase with the distance then the potential could not be compatible with bound states and, in such a case, it is very likely that the holographic bound cannot be fulfilled. This fact can be explained as follows: gravitational interaction is able to lower the effective degrees of freedom because it is always attractive so that many bound states can be formed and such bound states behave, in many respects, as single particles. If the potential decreases with the distance then the particles cannot form bound states because they prefer to be far apart. The second hypothesis will be to take the interaction potential depending only on the euclidean distance between the ith and the jth particles:

$$u = u\big(|\vec{q}_{(i)} - \vec{q}_{(j)}|\big). \tag{5}$$

This simplifying assumption is not very restrictive, the following computations can be generalized also to more general ansatz for the interaction potential. For example, if one assume that

$$u = u'\big(|\vec{q}_{(i)} - \vec{q}_{(j)}|\big) f(\theta_{(i)}, \theta_{(j)}, \xi_{(i)}, \xi_{(j)}), \qquad 0 < \alpha_1 \leqslant f(\theta_{(i)}, \theta_{(j)}, \xi_{(i)}, \xi_{(j)}) \leqslant \alpha_2, \tag{6}$$

where α_i, $i = 1, 2$, are two positive numerical constants, $\theta_{(i)}$ are the angular coordinates of $\vec{q}_{(i)}$ and $\xi_{(i)}$ are some internal coordinates characterizing the ith particle, then one will obtain qualitatively the same results provided u' is a non-decreasing function of the Euclidean distance between the ith and the jth particles.

In principle, since we have at our disposal the interaction potential, we can compute the entropy:

$$S_\beta = -\beta \partial_\beta \ln Z_\beta = \frac{c_1 V(U) I_\beta - \beta \partial_\beta I_\beta}{I_\beta}, \tag{7}$$

$$I_\beta = \int_{U^N} \prod^{i=1,\ldots,N} d^3 q_{(i)} \exp\left[-\beta \sum_{i,j | i \neq j} u\big(|\vec{q}_{(i)} - \vec{q}_{(j)}|\big)\right], \tag{8}$$

where U is any region contained in M and Eqs. (3) and (1) have been used. Now, the problem is to find which kind of potentials fulfils the condition

$$S_\beta = c_2 A(\partial U), \tag{9}$$

where $A(\partial U)$ is the area of the boundary ∂U of the region U and c_2 is a dimensional constant to be specified later on. The above holographic requirement can be thought as a differential equation for the integral I_β defined in Eq. (8):

$$\frac{c_1 V(U) I_\beta - \beta \partial_\beta I_\beta}{I_\beta} = c_2 A(\partial U). \tag{10}$$

The above equation can be easily solved as follows

$$I_\beta = k \beta^{[c_1 V(U) - c_2 A(\partial U)]}, \tag{11}$$

where k is an integration constant. Thus, we have to deduce which conditions on the potential (remember that I_β depends on the potential through Eq. (8)) the above form of I_β implies. At a first glance, it seems unlikely for Eq. (11) to be able to severely constrain the interaction potential since, roughly speaking, many functions can have

the same integral. In fact, by taking into account that Eq. (8) should hold for any fixed $\beta < 1$ and for any volume not too small (in a sense which will be clarified later on) contained in M and containing all the particles as well as the line segment joining them, it will be possible to find powerful restrictions on the form of u. In other words, with a suitable choice of the region contained in M (as we will now see, it will be important to consider a region whose area and volume have non-trivial scaling with ρ), Eq. (11) will disclose the physical content of the holographic principle.

2.1. Constraints on the parameters

The role of quantum mechanics will be to translate the fact that this model can be trusted only in a classical regime into inequalities between the parameters. The first constraint is related to the range of the argument $|\vec{q}_{(i)} - \vec{q}_{(j)}|$ of the potential which cannot be to small

$$\frac{\rho}{l_0} \leqslant |\vec{q}_{(i)} - \vec{q}_{(j)}| \leqslant 2\rho, \quad l_0 \gg 1, \tag{12}$$

l_0 being a large positive number to be fixed. The above inequality is related to the fact that the classical form of an interaction potential is valuable only above a certain length scale below which some unknown quantum effects set in. Thus, the number l_0 is related not only to the arising of quantum effects but also to the potential.

The second constraint is purely quantum mechanical in nature and is only related to holography: it deals with the minimum length below which we cannot resolve anymore distinct particles inside M due to quantum effects. A reasonable order of magnitude for such a scale, which is suggested by the holographic principle is

$$\rho_{\min} = \frac{a}{\rho^2}, \tag{13}$$

where a is a constant with the dimension of a volume. The point is that, according to the holographic bounds, ρ^2 should be an upper bound for the total entropy of M. The more the entropy, the more different quantum states are available inside M; the more different quantum states, the easier will be to distinguish distinct particles. In other words, if inside M only few quantum states are available, for example, only one, then it will be impossible to distinguish distinct particles with physical measurements since they will be in a single quantum entangled states which would be destroyed by an external action.[3]

It is worth to note that this hypothesis is rather natural since, as it is well known, in the limit of large quantum numbers the quantum effects can be neglected (see, for example, [10,11]).

3. The model

The first problem, in trying to understand which are the possible forms of the interaction potential compatible with Eq. (11), is that it is a rather unusual constraint. In order to analyze Eq. (11), it is possible to develop an approximate method which gives very accurate results when the number of particles is large enough.

The geometrical setting is the following. Let us denote with T_{ij} a cylinder connecting the particle i and the particle j. Let the diameter of the section of T_{ij} be equal to $2\rho_{\min}$ (which is defined in Eq. (13)) in such a way that inside the cylinder T_{ij} there are only the particle i and the particle j. Remember that, in any case, the distance between every pair of particles has to be greater than ρ/l_0 and smaller than 2ρ so that the minimum height for the cylinder will be ρ/l_0 and the maximum height will be 2ρ. Thus, the diameter of the section of the cylinders is

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[3] Another way to argue that the above order of magnitude is reasonable is to consider a length scale of order ρ/N where N is the total number of particles. Because of Eq. (1), one can assume $N \sim \rho^3$ so that the minimum radius below which we cannot resolve two particles anymore will be $\propto \rho^{-2}$. However, the first argument is more sound since it is "purely holographic" and it does not refer to Eq. (1).

related to the smallest geometric scale which is resolvable with physical measurements, while the minimum height is related to the minimum scale below which the potential cannot work well anymore. It is worth to note here that, if two cylinder intersect, for example, T_{ij} and T_{kl}, we can break any of the two cylinder into two cylinders (T_{ij} goes into T_{ip} and T_{pj} while T_{kl} goes into T_{kp} and T_{pl}) and put in the intersection a new particle p: we are allowed to do in this way since, at the end of the computations, we will make the limit of a very large number of particles. For this reason we can simply consider a net of non-intersecting cylinders.

Hence, we have the following inequalities for the volumes (V) and the areas (A) of the cylinders T_{ij}:

$$\pi a^2 \frac{1}{l_0 \rho^3} \leqslant V(T_{ij}) \leqslant 2\pi a^2 \frac{1}{\rho^3}, \qquad \pi a \frac{1}{l_0 \rho} \leqslant A(\partial T_{ij}) \leqslant \pi a \frac{1}{\rho}. \tag{14}$$

Extending the above inequalities to the union[4]

$$\Gamma = \bigcup_{\substack{i \neq j}}^{1 \leqslant i,j \leqslant N} T_{ij}, \tag{15}$$

of all $(N-1)N/2$ cylinders, we obtain

$$V_{\min}(\Gamma) = \frac{(N-1)N}{2} \pi a^2 \frac{1}{l_0 \rho^3} \leqslant V(\Gamma) \leqslant \frac{(N-1)N}{2} 2\pi a^2 \frac{1}{\rho^3} = V_{\max}(\Gamma), \tag{16}$$

$$A_{\min}(\partial \Gamma) = (N-1)N \pi a \frac{1}{l_0 \rho} \leqslant A(\partial \Gamma) \leqslant 2(N-1)N \pi a \frac{1}{\rho} = A_{\max}(\partial \Gamma). \tag{17}$$

The holographic principle (9) has to hold for any connected subset of M containing all the particles and, in particular, it has to hold for Γ defined in Eq. (15). Since u is supposed to be a non-decreasing function of the distance between i and j one gets:

$$-\beta u\left(\frac{\rho}{l_0}\right) \geqslant -\beta u\left(\left|\vec{q}_{(i)} - \vec{q}_{(j)}\right|\right) \geqslant -\beta u(2\rho). \tag{18}$$

On the other hand, Eq. (11) must hold, so that from Eqs. (16), (18) and (8) it follows (remember that $0 < \beta < 1$)

$$k\beta^{[c_1 V_{\max}(\Gamma) - c_2 A_{\min}(\partial \Gamma)]} \leqslant k\beta^{[c_1 V(\Gamma) - c_2 A(\partial \Gamma)]} = I_\beta \leqslant \exp\left[-\frac{(N-1)N}{2} \beta u\left(\frac{\rho}{l_0}\right)\right] V(\Gamma)^N$$

$$\leqslant \frac{[(N-1)N\pi a^2]^N}{\rho^{3N}} \exp\left[-\frac{(N-1)N}{2} \beta u\left(\frac{\rho}{l_0}\right)\right], \tag{19}$$

$$k\beta^{[c_1 V_{\min}(\Gamma) - c_2 A_{\max}(\partial \Gamma)]} \geqslant k\beta^{[c_1 V(\Gamma) - c_2 A(\partial \Gamma)]} = I_\beta \geqslant \exp\left[-\frac{(N-1)N}{2} \beta u(2\rho)\right] V(\Gamma)^N$$

$$\geqslant \frac{\left[\frac{(N-1)N}{2} \frac{\pi a^2}{l_0}\right]^N}{\rho^{3N}} \exp\left[-\frac{(N-1)N}{2} \beta u(2\rho)\right], \tag{20}$$

where $V_{\max}(\Gamma)$, $V_{\min}(\Gamma)$, $A_{\min}(\partial \Gamma)$ and $A_{\max}(\partial \Gamma)$ have been defined in Eqs. (16) and (17). From Eqs. (19) and (20) it follows

$$c_1 (\ln \beta)(N-1)N \frac{\pi a^2}{\rho^3} + \frac{(N-1)N}{2} \beta u\left(\frac{\rho}{l_0}\right) \leqslant c_2 (\ln \beta)(N-1)N \frac{\pi a}{l_0 \rho} + L_{\max} - 3N \ln \rho, \tag{21}$$

$$c_1 (\ln \beta)(N-1)N \frac{\pi a^2}{2 l_0 \rho^3} + \frac{(N-1)N}{2} \beta u(2\rho) \geqslant 2 c_2 (\ln \beta)(N-1)N \frac{\pi a}{\rho} + L_{\min} - 3N \ln \rho, \tag{22}$$

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[4] The union Γ is a sort of fat graph whose vertices are the particles and whose bold links are the cylinders.

$$L_{\max} = \ln\left\{\frac{[(N-1)N\pi a^2]^N}{k}\right\}, \qquad L_{\min} = \ln\left\{\frac{[(N-1)N\frac{\pi a^2}{2l_0}]^N}{k}\right\}. \tag{23}$$

This is an important point: if we would perform the above computations in a larger or a smaller[5] spherical region, the only change in Eqs. (21) and (22) would be the replacement of ρ with the new radius r which, therefore, can be identified with the argument of the potential. Thus, in Eqs. (21) and (22) we can rescale the argument of u obtaining the following bounds:

$$u(r) \leqslant \left(\frac{c_2(\ln\beta)2\pi a}{\beta l_0^2}\right)\frac{1}{r} - \left(\frac{c_1(\ln\beta)2\pi a^2}{\beta l_0^3}\right)\frac{1}{r^3} - \frac{6N}{\beta N(N-1)}\ln(l_0 r) + \Lambda_{\max}, \tag{24}$$

$$u(r) \geqslant \left(\frac{c_2(\ln\beta)8\pi a}{\beta}\right)\frac{1}{r} - \left(\frac{c_1(\ln\beta)8\pi a^2}{\beta l_0}\right)\frac{1}{r^3} - \frac{6N}{\beta N(N-1)}\ln\left(\frac{r}{2}\right) + \Lambda_{\min}, \tag{25}$$

$$\Lambda_{\max} = \frac{2}{\beta N(N-1)}L_{\max}, \qquad \Lambda_{\min} = \frac{2}{\beta N(N-1)}L_{\min}, \tag{26}$$

where $L_{\max}$ and $L_{\min}$ have been defined in Eq. (23). Eventually, by taking the limit for $N \to \infty$, the result is

$$-\frac{G_{\max}}{r} + \frac{C_{\max}}{r^3} \leqslant u(r) \leqslant -\frac{G_{\min}}{r} + \frac{C_{\min}}{r^3}, \tag{27}$$

$$G_{\max} = \left|\frac{c_2(\ln\beta)8\pi a}{\beta}\right|, \qquad G_{\min} = \left|\frac{c_2(\ln\beta)2\pi a}{\beta l_0^2}\right|, \tag{28}$$

$$C_{\max} = \left|\frac{c_1(\ln\beta)8\pi a^2}{\beta l_0}\right|, \qquad C_{\min} = \left|\frac{c_1(\ln\beta)2\pi a^2}{\beta l_0^3}\right|, \tag{29}$$

where

$$r > 3\frac{ac_1}{l_0 c_2}, \tag{30}$$

otherwise the hypothesis that u is a non-decreasing function would be violated.

4. Physical implications

First of all, the inequalities (24) and (25) will be consistent only when $\beta < 1$ in such a way that the terms in round brackets multiplying $1/r$ are negative. We observe from Eq. (27) that, for r big enough, the terms of order $1/r^3$ are negligible and u is constrained to lie between two Newtonian-like potentials. It is remarkable that the Newtonian terms only depends on purely "holographic" constants. That is, the constants in the round brackets multiplying $1/r$ only depend on β, a and c_2.

This is a highly non-trivial self-consistency check of the fact that the holographic principle can really fix the form of the gravitational interaction: the constant c_2 is the constant multiplying the area of M in the holographic constraint (9) and also a enters in the definition of the holographic length scale (13). Thus, the Newtonian part is constrained only by the holographic principle (as it should be), while the $1/r^3$ terms, which do not vanish for $N \to \infty$, contain the proportionality constant between the volume and the entropy in the limit of vanishing potential.

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[5] The new radius can be taken larger than ρ; it can also be smaller than ρ provided that all the particles are contained in the new spherical region and Eq. (30) is fulfilled, otherwise the results would change as it happens when one consider the gravitational potential outside and inside a sphere of constant density.

Another important aspect of the above model is the following. In our universe, the number of particles is very large but, actually, it is not infinite. For this reason, there is the possibility of a very small constant to be added to the potential. The inequalities (24) and (25) tell us that the magnitude of such a constant should lie between $\Lambda_{\min}$ and $\Lambda_{\max}$ defined in Eq. (26). It is very interesting to note that in $\Lambda_{\min}$ and $\Lambda_{\max}$, besides a geometrical factor, it appears a factor $(\log N)/N$ which (being $N \sim 10^{120}$ a reasonable estimate of the total number of particles in the universe) is of order 10^{-118}. Such a small factor could resolve the problem of the too large value QFT-computed cosmological constant: if the coupling of gravity with quantum fields is such that holography is preserved, then the quantized version of the holographic constraint (9) could provide the right factor to suppress the "bare" QFT cosmological constant. The holographic principle, by properly taking into account the effects of gravity on the degrees of freedom, could renormalize the "bare" QFT cosmological constant with a very small factor. An intuitive explanation of this fact is that QFT counts as distinct pair of degrees of freedom which, in fact, coupled by gravity behave as single degrees of freedom and should, therefore, not be overcounted.

A more formal way to understand this fact, which also clarify that the above argument could also hold at a quantum level, is the following. One of the main reason behind the fact that in QFT the computed cosmological constant is too large is that in the path integral approach one performs the diagrammatic expansion starting from classical vacua. On the other hand, classical vacua (that is, stable solutions of the classical equations of motions) are invariant if we add to the Hamiltonian a constant term so that they cannot provide us with an energy scale. The energy scale ("zero point energy") of QFT is a purely quantum effects which, when renormalized, is of the order of the UV cutoff and, therefore, too large. If the coupling with gravity is standard (via Lagrangian or Hamiltonian), then the above problem, at a first glance, will be only slightly softened for the same reasons as above. Instead, let us consider an equation like

$$-\beta \partial_\beta \ln Z_\beta = \frac{1}{4L_P^2} A(\partial M), \tag{31}$$

$$Z_\beta = \int \left[D\Phi^a Dg_{\mu\nu} D\pi_{\mu\nu} \right] \big|_{B.C. \to \partial M} \exp\left[-\beta \int\limits_M H\left(\Phi^a, g_{\mu\nu}, \pi_{\mu\nu}\right) d\mu \right], \tag{32}$$

where L_P is the Planck length, D is the standard notation for the "path" integration, Φ^a is a collective symbol to denote the set of quantum fields (besides gravity) and their conjugated momenta (whose phase space variables are denoted with $g_{\mu\nu}$ and $\pi_{\mu\nu}$), μ the Lebesgue measure on M and the symbol $B.C. \to \partial M$ means that we must impose suitable boundary condition on the fields as they approach the boundary of M. Gravity should be introduced by requiring that the metric needed to compute the curved partition function (32) is such that Eq. (31) is fulfilled.

Although to solve explicitly this problem seems to be a rather hopeless task, a very interesting result is now apparent: we are not free anymore to add an arbitrary constant to the density of Hamiltonian, since Eq. (31) is not invariant under the transformation

$$H \to H + \text{const}, \tag{33a}$$

in other words, an holographic equation like Eq. (31) set a scale for the zero point energy.

As it has been shown at the end of Section 3, when one takes into account that the number of particles is very large but finite, the gravitational potential $u(r)$ is determined up to a constant $\Lambda_0 \in [\Lambda_{\min}, \Lambda_{\max}]$ which may be naturally interpreted as the zero point energy $H(0, g_{\mu\nu} = \eta_{\mu\nu}, 0)$. Even if a rigorous quantum computation is required to obtain the exact value of Λ_0, it is nevertheless interesting to note that Eq. (31) tells that Λ_0 should be positive and of the order (such an estimate could be improved by a careful quantum computation)

$$\frac{1}{4L_P^2} A(\partial M) = -\beta \partial_\beta \ln \left(\int \left[D\Phi^a \, Dg_{\mu\nu} \, D\pi_{\mu\nu} \right] \exp \left[-\beta \int_M \Lambda_0 \, d\mu \right] \right)$$

$$= -\beta \partial_\beta \left(-\beta \Lambda_0 \int_M d\mu \right) = \beta V(M) \Lambda_0. \tag{34}$$

From the above equation we obtain

$$\Lambda_0 \sim H(0, g_{\mu\nu} = \eta_{\mu\nu}, 0) \sim \frac{1}{4\beta L_P^2} \frac{A(\partial M)}{V(M)} \sim M_P^4 \left(\frac{k_b T}{M_P} \right) \left(\frac{L_P}{L_U} \right),$$

where M_P is the Planck mass and $L_U = V(M)/A(\partial M)$ should be a length measuring the size of the space–time region past causally connected today. If we take as L_U the Hubble radius today and as T the cosmic temperature today we get

$$\Lambda_0 \sim M_P^4 \times 10^{-95}.$$

This result is very promising if we consider that, in this computation (even if QFT has been taken into account in a rather rough way) it has been achieved a striking reduction (of, at least, 30 orders of magnitude) of the values of the cosmological constant which are usually obtained in different context (QFT, SUSY QFT, SUGRA and so on) without introducing SUSY, extra dimensions or fine-tuning of any kind. Eq. (34) also suggests that, in the past, the cosmological "constant" should had been (much) greater. This implies a modification of Einstein equations. The necessity of some kind of modification of the Einstein equations is, today, widely recognized due, for example, to the experimental data on the accelerated expansion. The best explanation of the actual experimental data would be a "varying" cosmological constant much higher in the past than now (see, for example, [12]). On the other hand, there is not in the standard approaches a natural way to achieve this goal. Usually, non-minimally coupled scalar fields (whose physical origin is, however, unknown) are introduced in order to imitate the behavior of a "varying" Λ term (see, for example, [12]). In fact, the holographic principle can give rise to results in a much better agreement with experimental data providing, at the same time, with a more natural explanation of a Λ decreasing with time. On the theoretical side, it would be also very interesting to try to repeat the previous computation in curved backgrounds. In particular, we would like to give an interpretation of the type here proposed to the AdS/CFT correspondence: this could disclose, for example, a sharp relation between the cosmological constant and the number of the elementary degrees of freedom without using CFT. However, the difficulties related to a curved background have not allowed, up to now, this ambitious achievement.

5. Conclusion

In this Letter an alternative point of view to analyze the relations between gravity and holography has been proposed. It is commonly believed that the holographic principle will be a corollary of the final theory of quantum gravity which, on the other hand, is not available yet. For this reason, it is interesting to try to go in the opposite direction: is it possible to deduce the gravitational interaction from the holographic principle? Here, the first preliminary steps in this direction have been performed. It has been shown that the classical gravitational potential belongs to the rather narrow region allowed by the holographic principle. It has been clarified the physical mechanism responsible for the smallness of the observed cosmological constant with respect the one computed in QFT: QFT overcounts pairs of degrees of freedom which, in fact, coupled by gravity, behave as single degrees of freedom and should not be overcounted. The holographic principle could face with this overcounting thanks to a very small multiplying factor. Even if the computation are entirely classical, it has been argued that the same could hold at a quantum level; a promising estimate of the order of magnitude of the cosmological constant has also been obtained. The reason for this is that Eq. (31) is not invariant when we add to the density of Hamiltonian a constant.

In other words, the holographic principle encodes a natural scale for the cosmological constant which is likely to survive also at a quantum level.

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