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Linear Algebra and its Applications 428 (2008) 2805–2812
LINEAR ALGEBRA AND ITS APPLICATIONS
www.elsevier.com/locate/laa

Lineability in subsets of measure and function spaces ☆

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Received 21 December 2007; accepted 11 January 2008
Available online 25 February 2008
Submitted by R.A. Brualdi

Abstract

We show, among other results, that if λ denotes the Lebesgue measure on the Borel sets in [0, 1] and X is an infinite dimensional Banach space, then the set of measures whose range is neither closed nor convex is lineable in $ca(\lambda, X)$. We also show that, in certain situations, we have lineability of the set of X-valued and non-σ-finite measures with relatively compact range. The lineability of sets of the type $L^p(I)\setminus L^q(I)$ is studied and some open questions are proposed. Some classical techniques together with the converse of the Lyapunov Convexity Theorem are used.

AMS classification: Primary 28A20, 46E15; Secondary 46E30

Keywords: Lineability; Spaceability; Linear spaces; Measure space; Injective measure; Function spaces

1. Introduction and preliminaries

In the last years several results concerning the linear structure of certain subsets of functions verifying some *special* (and *apparently uncommon*) properties have been appearing in Mathemat-

☆ The first and fourth authors were supported by MTM 2006-03531.
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0024-3795/$ - see front matter
doi:10.1016/j.laa.2008.01.008

ical Analysis. Large vector spaces of continuous nowhere differentiable functions, everywhere surjective functions, or differentiable nowhere monotone functions have been constructed in the past. Moreover, new mathematical terminology has been also introduced. Following this terminology, we say that a subset M of functions satisfying such a property is *spaceable* if $M \cup \{0\}$ contains a closed infinite dimensional subspace. The set M will be called lineable if $M \cup \{0\}$ contains an infinite dimensional vector space. At times, we will be more specific, referring to the set M as μ-lineable if it contains a vector space of dimension μ. This terminology of *lineable and spaceable* was first introduced in [2,3,9,14].

Some of these special properties are not isolated phenomena. In [3] it was shown that the set of everywhere surjective functions is 2^c-lineable and that the set of differentiable functions on $\mathbb{R}$ which are nowhere monotone is lineable in $\mathscr{C}(\mathbb{R})$. Fonf et al. showed [10] that the set of nowhere differentiable functions on [0, 1] is spaceable in $\mathscr{C}[0, 1]$. Some of these pathological behaviors occur in really interesting ways. For instance Hencl [15] showed that any separable Banach space is isometrically isomorphic to a subspace of $\mathscr{C}[0, 1]$ whose non-zero elements are nowhere approximately differentiable and nowhere Hölder. Also, in [17], the third and fourth authors showed that, in certain situations, there is lineability in the set of bounded linear and non-absolutely summing operators. They also provided examples of large subspaces inside $\Pi_p(E, F) \setminus I_p(E, F)$, the set of non-$p$-integral p-summing operators. For further new results regarding Banach spaces and Banach algebras of functions enjoying some of these so called *pathological* properties we refer the interested reader to [1,5,11,12] and, for results related to lineability and non-measurability, we refer to [13].

Our work here continues this ongoing search for sets of functions enjoying special properties. Our main results in Section 2 are related to some pathologies and special properties of measures like, for instance, the converse of Lyapunov convexity theorem. The main results in this section will be the following.

Proposition 2.2. *Let $(\Omega, \Sigma, \lambda)$ be the measure space on the unit interval and $1 \leqslant p < \infty$. Then the set of ℓ_p-valued measures with relatively compact range and such that their variation measure takes the value infinity on every non-null set is lineable in $cca(\Omega, \Sigma, \ell_p)$.*

Theorem 2.4. *Let λ be the Lebesgue measure on the Borel sets in* [0, 1]*, and let X be an infinite dimensional Banach space. Then the set of injective measures is lineable in $ca(\lambda, X)$.*

Corollary 2.5. *Let λ be the Lebesgue measure on the Borel sets in* [0, 1]*, and X be an infinite dimensional Banach space. Then the set of measures whose range is neither closed nor convex is lineable in $ca(\lambda, X)$.*

Some of the techniques we use to prove the above results are inspired in arguments from [4].

Section 3 will deal with the lineability of subsets of classical ℓ_p spaces and sets of the type $L^p(I) \setminus L^q(I)$ for I either a bounded or unbounded interval, showing, by means of classical real analysis techniques, that:

Theorem 3.3. *If $1 \leqslant p < q$ then the set $L^p[0, 1] \setminus L^q[0, 1]$ is c-lineable.*

Theorem 3.4. *If $p > q \geqslant 1$ and I any unbounded interval then the set $L^p(I) \setminus L^q(I)$ is c-lineable.*

Theorem 3.5. *If $p > q \geqslant 1$ then the set $\ell_p \setminus \ell_q$ is c-lineable.*

Finally, we will conclude by proposing some questions and open problems. Most of the notation is from the category of measure and function spaces and rather usual, otherwise we will give definitions when necessary.

2. Some pathologies in measure spaces

It was proved in [19, p. 90] and in [16] that if X is an infinite dimensional Banach space then there exists a non-trivial X-valued measure with relatively compact range such that its variation measure assumes the value infinity on every non-null set. Let (Ω, Σ) be a measurable space and let X be an infinite dimensional Banach space. We denote by $ca(\Omega, \Sigma, X)$ the Banach space of all countably additive measures $\mu : \Sigma \longrightarrow X$ endowed with the semi-variation norm

$$\|\mu\| = \sup\{\|\mu(E)\| : E \in \Sigma\}.$$

Also, let $cca(\Omega, \Sigma, X)$ denote the closed subspace of $ca(\Omega, \Sigma, X)$ consisting of measures with relatively norm compact range. Given a positive measure λ, we denote by $ca(\lambda, X) = \{\mu \in ca(\Omega, \Sigma, X) : \mu \ll \lambda\}$ where, as usual, $\mu \ll \lambda$ means the absolute continuity of μ with respect to λ.

Before stating and proving the main results of this section, recall that, given a measure μ, its variation is defined as

$$|\mu|(E) = \sup\left\{\sum_{i=1}^{m} \|\mu(E_i\| : E_i\text{'s pairwise disjoint and } \bigcup_{i=1}^{m} E_i = E\right\}.$$

Let us also recall the following technical lemma, whose proof can be found in [17].

Lemma 2.1. *Let $\{a_n\}_{n\in\mathbb{N}}$ be a sequence of positive real numbers. If $\sum_{n=1}^{\infty} a_n = +\infty$ then there exists $\{A_i\}_{i\in\mathbb{N}} \subseteq \mathbb{N}$ such that*:

(i) $|A_i| = \omega_0$ *for each $i \in \mathbb{N}$, where $|A|$ denotes the cardinality of A, and $\omega_0 = |\mathbb{N}|$,*
(ii) $A_i \cap A_j = \varnothing$ *if $i \neq j$, and*
(iii) $\sum_{m\in A_i} a_m = +\infty$ *for each $i \in \mathbb{N}$.*

Throughout this paper we will apply the previous lemma several times to the harmonic series $\{\frac{1}{n}\}_{n\in\mathbb{N}}$. The following proposition, where we consider classical ℓ_p spaces, will serve as an example.

Proposition 2.2. *Let $(\Omega, \Sigma, \lambda)$ be the measure space on the unit interval and $1 \leqslant p < \infty$. Then the set of ℓ_p-valued measures with relatively compact range and such that their variation measure takes the value infinity on every non-null set is lineable in $cca(\Omega, \Sigma, \ell_p)$.*

Proof. We will denote by e_n the usual unit vector of ℓ_p, let $r_n : [0, 1] \longrightarrow \{-1, 1\}$ be the sequence of Rademacher functions and we denote with λ the Lebesgue measure on the Borel sets in $[0, 1]$. Now, consider a sequence $\{A_n\}_{n\in\mathbb{N}}$ of subsets of $\mathbb{N}$ as in Lemma 2.1. Next, let us define $\mu_n : \Sigma \longrightarrow \ell_p$ by

$$\mu_n(E) = \sum_{j\in A_n} \left[\left(\int_E r_j(t)\mathrm{d}\lambda(t)\right) \cdot \frac{e_j}{j^{\frac{1}{p}}}\right]. \tag{1}$$

First of all let us notice that each μ_n is in $cca(\Omega, \Sigma, \lambda)$. In order to do this, recall from [8] that a subset K of ℓ_p is relatively norm compact if and only if K is norm bounded and

$$\lim_{n\to\infty}\sum_{i\geqslant n}|a_i|^p=0$$

uniformly on $a \in K$.

Furthermore fix $n \in \mathbb{N}$ and notice that, trivially, $\mu_n(\Sigma)$ is norm bounded (it actually is weakly compact). Secondly we need to show that

$$\lim_{m\to\infty}\sum_{i\in A_n, i\geqslant m}\frac{1}{i}\left|\int_E r_i(t)\mathrm{d}\lambda(t)\right|^p=0$$

uniformly for $E \in \Sigma$.

(1) The case $p = 1$ follows from the fact that ℓ_1 has the Schur property.
(2) For $1 < p < 2$, we recall a well known result by Rosenthal [18] stating that every operator $L_\infty[0, 1] \longrightarrow \ell_p$ is compact.
(3) If $p \geqslant 2$, we fix $m \in \mathbb{N}$. Then:

$$\sum_{i\in A_n, i\geqslant m}\frac{1}{i}\left|\int_E r_i(t)\mathrm{d}\lambda(t)\right|^p\leqslant\frac{1}{m}\sum_{i\in A_n, i\geqslant m}\left|\int_E r_i(t)\mathrm{d}\lambda(t)\right|\leqslant\frac{1}{m}\lambda(E)\leqslant\frac{1}{m}.$$

Therefore the sequence $\{\mu_n\}_n$ belongs to $cca(\Omega, \Sigma, X)$.

Now, suppose that there exist $n \in \mathbb{N}$, $A \in \Sigma$ with $\lambda(A) > 0$ and with $|\mu_n|(A)$ finite. Since ℓ_p has the Radon–Nikodym property (see, e.g. [7]) then there exists a Bochner integrable function $g : A \longrightarrow \ell_p$ such that

$$\mu_n(E)=\int_E g(t)\mathrm{d}\lambda(t)\quad\text{for all } E\in\Sigma \text{ with } E\subset A.$$

Using the original expression of μ_n, (1), we obtain

$$g(t)=\sum_{j\in A_n} r_j(t)\cdot\frac{e_j}{j^{\frac{1}{p}}}\quad\text{a.e. } t\in A.$$

In particular, since $|r_j(t)| = 1$ for each $t \in [0, 1]$ we obtain (Lemma 2.1) that

$$\|g(t)\|_{\ell_p}=\sum_{j\in A_n}\frac{1}{j}=\infty,$$

a clear contradiction.

Also, $\{\mu_n\}_n$ is a linear independent sequence in $cca(\Omega, \Sigma, \ell_p)$ since the μ_n's have disjoint supports. Finally, notice that every element of $\operatorname{span}\{\mu_n : n \in \mathbb{N}\}_n$ has the property that its variation measure is infinity for non-null elements. It suffices to check this for a linear combination of two elements. Let $c_1, c_2 \in \mathbb{R}$ and $n_1, n_2 \in \mathbb{N}$. Since

$$(c_1\mu_{n_1}+c_2\mu_{n_2})(E)=\sum_{j\in A_{n_1}\cup A_{n_2}}\int_E s_j(t)\mathrm{d}\lambda(t)\cdot\frac{e_j}{j^{\frac{1}{p}}},\tag{2}$$

where $s_j(t) = c_i r_j(t)$ if $j \in A_{n_i}$ $(i = 1, 2)$. Then, using again the Radon–Nikodym property of ℓ_p spaces we obtain that, if there exists $A \in \Sigma$ with $\lambda(A) > 0$ so that $|c_1\mu_{n_1} + c_2\mu_{n_2}|(A)$ is finite then there should exist a Bochner integrable function $\tilde{g} : A \longrightarrow \ell_p$ with

$$(c_1\mu_{n_1}+c_2\mu_{n_2})(E)=\int_E\tilde{g}(t)\mathrm{d}\lambda(t).$$

Therefore, using (2) and applying Lemma 2.1 again, we have

$$\|\tilde{g}(t)\|_{\ell_p} = |c_1| \sum_{j\in A_{n_1}} \frac{1}{j} + |c_2| \sum_{j\in A_{n_2}} \frac{1}{j} \quad \text{a.e. } t \in [0, 1],$$

which clearly is a contradiction. □

Definition 2.3. Let (Ω, Σ) be a measurable space, λ a positive measure on Σ, and X an infinite dimensional Banach space. A measure $\mu \in ca(\lambda, X)$ is said to be *injective* when for each $\phi, \psi \in L_\infty(\lambda)$,

$$\text{if } \int \phi \mathrm{d}\mu = \int \psi \mathrm{d}\mu \quad \text{then } \phi = \psi \quad \lambda - \text{a.e.}$$

In the previous definition we considered only infinite dimensional Banach spaces, since Halmos proved that every measure μ from a σ-algebra Σ to a finite dimensional Banach space is always *semiconvex* (i.e. for each $E \in \Sigma$ there exists $F \in \Sigma$ such that $\mu(F) = \frac{\mu(E)}{2}$); of course, no semiconvex measure can be injective.

Theorem 2.4. *Let λ be the Lebesgue measure on the Borel sets in* $[0, 1]$, *and let X be an infinite dimensional Banach space. Then the set of injective measures is lineable in $ca(\lambda, X)$.*

Proof. The proof will follow in the same manner as previously. First, let $r_n : [0, 1] \longrightarrow \{-1, 1\}$ be the sequence of Rademacher functions. Now, consider a sequence $\{A_n\}_n$ verifying (i) and (ii) from Lemma 2.1. Since X is an infinite dimensional Banach space, X contains a basic sequence, let us call it $\{x_n\}_n$ (see, e.g. [6]). Now, define $\mu_n : \Sigma \longrightarrow X$ by

$$\mu_n(E) = \sum_{j\in A_n} \left(\int_E r_{p_n(j)}(t) d\lambda(t) \right) \cdot \frac{x_j}{2^j},$$

where p_n is a bijection $p_n : A_n = \{a_n(i)\}_i \leftrightarrow \mathbb{N}$ given by $p_n(j) = i$ where $a_n(i) = j$. Now, notice that each μ_n is an injective measure. Indeed, fix $n \in \mathbb{N}$ and consider $\phi, \psi \in L_\infty(\lambda)$ such that $\int \phi d\mu_n = \int \psi d\mu_n$. Then

$$\sum_{j\in A_n} \left(\int \phi r_{p_n(j)} \, \mathrm{d}\lambda \right) \frac{x_j}{2^j} = \sum_{j\in A_n} \left(\int \psi r_{p_n(j)} \, \mathrm{d}\lambda \right) \frac{x_j}{2^j}.$$

Since $\{x_n\}_n$ is a basic sequence we obtain $\int \phi r_{p_n(j)} \mathrm{d}\lambda = \int \psi r_{p_n(j)} \mathrm{d}\lambda$ for each $j \in A_n$, and from the fact that the sequence of Rademacher functions $\{r_n\}_n$ is total in $L_1(\lambda)$ we have $\phi = \psi$ λ − a.e.

It is a simple exercise to show that, by construction, the sequence $\{\mu_n\}_n$ is linearly independent. Finally, one can easily check (using the same argument as in Proposition 2.2) that every non-null element of $\text{span}\{\mu_n : n \in \mathbb{N}\}_n$ is injective as well. □

A classical result of Lyapunov states that, given a finite dimensional Banach space X we have that, for each σ-algebra Σ and for each measure $G : \Sigma \longrightarrow X$ such that G is σ-additive, has bounded variation and is atomless (or not atomic measure), then the range of G (i.e. $\{G(E) : E \in \Sigma\}$) is a compact convex set of X. Wnuk [20] proved that if X is an infinite dimensional Banach space then there exists a measure whose range is neither closed nor convex. Since it is well known that every injective measure has range neither closed nor convex, we can now obtain the following result, consequence of Theorem 2.4.

Corollary 2.5. *Let λ be the Lebesgue measure on the Borel sets in* $[0, 1]$, *and let X be an infinite dimensional Banach space. Then the set of measures whose range is neither closed nor convex is lineable in $ca(\lambda, X)$.*

3. Subspaces of $L^p(I)\backslash L^q(I)$, sequence spaces and non linear properties

In this section we will focus our attention on the lineability of spaces of the type $L^p(I)\backslash L^q(I)$ for I either a bounded or unbounded interval. We will begin the section by considering the set $L^p(I)\backslash L^q(I)$ for I bounded. Without loss of generality we will restrict ourselves to $I = [0, 1]$. Before stating the first result of this section we need the following lemmas, of simple proof.

Lemma 3.1. *Let $k \in \mathbb{N}$, $0 < r_k < r_{k-1} < \cdots < r_2 < r_1$ and let $\alpha_1, \alpha_2, \ldots, \alpha_k$ be all non-zero real numbers. Then we have that*

(1) *If $\alpha_1 > 0$ then there exists $\varepsilon \in (0, 1)$ such that $\frac{\alpha_1}{x^{r_1}} > -\sum_{j=2}^{k} \frac{\alpha_j}{x^{r_j}}$ for every $x \in (0, \varepsilon)$.*
(2) *If $\alpha_1 < 0$ then there exists $\varepsilon \in (0, 1)$ such that $\frac{\alpha_1}{x^{r_1}} < -\sum_{j=2}^{k} \frac{\alpha_j}{x^{r_j}}$ for every $x \in (0, \varepsilon)$.*

Proof. Without loss of generality we may assume that $\alpha_1 > 0$ (since the other case is similar). Then, since $\lim x \to 0^+(-\sum_{j=2}^{k} \alpha_j x^{r_1 - r_j}) = 0$, we have that there exists $\varepsilon \in (0, 1)$ such that $-\sum_{j=2}^{k} \alpha_j x^{r_1 - r_j} < \alpha_1$, for every $x \in (0, \varepsilon)$. Multiplying the previous inequality by x^{-r_1} we obtain the desired result. □

Lemma 3.2. *Let $k \in \mathbb{N}$, $0 < r_k < r_{k-1} < \cdots < r_2 < r_1$ and let $\alpha_1, \alpha_2, \ldots, \alpha_k$ be all non-zero real numbers. For every $A > 0$, there exists $\varepsilon \in (0, 1)$ such that $\left|\sum_{j=1}^{k-1} \frac{\alpha_j}{x^{r_j}}\right| > \frac{A}{x^{r_k}}$ for every $x \in (0, \varepsilon)$.*

Proof. It is a simple exercise to check that the function $\sum_{j=1}^{k-1} \frac{\alpha_j}{x^{r_j - r_k}}$ is unbounded on any neighborhood of the form $(0, \delta)$, where $\delta > 0$. Moreover, $\lim x \to 0^+ \sum_{j=1}^{k-1} \frac{\alpha_j}{x^{r_j - r_k}} = \alpha_1 \cdot \infty$. Therefore, $\lim x \to 0^+ \left|\sum_{j=1}^{k-1} \frac{\alpha_j}{x^{r_j - r_k}}\right| = \infty$. Then, if $A > 0$, there exists $\varepsilon \in (0, 1)$ such that $\left|\sum_{j=1}^{k-1} \frac{\alpha_j}{x^{r_j - r_k}}\right| > A$, for every $x \in (0, \varepsilon)$. Multiplying this previous expression by x^{-r_k} we arrive at the wished inequality. □

Theorem 3.3. *Let $1 \leqslant p < q$. The set $L^p[0, 1] \setminus L^q[0, 1]$ is c-lineable.*

Proof. Let us consider the set of functions on $(0, 1)$ given by

$$\mathscr{B}_{p,q} = \left\{ \frac{1}{x^r} : x \in (0, 1), \frac{1}{q} < r < \frac{1}{p} \right\}$$

and let $V = \operatorname{span}\{\mathscr{B}_{p,q}\}$. We will show that $V \setminus \{0\} \subset L^p[0, 1] \setminus L^q[0, 1]$ and $\dim(V) = c$.

First of all, notice that $\mathscr{B}_{p,q} \subset L^p[0, 1] \setminus L^q[0, 1]$. Now, take $g \in V$, it would be

$$g(x) = \sum_{j=1}^{k} \frac{\alpha_j}{x^{r_j}},$$

where $\alpha_j \in \mathbb{R}$, $\frac{1}{q} < r_j < \frac{1}{p}$ for every $j \in \{1, 2, \ldots, k\}$. Let us also assume, without loss of generality, that $r_1 > r_2 > \cdots > r_k$. Let us further suppose that $g = 0$ and that $\alpha_1 \neq 0$. Taking ε as in Lemma 3.1 we obtain that $g(x) \neq 0$ over the interval $(0, \varepsilon)$, thus $\alpha_1 = 0$. Proceeding inductively in this manner, and applying Lemma 3.1 $(k - 1)$ times we obtain that $\alpha_j = 0$ for every $j \in \{1, 2, \ldots, k\}$. This shows that $\dim(V) = c$. Next, let us show that, if $g \neq 0$ then $g \in L^p[0, 1] \setminus L^q[0, 1]$. It suffices to show that $g \notin L^q[0, 1]$. Let us now use Lemma 3.2 (for $A = 1$). We have that there exists $\varepsilon \in (0, 1)$ such that

$$\int_0^1 |g(x)|^q \mathrm{d}x > \int_0^\varepsilon |g(x)|^q \mathrm{d}x > \int_0^\varepsilon \frac{1}{x^{q \cdot r_k}} \mathrm{d}x = \infty$$

and we are done. □

Of course we can also infer dual results for the sets $L^p(I) \setminus L^q(I)$ for I unbounded and for the difference of the classical ℓ_p spaces. Thus, we can state the following results, whose proofs follow in a similar manner as the one above.

Theorem 3.4. *Let $p > q \geqslant 1$ and let I be any unbounded interval. The set $L^p(I) \setminus L^q(I)$ is c-lineable.*

Theorem 3.5. *Let $p > q \geqslant 1$. The set $\ell_p \setminus \ell_q$ is c-lineable.*

For Theorem 3.4 we can, without loss of generality, consider $I = [1, \infty)$ and the vector space given by

$$\operatorname{span}\left\{\frac{1}{x^r} : x \in I, r \in \left(\frac{1}{p}, \frac{1}{q}\right)\right\}.$$

Also, for Theorem 3.5 it suffices to consider the following vector space:

$$\operatorname{span}\left\{\left\{\frac{1}{n^r}\right\}_{n \in \mathbb{N}} : r \in \left(\frac{1}{p}, \frac{1}{q}\right)\right\}.$$

We finish this section by pointing out a remark about sequence spaces and a question originally posed by Aron and Gurariy in 2003.

Remark 1. Aron and Gurariy posed the question of whether there exists an infinite dimensional and closed subspace of ℓ_∞ every non-zero element of which has a finite number of zero coordinates. If we denote by P the set of odd prime numbers, and (if $p \in P$) we call

$$x_p = \left(\frac{1}{p}, \frac{1}{p^2}, \frac{1}{p^3}, \frac{1}{p^4}, \ldots\right) \in \ell_\infty,$$

it is not hard to see that

$$\operatorname{span}\{x_p : p \in P\}$$

is an infinite dimensional manifold enjoying the wished property. However, the problem originally posed by Aron and Gurariy concerning *spaceability* still remains open.

Acknowledgement

The authors wish to express their gratitude to the Referee whose thorough analysis and insightful remarks improved the text.

References

[1] A. Aizpuru, C. Pérez-Eslava, J.B. Seoane-Sepúlveda, Linear structure of sets of divergent sequences and series, Linear Algebra Appl. 418 (2–3) (2006) 595–598.
[2] R.M. Aron, D. García, M. Maestre, Linearity in non-linear problems, Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat. 95 (2001) 7–12.
[3] R.M. Aron, V.I. Gurariy, J.B. Seoane-Sepúlveda, Lineability and spaceability of sets of functions on $\mathbb{R}$, Proc. Amer. Math. Soc. 133 (2005) 795–803.
[4] R. Anantharaman, J. Diestel, Sequences in the range of a vector measure, Comment. Math. Prace Mat. 30 (2) (1991) 221–235.
[5] R.M. Aron, D. Pérez-García, J.B. Seoane-Sepúlveda, Algebrability of the set of non-convergent Fourier series, Studia Math. 175 (2006) 83–90.
[6] J. Diestel, Sequences and Series in Banach Spaces, Springer-Verlag, New York, 1984.
[7] J. Diestel, J.J. Uhl Jr., Vector measures, Mathematical Surveys, American Mathematical Society, Providence, R.I. 15 (1977).
[8] N. Dunford, J.T. Schwartz, Linear Operators. I. General Theory, Interscience Publishers Inc., New York Interscience Publishers, Ltd., London, 1958.
[9] P. Enflo, V.I. Gurariy, On lineability and spaceability of sets in function spaces, Preprint.
[10] V. Fonf, V.I. Gurariy, V. Kadeč, An infinite dimensional subspace of $\mathscr{C}[0, 1]$ consisting of nowhere differentiable functions, C.R. Acad. Bulgare Sci. 52 (11–12) (1999) 13–16.
[11] D. García, B.C. Grecu, M. Maestre, J.B. Seoane-Sepúlveda, Infinite dimensional Banach spaces of functions with nonlinear properties. Preprint.
[12] F.J. García-Pacheco, N. Palmberg, J.B. Seoane-Sepúlveda, Lineability and algebrability of pathological phenomena in analysis, J. Math. Anal. Appl. 326 (2) (2007) 929–939.
[13] F.J. García-Pacheco, J.B. Seoane-Sepúlveda, Vector spaces of non-measurable functions, Acta Math. Sinica (English Series) 22 (6) (2006) 1805–1808.
[14] V.I. Gurariy, L. Quarta, On lineability of sets of continuous functions, J. Math. Anal. Appl. 294 (2004) 62–72.
[15] S. Hencl, Isometrical embeddings of separable Banach spaces into the set of nowhere approximatively differentiable and nowhere Hölder functions, Proc. Amer. Math. Soc. 128 (2000) 3505–3511.
[16] L. Janicka, N. Kalton, Vector measures of infinite variation, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. 25 (3) (1977) 239–241.
[17] D. Puglisi, J.B. Seoane-Sepúlveda, Bounded linear non-absolutely summing operators, J. Math. Anal. Appl. 338 (2008) 292–298.
[18] H.P. Rosenthal, On quasi-complemented subspaces of Banach spaces, with an appendix on compactness of operators from $L_p(\nu)$ to $L_r(\nu)$, J. Funct. Anal. 4 (1969) 176–214.
[19] E. Thomas, The Lebesgue–Nikodym theorem for vector valued measured, Mem. AMS 139 (1974).
[20] W. Wnuk, The converse of Lyapunov convexity theorem. Comment, Math. Prace Mater. 21 (2) (1980) 389–390.