Effect of Model Plant Mismatch on MPC Performance and Mismatch Threshold Determination

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Abstract

Existing model-plant mismatch detection and isolation methods mainly employ correlation analysis approaches to detect the sub-model that have statistically significant mismatch from the plant. However, statistical significance does not ensure that any control performance deterioration is due to the detected mismatch. A small but statistically significant mismatch may not have any impact on the performance of the controller. This paper presents the effect of model plant mismatch (MPM) on the performance of model predictive controller (MPC) and a systematic approach to determine the thresholds of mismatches above which the performance deterioration can be considered significant. A simulation case study of the Wood and Berry distillation model with MPC is used as a case study to demonstrate the efficacy of the proposed approach. For the stated case study, a 70% increase in the overall integral error (OIE) for set-point tracking problem is found to be an acceptable limit for the MPC performance deterioration. The thresholds of gain, time delay and time constant MPM for 70% increase in OIE for Wood and Berry simulation case study are found to be {(-20% +40%) (-13% +7%); (-40% +31%) (-7% +27%)}, {(-40% +40%) (-40% +22%); (-20% +17%)(-29% +27%)} and {(-40% +15%) (-20% +40%); (-23% +40%)(-28% +36%)}, respectively.

Keywords: MPC; Model Plant Mismatch; Gain Mismatch; Time Delay Mismatch; Time Constant Mismatch; Controller performance;

1. Introduction

Model predictive controllers (MPC) are currently the preferred multivariable controllers in most process industries because of their optimal control outputs and systematic method of handling constraints and interactions [1]. MPC uses the dynamic model of the plant it controls to predict the future output which is later used together with the desired output trajectory and set of constraints in the calculation of the optimal controller output [1-4]. The performance of MPC, therefore highly depends on the accuracy of the dynamic model employed. Over time, the dynamics of the plant may change due to ageing, fouling, changes in operating regimes, etc. This causes the dynamic model of the plant to deviate from the original dynamics of the model used in the MPC design. This deviation is known as model plant mismatch (MPM). MPC may tolerate small MPM, however increased MPM can deteriorate the control performance significantly in some cases even leading to shutdowns [2].

MPM is normally addressed by re-identification. However, complete plant re-identification is costly and time consuming in most industrial MIMO scenarios. Therefore, detection of the existence of MPM and isolation of the sub-models that have mismatch significant enough to cause performance deterioration of the MPC have been the interest of several researchers [5-8].

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The most common approach to detect and isolate the mismatched sub-models is based on correlation analysis of the plant output and plant residual. The methods usually identify the correct sub-models that have statistically significant mismatches from the plant. However, statistical significance does not ensure that any control performance deterioration is due to the detected mismatch. A small model plant mismatch which causes insignificant deterioration in the controller performance may be detected with a statistical significant level if the detection method is effective. However, re-identification of sub-models with such mismatches is a waste of resource and money since it doesn’t bring any significant improvement in the controller performance.

This study investigates the effect of model plant mismatch on the performance of MPC and suggests a systematic approach to determine the thresholds of mismatches above which the performance deterioration can be considered significant. Simulation of the Wood and Berry distillation model with MPC is used as a case study to demonstrate the effectiveness of the proposed method. The result gives good insights into the effect of MPM and a systematic approach for reinforcing the current detection and isolation approaches. The study considers plants that can be adequately expressed using First Order Plus Time Delay (FOPTD) models.

2. Methodology

The Wood and Berry distillation column model together with MPC is used as a case study to investigate the effect of MPM and determine the mismatch thresholds above which the performance deterioration is unacceptable. The transfer function model of Wood and Berry distillation column are given in equation (1), other details can be found in [9]. The tuning parameters of the MPC are given in Table 1. A matrix of model-plant mismatches is determined and the closed loop response data for step changes in set-point is generated after introducing this mismatches in the plant. The performance of the MPC is measured using Overall Integral Error (OIE) which is defined in this paper as the infinite norm of the Integral absolute error (IAE), Integral squared error (ISE) and Integral time absolute error (ITAE) for set-point tracking problem. The definition for IAE, ISE and ITAE are available in [12].

\[
\begin{bmatrix}
X_D(s) \\
X_B(s)
\end{bmatrix} =
\begin{bmatrix}
12.8e^{-s} & -18.9e^{-3s} \\
16.7s + 1 & 21s + 1 \\
6.6e^{-7s} & -19.4e^{-3s} \\
10.9s + 1 & 14.4s + 1
\end{bmatrix}
\begin{bmatrix}
R(s) \\
S(s)
\end{bmatrix} +
\begin{bmatrix}
3.8e^{-8.1s} \\
14.9s + 1 \\
4.9e^{-7s} \\
13.2s + 1
\end{bmatrix}
F(s)
\]

where
\(X_D\) = distillate compositions
\(X_B\) = bottom compositions
\(R\) = reflux flow rate
\(S\) = steam flow rate
\(F\) = feed flow rate

For this project, the disturbance \(F\) is set to 0 to prevent it from affecting the plant performance. This will ensure that any change in performance comes due to MPM only.

Table 1. Tuning Parameters of the MPC.

<table>
<thead>
<tr>
<th>MPC parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPC control interval</td>
<td>1</td>
</tr>
<tr>
<td>Prediction horizon</td>
<td>20</td>
</tr>
<tr>
<td>Control horizon</td>
<td>5</td>
</tr>
<tr>
<td>Input weight</td>
<td>0.5</td>
</tr>
<tr>
<td>Input rate weight</td>
<td>0.2</td>
</tr>
<tr>
<td>Output weight</td>
<td>0.5</td>
</tr>
</tbody>
</table>

The mismatch parameters, i.e., the gain, time delay and time constant mismatches are included in the plant model as shown in equation (2) where \(K\), \(r\) and \(\theta\) are the same parameters as used in the plant model for the design of the MPC. The values of \(\Delta K\), \(\Delta r\) and \(\Delta \theta\) represent the gain, time constant and time delay mismatches in the plant.

\[
G_p = \frac{(K + \Delta K)e^{-(\theta + \Delta \theta)s}}{(r + \Delta r)s + 1}
\]
The mismatch percentages range investigated in this paper is -40% to +40%. The definitions for IAE, ISE and ITAE are available in [1]. However, since it is difficult to make conclusions based on three different parameters having different values. Therefore, in this paper we define one overall performance measure known in this paper as overall integral error (OIE).

First we normalize the integral errors by dividing them by their corresponding values with no MPM. To do this, assume the integral errors of a properly tuned controller for a given set point change are $IAE_0$, $ISE_0$ and $ITAE_0$. The normalized integral errors are defined as:

\[ IAEN = \frac{IAE}{IAE_0} \]  
\[ ISEN = \frac{ISE}{ISE_0} \]  
\[ IAEN = \frac{ITAE}{ITAE_0} \]  

The overall integral error (OIE) is defined in this paper as the $\ell_\infty$-norm of the three normalized integral errors.

\[ OIE = \max\{ IAEN, ISEN, ITAE \} \]

3. Results and Discussions

In this section, the effect of MPM in a single sub-model on the performance of the MPC is presented. The closed-loop response of the system when there is no model plant mismatch is shown in Fig. 1. The three integral errors for the closed-loop response for set-point changes for no mismatch condition are given in Table 2.

<table>
<thead>
<tr>
<th>$IAE$</th>
<th>$ISE$</th>
<th>$ITAE$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.9518</td>
<td>4.5049</td>
<td>2.0657</td>
</tr>
<tr>
<td>3.6579</td>
<td>9.0631</td>
<td>16.2560</td>
</tr>
</tbody>
</table>

Fig. 1. The closed-loop response for step change in set point when there is no mismatch.

3.1. Effect of Gain Mismatch

The effect of gain mismatch in a given sub-model on the MPC overall performance are shown in Fig. 2. The plot in Fig. 2a represents the overall integral error (OIE) for set-point change when there is -40% to 40% model plant mismatch in the transfer function $G_{11}$. The other plots b, c, and d similarly represent the OIE for $G_{12}$, $G_{21}$ and $G_{22}$, respectively.
It is observed from Fig. 2 that the performance deterioration due to MPM does not follow the same pattern for different sub-models. The direction of the mismatch, i.e. whether the plant parameter increases or decreases with respect to the original model, also plays a very important role. From Fig. 2a, an increase of 40% in gain of the plant in $G_{11}$ from the original plant model doesn’t have any significant deterioration on the control performance in both outputs, whereas 40% decrease in the gain of $G_{11}$ increases the overall integral error by around 400% in $x_D$. On the other hand, 40% decrease of the plant gain from the model in transfer function $G_{21}$ doubles the overall integral error of the MPC on $x_D$, while 40% increase raises the overall integral error 4 times. It is also noted from Fig. 2 that, the most significant deterioration occurs when the plant gain for $G_{22}$ decreases by 40%.

The closed-loop response for a step change in set points by 0.1 in the latter case is shown in Fig. 3. It is observed from Fig. 3 that, there is a significant steady state offset in the response of $x_B$.

3.2. Effect of Time Delay Mismatch

The effect of time delay mismatch on the performance of the MPC is shown in Fig. 4. It is observed from Fig. 2 that the effect of time delay of $G_{11}$ on the performance of the MPC is negligible.
The increase in OIE for -40% mismatch in time delay from the original model is, i.e. when the plant time delay decreases by 40%, is around 2% for +40% it is around 24%. This may be due to the fact that the original time delay of G11 is the smallest of all the four time delays, i.e. 1 min. It should be noted from equation (1) that the maximum time delay of the model is 7 min and it occurs at G21. It is observed that the maximum performance deterioration due to MPM in time delay occurs at G21 and the OIE increases by 350% at +40% mismatch in time delay.

3.3. Effect of Time Constant Mismatch

The performance deterioration of the MPC for set-point change due to MPM in time constant is indicated in Fig. 5. The performance of MPC in tracking set-point of \( x_D \) is most affected by time constant mismatches in sub-models G11 and G12 as observed in Fig. 5(a) and 5(b). On the other hand, the performance of MPC in tracking set-point of \( x_B \) is most affected by time constant mismatches in G21 and G22 as shown in Fig. 5(c) and 5(d). This result is expected since G21 and G22 are the transfer functions directly related to the respective outputs. This trend is also observed in gain mismatch and time delay mismatches in Fig. 2 and 4.

The worst MPC performance deterioration due to time constant occurs when the time constant of the plant decreases by 40% in sub-model G12 leading to increase of the OIE by 400%. The closed-loop response for a set-point change of 0.1 in \( x_B \) and \( x_B \) for the latter scenario is shown in Fig. 6.
3.4. Determination of Mismatch Threshold

It is very useful in mismatch detection and isolation to have some estimate of a threshold of MPM above which the MPC performance will not be acceptable. In this research, with the MPC performance measured in OIE, a maximum of 70% increase in the overall integral error for set-point tracking is found to be a tolerable range for the MPC performance. Fig. 7 shows the performance of MPC for set-point tracking of \( x_D \) and \( x_B \) for -7% and +27% MPM in gain mismatch of \( G_{22} \) that causes 70% increase in the OIE.

The threshold for a maximum of 70% increase in OIE for gain, time delay and time constant mismatch are obtained to be as given by equations (7) (8) and (9), respectively.

\[
\text{Thresholds for gain MPM} = \begin{bmatrix} (-20\% + 40\%) & (-13\% + 7\%) \\ (-40\% + 31\%) & (-7\% + 27\%) \end{bmatrix}
\]

\[
\text{Thresholds for time delay MPM} = \begin{bmatrix} (-40\% + 40\%) & (-40\% + 22\%) \\ (-20\% + 17\%) & (-29\% + 27\%) \end{bmatrix}
\]

\[
\text{Thresholds for time constant MPM} = \begin{bmatrix} (-40\% + 15\%) & (-20\% + 40\%) \\ (-23\% + 40\%) & (-28\% + 36\%) \end{bmatrix}
\]
4. Conclusion

The effect of MPM on the performance of MPC is studied and thresholds above which the controller performance are not acceptable is determined for Wood and Berry distillation column model. A new performance measure that reflects the overall measure of IAE, ISE and ITAE is defined by taking the $\|\cdot\|_\infty$-norm of the three normalized integral errors. For Wood and Berry case study, a 70% increase in OIE for set-point tracking problem is found to be an acceptable limit for the MPC performance deterioration. The thresholds of gain, time delay and time constant MPM for 70% increase in OIE for Wood and Berry simulation case study is found to be \{(-20\% +40\%) (-13\% +7\%); (-40\% +31\%) (-7\% +27\%); (-20\% +17\%) (-29\% +27\%)\} and \{(-40\% +15\%) (-20\% +40\%); (-23\% +40\%) (-28\% +36\%)\}, respectively. The study presents a systematic approach for determining the thresholds of gain, time delay and time constant mismatch. In addition, it gives insight into the effect of MPM on the performance of MPC.

References