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Geometric error analysis of compressor blade based on reconstructing leading and trailing edges smoothly

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Abstract

In the process of blade design, the shape of blade, which causes great influence on aerodynamic performance, is the most difficult part for designer. The shape of blade changes because of the cutting force, residual stress and deformation. To analyze the precise error of this, this paper presents a new algorithm for reconstructing cross section curve of blade from measuring points. The Gauss-Newton method and dichotomy are used to fit the circle of leading and trailing edge, which is the most important part of blade. Then the deformation error, contour error and parameter error are calculated with relative algorithm based on the new cross section curve. Also, a series of points and a blade model are utilized to demonstrate the effectiveness of this novel algorithm on reconstructing cross section curve and analyzing machining error.

Keywords: cross section curve reconstructing; leading edge fitting; blade error analyzing.

1. Introduction

As the hardest machining components among the aviation industry, aero-engine is used in the field of Nation Defense widely, and plays an important role in commercial areas. As an important part of the aero-engine, the blades are particularly important for jet engines, in order to satisfy the mechanical property, structure design and aerodynamic performance, blades need to be obtained with high manufacture precision.

Just as the history which is the development of blades shows, the development of new materials and manufacturing techniques makes blades more durable and lighter. Scholars have been studied in this area for many years. Such design improves the performance of gas turbine engine significantly, and makes a great contribution to obtain a higher contribution ratio and lower fuel consumption [1].

In the process of blade design, the shape of blade, which causes great influence on aerodynamic performance, is the most difficult part for designer. It is used to change the direction of the air flow. Moreover, for the fan blades and compressor blades, it also plays a role of boosting and decelerating. In evaluation of the aerodynamic performance of blades, improving the efficiency of air compressing, reducing the consumption of secondary flow and increasing stall margin are very important for aerodynamic performance. The goal of any blade-design method is to find a geometry that satisfies flow requirements with minimum loss, tolerable mechanical stresses, minimum disturbances downstream and upstream, and in the case of compressors adequate stall margin, among others. The design of blade geometries is a very important step for the design of efficient turbomachines, as the blade design process directly influences the blade-row efficiency and thus the overall machine efficiency.

Many researchers have studied on aerodynamic performance of blade influenced by the geometric precision of leading and trailing edges. Hamakhan [2] redesigned the original shape of the blade. Aiming at the shape of the original leading edge, he sharpened the edge and ensured the smooth attachment under the premise of constant chord length. After testing the aerodynamic performance of the blade under different attack angle (-10°, -5°, +5°, +10°), he found that the
blade with slender leading edge did a better job in avoiding flow separation.

In the same way, Yin [3] proposed a method of continuous curvature leading edge of compressor blade. Based on cubic Bezier spline, the design only required three independent variables. Compared with circular and elliptical leading edge, the working range is enlarged by 1.2° and 2.5°, respectively. Thus the improvement of efficiency and working range was achieved.

Therefore, precise modeling technology is very important in machining and subsequent measuring. The main purpose of reconstructing cross section curve of blade, which is the curve fitted by measured points from machined blade, is to extract the shape parameters which will be compared with the theoretical parameters later. Combined with aerodynamic performance experiment, we can analyze the influence of processing error on the aerodynamic performance and aerodynamic sensitivity at different parts of the blade [4].

In this paper, based on process of the data measured in the process of machining blade, a new blade cross section curve reconstruction algorithm is proposed. Under the condition of precise measurement, this algorithm can obtain minimum fitting error while satisfy the smoothness and continuity of cross section curve as far as possible. Then the aerodynamic performance can be analyzed in the future based on the error analysis.

2. Parameters of blade

In general, the fan blades and compressor blades are the most important parts in aero-engine. Although they are in different size, substantially they contain different cross section curves, which are the basic airfoil bended by mean line. Because of the free-form surface, the structure of the blade is very complex. In order to grasp the shape of blade precisely, the designer needs to design a reasonable mean line to meet the aerodynamic characteristics of the blades. Then the original airfoil, which is symmetrical not bended, is selected to obtain thickness distribution function of airfoil.

After that, different thickness is superimposed to the both side of mean line. All these theories produce one cross section curve of blade. The other cross section curves of blade are calculated from different aerodynamic performance corresponding to the height. Finally, with all these curves stacked in a stacking line, the geometry of blade is generated. Therefore, mean line and thickness distribution are the most important parameters. All these settings are to design better parameters of airfoil which determine the aerodynamic performance of blade. The geometric profile is shown in Fig.1 and some parameters of airfoil are shown in Fig.2.

2.1. Geometric profile

1. Leading edge surface. The region contacted by free stream first used to divide free stream into different surfaces.
2. Suction surface. Named as convex surface. The flow accelerates on this surface and produce a low pressure.
3. Pressure surface. Named as concave surface. The pressure here is much higher than suction surface region.
4. Trailing edge surface, where flow from suction surface and pressure surface join together.

Fig. 1. The introduction of blade terminology

5. Cross section curve. An intersection curve produced by a plane and blade model, which consists of four curves.

2.2. Parameters of airfoil

1. Mean line $M(s)$. A continuous curve formed by the center points of inscribed circles of cross-section curve. It is the datum of blade design, which changes the direction of flow.
2. Leading edge point $A$ and trailing edge point $B$. The separation point of free steam when attack angle is 0 is the leading edge point. The flow converges at the point of trailing edge is the trailing edge point.
3. Chord $b$. The longest line of cross section curve of blade. The other cross section curves of blade are calculated from different aerodynamic performance corresponding to the height. Finally, with all these curves stacked in a stacking line, the geometry of blade is generated. Therefore, mean line and thickness distribution are the most important parameters. All these settings are to design better parameters of airfoil which determine the aerodynamic performance of blade. The geometric profile is shown in Fig.1 and some parameters of airfoil are shown in Fig.2.

3. The algorithm of reconstructing cross section curve

Calculating parameters is an important step for error analysis, which is based on cross section curve of airfoil. For design model or theory model, cross section curve is a lap of curves from the section of blade model. With regard to measuring data, the cross section curve is not existed. For
getting the parameters of measuring model, an effective algorithm to reconstruct cross section curve of measuring model is needed. The points measured by CMM is scattered in space and how to sort these points into several cross section curves is the focus of this paper.

3.1. Classify the measuring points

For machining conveniently, technologist divide blade surface into suction surface, pressure surface, leading edge surface and trailing edge surface. Consider this, when analyzing machining error, the same method is used. The measuring points are divide into four different regions, which are convex points, concave points, leading edge points and trailing edge points, according to the blade surface. It is quite simple to classify the measuring points.

3.2. Filter convex points and concave points

After machining, the part surface is not smooth for the cutting mark. Even consider the removing cutting mark process, the surface is still not as smooth as what we expect. The high precise CMM can record the rough surface truthfully. The curvature of curve, interpolated by measuring points with cubic NURBS, varies intensely. The weak smoothness will affect the following calculation. Therefore, the convex points and concave points are fitted first. The fitting method won’t be discussed here.

3.3. Fit convex curve, concave curve and extract mean line

After filtering, interpolating a sequence of convex points and concave points with cubic NURBS and generating convex curve and concave curve, which are much smoother. Applying the extraction algorithm of mean line in the above section, the original measuring mean line \( M_1 \) is generated as shown in Fig. 3.

Besides, extend the new convex curve, concave curve and extract another mean line \( M_2 \), which is longer than before.

3.4. Fit leading and trailing edges

So much preparatory work is for fitting leading and trailing edges. For machining conveniently, the shape of leading and trailing edges are treated as circle when design the airfoil. The practice may sacrifice aerodynamic performance in some way, but improve the machining accuracy because it is easy to machine a circle relatively. Therefore, when fitting leading and trailing edges, the circle shape is what we consider.

Considering the continuity of cross section curve, the measuring points are not the only constraint. The convex and concave curves are also the constraint condition. That is the tangents of endpoints from both leading edge and convex or concave curve are in the same direction. With these constraints, the fitted error of circle needs to be minimum, which is expressed as follows.

\[
\begin{aligned}
\min_{\mathbf{u}} f(\mathbf{u}) &= \sum_{i=1}^{n} e_i^2(\mathbf{u}) = \sum_{i=1}^{n} \left[ (P_i - O_i)^2 + (P_i - O_i)^2 - R_i^2 \right]^2 \\
\text{s.t.} &\text{The fitted circle is tangent to convex and concave curves}
\end{aligned}
\]

where, \( \mathbf{u} = (O_x, O_y, R)^T \). This optimization problem is three-variable non-liner fitting problem.

For the similarity of leading edge and trailing edge, the analysis below only considers the leading edge. The method to fit trailing edge is the same. Considering the relation of fitted circle, convex, concave curves, the characteristic of mean line and thickness distribution function, the arc length parameter \( s \) of mean line can express the center of fitted circle and radius. With this relation, the fitted circle only moves through mean line. The value of radius is the thickness of blade, which is found in thickness distribution function.

After these simplifications, the fitted problem can be expressed in this way.

\[
\min_{s} f(s) = \sum_{i=1}^{n} e_i^2(s) = \sum_{i=1}^{n} \left[ (P_i - O_i(s))^2 + (P_i - O_i(s))^2 - R(s)^2 \right]^2
\]

where, \( s \) is the arc length parameter. \( O_i(s), O_i(s), R(s) \) are the center coordinate of fitted circle and radius function about \( s \).

If dividing the interval of mean line evenly, the fitted error follows some rule. In a small region, there is the minimum error. Therefore, the optimization problem can be solved with dichotomy or Gauss-Newton method. The two methods are introduced as follows.

- Fit circle with dichotomy
  Step1. Initial conditions choosing.
  Arc length parameter \( s_0 \) of the left endpoint of interval is the endpoints from measuring mean line \( M_1 \) near the leading points. Arc length parameter \( s_f \) of the right endpoint of interval is the endpoints from measuring mean line \( M_1 \) near the leading points shown in Fig. 3.
  Step2. Median calculating.

\[
s_m = (s_0 + s_f) / 2
\]
With the median arc length parameter \( s_m \), the value of radius can be found in thickness distribution function \( R(t) \). Then, the coordinate of center \( o_n \) is found in mean line \( M_2 \) with \( s_m \). The fitted error \( f_m \) is calculated with fitted circle created by center and radius. Then, the errors \( f_{m-1} \) and \( f_{m+1} \) is calculated with \( s_{m-1} \) and \( s_{m+1} \) respectively, which is the neighborhood of \( s_m \). All these errors are used to determine the monotonicity of \( s_m \).

Step 3. Monotonicity determining.

If \( f_{m+1} > f_m > f_{m-1} \), \( s_m \) is in monotone decreasing interval.

The value of \( s_m \) is assigned to \( s \). Go back to step 2 and iterate.

If \( f_{m+1} < f_m < f_{m-1} \), \( s_m \) is in monotone increasing interval.

The value of \( s_m \) is assigned to \( s \). Go back to step 2 and iterate.

If \( f_{m+1} > f_n \) and \( f_n < f_{m-1} \), \( f_n \) is minimum in the neighborhood of \( s_m \). The error of circle fitted by \( s_m \) is minimum. The fitting problem is accomplished. Quit the iteration.

- Fit circle with Gauss-Newton method.

Step 1. Make sure the variable is arc length parameter \( s \) of mean line \( M_2 \).

Step 2. Solving the Jacobian matrix about \( \partial e(s)/(\partial s) \).

\[
J(s) = \frac{[(P_{ts} - O_t(s)) - O_t'(s) + (P_{ds} - O_d(s)) - O_d'(s)]}{\sqrt{[(P_{ts} - O_t(s))^2 + (P_{ds} - O_d(s))^2]}} - R'(s)
\]

Step 3. Structure iterative formula about \( s \).

\[
s^{(k+1)} = s^{(k)} - (J^TJ)^{-1}J^Te(s^{(k)})
\]

where, \( e(s) = (e_t(s), \ldots, e_d(s))^T \).

Step 4. Initial value choosing.

The arc length parameter \( s_1 \), which is the endpoint of mean line \( M_1 \) near the leading edge, is chosen as initial value shown in Fig. 3.

Step 5. Iteration calculating.

Using iterative formula with \( s_1 \). After several times, the fitted circle is more precise with iterative arc length parameter.

3.5. Complete cross section curve

With fitted circle of leading edge, the circle of trailing edge is fitted in the same way. And the optimum fitting arc length parameters of leading and trailing edges are \( s_l \) and \( s_t \), respectively shown in Fig. 4.

With the center point \( Pol \) of leading edge from mean line \( M_2 \) by arc length parameter \( sl \), the nearest points in convex and concave curves, \( Pvt \) and \( Pct \), are calculated, respectively.

In the same way, the center point \( Pot \) near trailing edge in mean line and the nearest points \( Pvl \) and \( Pcl \) in convex and concave curves are calculated. With these four points in convex and concave curves and two points in mean line \( M_2 \), the cross section curve is reconstructed smoothly.

4. The algorithm of geometric error of blade

The direct method to test quality of blade is whether geometric error of blade satisfies the design tolerance. To make the geometric profile of blade becomes significant, high precision of measurement is the only way to show the original feature of blade. CMM short for coordinate measuring machine is used to measure the blade section by section for its high precision. The measuring points, which are produced by CMM, are used to analyze the geometric error of blade. Geometric error of blade can be divided into deformation error, contour error and parameter error. All these errors are based on the measuring points but focus on different aspect. They will be illustrated below.

4.1. Deformation error

Deformation error can be separated into twist error and offset error, which mean the deviation between measuring section and theory section. In other words, the measuring section can fit the theory section in a minimum error with translation and rotation.

From this principal, we can get the method to calculate deformation error, which is to calculate a translation matrix and a rotation matrix to transform the measuring points into a better position. This means registration. Because of the measuring strategy, we only consider the two-dimension registration.

The parameters of registration can be unified as \((0,0,γ,Δx,Δy,0)\). After registration, the measuring points are
\( Q = f(P) = R \cdot P + T \)  

(6)

where, \( Q = (x', y', z')^T \) is the points after registration, \( P = (x, y, z)^T \) is the original measuring points, \( T = (\Delta x, \Delta y, 0)^T \) is the translation matrix, \( R \) is the rotation matrix, which can be expressed as follow.

\[
R = R_\gamma R_\beta R_\alpha = \begin{bmatrix}
\cos \gamma & -\sin \gamma & 0 \\
\sin \gamma & \cos \gamma & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

(7)

Finally, using ICP algorithm short for iterative closest point to calculate the transformation matrix. The concrete steps of ICP can refer to [6].

4.2. Contour error

The calculation of contour error is simple for CAD software. Even in CMM, calculation of contour error is integrated. When using CAD software, we need to import the design model \( M \) and measuring points \( Q_i \) \((i = 1, 2 \ldots n)\). Then, find the closest points \( K_i \) \((i = 1, 2 \ldots n)\) of measuring points in design model and calculate the distance \( d \) of corresponding points, which is the error of the corresponding points.

4.3. Parameter error

It is the error between measuring parameter and theory parameter. Before calculating error, the precise parameter calculation of airfoil is considered. Note that the parameters of airfoil are based on cross section curve reconstructed before.

Not all the parameters mentioned in the above section need to calculate error. The most important is how to prove the efficiency of the algorithm about reconstructing cross section curve.

Error of chord \( e(b) = b' - b \)

Error of maximum thickness \( e(D) = D' - D \)

Error of position of maximum thickness \( e(e) = e' - e \)

5. Verification

A cross section curve and its measuring points are prepared as the test objects to demonstrate the effectiveness of reconstructing algorithm and error analysis.

5.1. Analysis of reconstructing cross section curve

Applying the reconstructing algorithm and comparing the difference between dichotomy and Gauss-Newton method. The reconstructed cross section curve is shown in Fig. 5.

From the table 1, both dichotomy and Gauss-Newton method achieve high precision. But the rate of convergence of Gauss-Newton method is quicker than dichotomy obviously. Analysis of airfoil error

- Deformation error calculating

The theory model and measuring points are shown in Fig. 6. It is obvious that measuring points do not fit theory model appropriately.

After 90 times iteration shown in Fig. 8, the rotation matrix and translation matrix are calculated. From them, the twist error \( \gamma \), offset error \( x \) and \( y \) are calculated as \( \gamma = -0.03933 \), \( x = 0.165532 \) and \( y = -0.083344 \), respectively.

The theory model and measuring points are shown in Fig. 7 after registration.

- Contour error calculating

The histogram of contour error is shown in Fig. 9 after calculating.

<table>
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<tr>
<th>x</th>
<th>s(k)-s(k-1)</th>
<th>s</th>
<th>s(k)-s(k-1)</th>
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<tbody>
<tr>
<td>0.017968147</td>
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<td>0.01836673</td>
<td>0.000368526</td>
</tr>
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<td>0.02</td>
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<table>
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<tr>
<th>Theory value</th>
<th>Measuring value</th>
<th>Error</th>
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<td>Chord</td>
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<td>137.0265mm</td>
</tr>
<tr>
<td>Maximum thickness</td>
<td>4.05995mm</td>
<td>4.325829mm</td>
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<tr>
<td>Position of maximum thickness</td>
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<td>0.599286</td>
</tr>
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</table>

Fig. 5. Result of reconstructing cross section curve and extracting parameter of airfoil, (a) leading edge; (b) trailing edge
Analyzing the distribution of contour errors, we find that most errors distribute around 0.15mm, which is the machining error of convex and concave surface. Generally, the larger error occurs in leading and trailing surface, which is about 0.27mm. From the figure, the machining error need to be controlled.

- Parameter error calculating

Chord and maximum thickness of airfoil are considered to analyze error. Applying the algorithm mentioned above, the parameter errors extracted from measuring cross section curve are calculated, as shown in Table 2.

Comparing parameter error and contour error, we find that the chord error is twice larger than leading and trailing edge error and maximum thickness error is twice larger than convex and concave error because of their geometric relation. In this case, controlling the parameters, which influence aerodynamic performance, is controlling the machining error of relative region.

6. Conclusion

Considering the aero-engine blade as the research object, this paper mainly studied the reconstruction method of the cross section curve of blade and error analysis method. The methods of the deformation error, contour error and parameter error are given as well. Among them, the reconstruction of the cross section curve involves the reconstruction of leading and trailing edges, concave and convex curves. Filtering and interpolation methods are used in the reconstruction of concave and convex curves. The constraints of the concave and convex curves are used in the reconstruction of leading and trailing edges. All the methods are used to fit the circle in order to minimize fitting error of leading and trailing edges, and complete reconstruction of the cross section curve. In blade error analysis, comparing the theoretical and measuring cross section curves, machining error in this section is obtained. Then, a set of measuring points and theory model are given to prove the effectiveness of the algorithm. Practically, the algorithm satisfies the requirements of smoothness and continuity of the reconstructed cross section curve perfectly. The G1 continuity at separation points of leading and trailing edges is achieved as well. After this, the error can be used to analyze the loss of aerodynamic performance.

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