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Invariant distributions on a non-isotropic pseudo-Riemannian symmetric space of rank one

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ABSTRACT

We investigate the structure of invariant distributions on a non-isotropic non-Riemannian symmetric space of rank one. Especially, the J-criterion related to the generalized Gelfand pair is shown for this space without imposing the condition on the eigenfuction of the Laplace–Bertrami operator.

1. INTRODUCTION

In this paper, the problem [11] to understand the double coset space $H\backslash G/H$ in terms of functions of distribution class for a homogeneous space G/H is examined in one specific case study for the space $G/H = SL(n+1,\mathbf{R})/GL^+(n,\mathbf{R})$. If one deals with the class of arbitrary functions, then the $H \times H$-orbit decomposition on G gives enough information. This is the case for the symmetric spaces over the finite field (cf. [15]) and the set of characteristic functions of orbits forms a linear basis of invariant functions. If one deals with the regular functions on the algebraic group, it is enough to look at the ring of invariants. More generally, if one considers bi H-invariant continuous functions, then the lower-dimensional orbits can be ignored.

We here consider distributions. The group case, the famous result of Harish-Chandra tells us that every invariant eigendistribution is locally integrable and

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that there is no singular invariant eigendistribution. For Riemannian symmetric spaces, every invariant eigendistribution is real analytic. However, for the case of non-Riemannian symmetric spaces, the natural function space for invariant eigendistributions (spherical functions) is the class of distributions (or its generalization to hyperfunctions). A lot of works have been done for spherical functions and also the eigenspace representations, which imposes the eigenspace condition without invariance. Here we will consider invariant distributions without the eigenfunction condition of (indefinite) Laplacian. In the case of invariant eigendistributions, the space is finite-dimensional, and especially for the rank-one case we can write down explicitly such distributions using, e.g., hypergeometric functions. On the other hand, the space of invariant distributions is infinite-dimensional, and has no ring structure in general.

Our target space G/H is a symmetric space, however, we use the related homogeneous spaces, having a larger symmetry. In the case of the analysis on the tangent space of the symmetric space, such an enlargement of the symmetry, which has been discussed in [5] (see also [7]), can be formulated on the tangent space itself, but on the symmetric spaces it is necessary to introduce such homogeneous spaces to formulate the enlargement of the symmetry.

Anyway, for the symmetric space or its related (spherical) homogeneous space, the double coset space $H\backslash G/H$ is not a manifold especially near the origin. It is neither smooth nor Hausdorff, in general. Since we are considering distributions, the analysis at the origin is complicated on such a space. For a non-Archimedean (p-adic) local field, the structure of distributions are easier than for real numbers $\mathbf{R}$, e.g., the extensions and decomposition of distributions reduces some problem to the orbit decompositions, and the structure of the distributions supported on a lower-dimensional submanifold is simpler (there is no derivative of Dirac delta), cf. [1]. In our Lie group case, analysis around some singular locus will be more subtle, and it is a point of discussion. We apply the result [7] on the tangent space of G/H to prove the extra symmetry which every invariant distribution has. This enables us to eliminate a contribution of some singular loci on the non-isotropic symmetric spaces. This is given in Section 3. In Section 2, we summarize several geometric facts, which will be used in the later sections or help us to understand the double coset space.

As an application of the main theorem in Section 3, we discuss the property called *generalized Gelfand pair* in Section 4. This notion is related to the uniqueness of the decomposition of the left regular representations of $L^2(G/H)$, see [14]. So-called J-criterion is one of the well-known sufficient conditions to be a generalized Gelfand pair. The J-criterion for a class of semisimple symmetric spaces of rank one is examined in [13]. Here we slightly generalize the statement by dropping the eigenfunction condition. Note that the J-criterion holds while the geometric counterpart does not hold. It suggests a careful study between the orbit spaces and the space of invariant distributions.

2.1. Enlargement of the symmetry of the space

Let $G = SL(n+1, \mathbf{R})$, and σ be the involution of G defined by the conjugation by the matrix $\mathrm{diag}(-1, 1, \ldots, 1)$. The fixed point subgroup is

$$G^\sigma = \left\{ \begin{pmatrix} (\det h)^{-1} & 0 \\ 0 & h \end{pmatrix} \,\middle|\, h \in GL(n, \mathbf{R}) \right\},$$

which is often to be identified with $GL(n, \mathbf{R})$. The identity component of G^σ is denoted by H, which is isomorphic to $GL^+(n, \mathbf{R})$. The homogeneous space G/H is a non-isotropic non-Riemannian symmetric space of rank one, which is our main concern in this paper. We introduce $H_1 = SL(n, \mathbf{R}) \subset H = GL^+(n, \mathbf{R})$, then we have $G^\sigma/H = \mathbf{Z}/2\mathbf{Z}$, $G^\sigma/H_1 \cong GL(1, \mathbf{R}) = \mathbf{R}^\times$, $H/H_1 \cong GL^+(1, \mathbf{R})$, and $H \cong H_1 \times GL^+(1, \mathbf{R})$. Then we have $G/H = (G/H_1)/(H/H_1) = (G/H_1)/GL^+(1, \mathbf{R})$. It is easy to see that the normalizer of H_1 in G is G^σ.

We will describe the homogeneous space G/H_1 as follows. Let $X_1 = \{(x, y) \mid x, y \in \mathbf{R}^{n+1}, \langle x, y\rangle = 1\}$. Here we regard $x = {}^t(x_1, x_2, \ldots, x_{n+1}) = {}^t(x_1, x')$, $y = {}^t(y_1, y_2, \ldots, y_{n+1}) = {}^t(y_1, y')$ as column vectors. The action of G on X_1 is defined by

$$g.(x, y) = (gx, {}^tg^{-1}y), \quad x, y \in \mathbf{R}^{n+1}, \; g \in G.$$

It is easy to see that this action is transitive. We set $x_1 = {}^t({}^te_1, {}^te_1)$ with $e_1 = {}^t(1, 0, \ldots, 0) \in \mathbf{R}^{n+1}$. Then the isotropy subgoup of G at x_1 is H_1, and we have a natural isomorphism $X_1 = G/H_1$.

We define the function $Q_0(x, y) = x_1y_1 + x_2y_2 + \cdots + x_{n+1}y_{n+1}$ on $\mathbf{R}^{2(n+1)}$. We denote by $\widetilde{G} = SO_0(Q_0) \cong SO_0(n+1, n+1)$ the identity component of the orthogonal group $O(Q_0)$ corresponding to the quadratic form Q_0. Then $\widetilde{G}$ also acts on X_1 transitively. The isotropy subgroup H_2 of $\widetilde{G}$ at x_1 is isomorphic to $SO_0(n, n+1)$. Then $X_1 \cong G/H_1 \cong \widetilde{G}/H_2$. The role of this isomorphism for the harmonic analysis has been emphasized in [3, Example 5.2]. This isomorphism means that the space G/H_1 has a larger symmetry $\widetilde{G}$. The expression $X_1 = G/H_1$ is not a symmetric space but a (real form of) spherical homogeneous space. The expression $X_1 = \widetilde{G}/H_2$ is an isotropic symmetric space of rank one.

2.2. Invariants and orbits

The function $Q(x, y) = 1 - x_1y_1 = {}^tx'y'$ on X_1 is H-invariant. The functions x_1 and y_1 on X_1 are H_1-invariant. The map

$$q : X_1 \ni (x, y) \mapsto (x_1, y_1) \in \mathbf{R}^2$$

is real-analytic, surjective, and H_1-invariant. As is seen later, the map q almost classifies the set of H_1-orbits on X_1, and any H_1-invariant continuous function on X_1 is a pull back by the map q of a continuous function on $\mathbf{R}^2$. For distributions, the question is more subtle.

Let $\widetilde{H} = SO_0(Q_1)$ be the identity component of the orthogonal group $O(Q_1)$ corresponding to the quadratic form $Q_1(x', y') = x_2 y_2 + \cdots + x_{n+1} y_{n+1}$ on $\mathbf{R}^{2n}$. Then $\widetilde{H}$ is identified with a subgroup of $\widetilde{G} = SO_0(Q_0)$. The group $\widetilde{H}$ preserves the invariants x_1 and y_1. Now we give several orbit decompositions.

(1) H_1-orbit decomposition on X_1. The map $q = (x_1, y_1)$ almost classifies the H_1-orbits on X_1. Say, for $(x_1, y_1) \in \mathbf{R}^2$ with $x_1 y_1 \neq 1$, the fiber $q^{-1}(x_1, y_1)$ is an H_1-orbit if $n \geqslant 2$. For $x_1 \neq 0$, the fiber $q^{-1}(x_1, x_1^{-1})$ splits into four H_1-orbits $\{(x', y') \mid {}^t x' y' = 0, x' \neq 0, y' \neq 0\}$, $\{(0, y') \mid y' \neq 0\}$, $\{(x', 0) \mid x' \neq 0\}$, and $\{(0,0)\}$ if $n \geqslant 3$. If $n = 2$, then the fiber $q^{-1}(x_1, x_1^{-1})$ consists of H_1-orbits $\{(x', y') = (x_2, x_3, y_2, y_3) \mid x' \neq 0,\ y_2 = -t x_3, y_3 = t x_2\}$ with $t \neq 0$, $\{(0, y') \mid y' \neq 0\}$, $\{(x', 0) \mid x' \neq 0\}$, and $\{(0,0)\}$.

(2) $GL^+(n, \mathbf{R})$-orbit decomposition on X_1. We consider the action of $GL^+(n, \mathbf{R})$ on X_1 by

$$(x_1, x', y_1, y') \mapsto (x_1, hx', y_1, {}^t h^{-1} y') \quad \text{for } h \in GL^+(n, \mathbf{R}),\ (x, y) \in X_1.$$

The orbit decomposition on X_1 under $GL^+(n, \mathbf{R})$ is the same as that under H_1, except for the case $n = 2$ and the fiber $q^{-1}(x_1, x_1^{-1})$, which is decomposed into five $GL^+(n, \mathbf{R})$-orbits $\{(x', y') = (x_2, x_3, y_2, y_3) \mid x' \neq 0, \pm(x_2 y_3 - x_3 y_2) > 0\}$, $\{(0, y') \mid y' \neq 0\}$, $\{(x', 0) \mid x' \neq 0\}$, and $\{(0,0)\}$.

(3) $\widetilde{H}$-orbit decomposition on X_1. Since the map is $\widetilde{H}$-invariant, the fiber $q^{-1}(x_1, y_1)$ for $(x_1, y_1) \in \mathbf{R}^2$ with $x_1 y_1 \neq 1$ is an $\widetilde{H}$-orbit if $n \geqslant 2$. The fiber $q^{-1}(x_1, x_1^{-1})$ consists of two $\widetilde{H}$-orbits, $\{0\}$ and $\{(x', y') \neq 0 \mid {}^t x' y' = 0\}$.

(4) H-orbit decomposition on X. This is equivalent to $GL^+(n, \mathbf{R}) \times GL^+(1, \mathbf{R})$-orbits on X_1. In this case, the map $Q: X_1 \ni (x, y) \mapsto 1 - x_1 y_1 \in \mathbf{R}$ almost classifies orbits. In fact, for $t \neq 0, 1$, the fiber $Q^{-1}(t)$ consists of two orbits $\{(x, y) \in X_1 \mid x_1 y_1 = 1 - t, \pm x_1 > 0\}$ if $n \geqslant 2$. $Q^{-1}(1)$ consists of five orbits $\{(x, y) \in X_1 \mid \pm x_1 > 0,\ y_1 = 0\}$, $\{(x, y) \in X_1 \mid x_1 = 0, \pm y_1 > 0\}$, and $\{(x, y) \in X_1 \mid x_1 = y_1 = 0\}$ if $n \geqslant 2$. For $x_1 \neq 0$, the fiber $Q^{-1}(0)$ splits into eight orbits $\{(x_1, x', x_1^{-1}, y') \mid \pm x_1 > 0, {}^t x' y' = 0, x' \neq 0, y' \neq 0\}$, $\{(x_1, 0, x_1^{-1}, y') \mid \pm x_1 > 0, y' \neq 0\}$, $\{(x_1, x', x_1^{-1}, 0) \mid \pm x_1 > 0, x' \neq 0\}$, and $\{(x_1, 0, x_1^{-1}, 0) \mid \pm x_1 > 0\}$ if $n \geqslant 3$. If $n = 2$, then the fiber $q^{-1}(0)$ consists of ten orbits $\{(x_1, x_2, x_3, x_1^{-1}, y_2, y_3) \mid \pm x_1 > 0,\ x' \neq 0,\ \pm(x_2 y_3 - x_3 y_2) > 0\}$, $\{(x_1, 0, x_1^{-1}, y') \mid \pm x_1 > 0,\ y' \neq 0\}$, $\{(x_1, x', x_1^{-1}, 0) \mid \pm x_1 > 0, x' \neq 0\}$, and $\{(x_1, 0, x_1^{-1}, 0) \mid \pm x_1 > 0\}$.

(5) $\widetilde{H} \times GL^+(1, \mathbf{R})$-orbit decomposition on X_1. For $n \geqslant 2$ and $t \neq 0$, the orbit decomposition of the fiber $Q^{-1}(t)$ is the same as that of (4). The fiber $Q^{-1}(0)$ splits into four orbits $\{(x_1, x', x_1^{-1}, y') \mid \pm x_1 > 0,\ {}^t x' y' = 0,\ (x', y') \neq 0\}$, and $\{(x_1, 0, x_1^{-1}, 0) \mid \pm x_1 > 0\}$ if $n \geqslant 3$. If $n = 2$, then the fiber $Q^{-1}(0)$ consists of four orbits $\{(x_1, x_2, x_3, x_1^{-1}, y_2, y_3) \mid \pm x_1 > 0, (x', y') \neq 0\}$, and $\{(x_1, 0, x_1^{-1}, 0) \mid \pm x_1 > 0\}$.

The geometric J-criterion (for an involution θ) is the statement that $Hg^{-1}H = H\theta(g)H$ for all $g \in G$. Note that for the involution $\theta(g) = {}^t g^{-1}$, the space G/H

does not satisfy the geometric J-criterion since the H-orbit $\{(x_1, 0, x_1^{-1}, y') \mid \pm x_1 > 0,\ y' \neq 0\}$ is mapped to $\{(x_1, x', x_1^{-1}, 0) \mid \pm x_1 > 0,\ x' \neq 0\}$, respectively, by $HgH \mapsto H\theta(g)^{-1}H$. Nevertheless, we will prove the (original) J-criterion (for distributions) in Section 4.

3. BI-INVARIANT DISTRIBUTIONS

We denote by $C^{-\infty}$ the set of functions of distribution class. We simply call a function of distribution class a *distribution.*

We start from a direct conclusion of the enlargement of the symmetry of the space G/H_1.

Lemma 1. *There are natural identifications between the set of distributions with the following properties.*

(i) *An H-bi-invariant distribution on G.*
(ii) *A left H and right H/H_1-invariant distribution on G/H_1.*
(iii) *A left $GL^+(n, \mathbf{R}) \times GL^+(1, \mathbf{R})$-invariant distribution on X_1.*

Proof. Since H normalizes H_1, H acts on G/H_1 from the right as

$$G/H_1 \ni gH_1 \mapsto ghH_1 \in G/H_1,$$

which induces the action of $H/H_1 \cong GL^+(1, \mathbf{R})$ on X_1 as $(x, y) \mapsto ((\det h)^{-1}x, (\det h)y)$, that is, $(x, y) \mapsto (t^{-1}x, ty)$ by $t > 0$. On the other hand, the action of H on G/H_1 from the left is

$$(x_1, x', y_1, y') \mapsto \big((\det h)^{-1}x_1, hx', (\det h)y_1, {}^t h^{-1}y'\big).$$

Then, an H-bi-invariant function on G is identified with a function on X_1 invariant under the action

$$(x, y) \mapsto (tx, t^{-1}y), \quad (x_1, x', y_1, y') \mapsto (x_1, hx', y_1, {}^t h^{-1}y'),$$

for all $t > 0$ and $h \in GL^+(n, \mathbf{R})$. Then $GL^+(1, \mathbf{R}) \times GL^+(n, \mathbf{R})$ is a subgroup of $\widetilde{G}$. $\square$

It is somewhat mysterious that the left-right action of H on G turns to be equivalent to the left action of $H \times GL^+(1, \mathbf{R}) \subset \widetilde{G}$.

The next theorem shows the enlargement of the symmetry on invariant distributions, which has not been predicted by the geometry, e.g., orbit structures.

Theorem 2. *Let $n \geqslant 3$. Then a $GL^+(n, \mathbf{R})$-invariant distribution on X_1 is $\widetilde{H}$-invariant.*

Proof. The invariance is local, so we may consider the restrictions on the open subset $\{Q < 1\}$ and $\{Q > 0\}$.

(i) On the open subset $Q < 1$. We can take the coordinates (x_1, x', y') on the open subset $\{(x, y) \in X_1 \mid Q < 1, x_1 \neq 0\} \cong \{(x', y') \in \mathbf{R}^{2n} \mid {}^t x' y' < 1\} \times \{x_1 \neq 0\}$. Actually, $y_1 = (1 - {}^t x' y')/x_1$. The action of $\widetilde{H}$ is only on the variables (x', y'). It has been proved in Theorem 1 of [7] that the D-modules for H-invariants equals to that for $\widetilde{H}_1$. It means that any H-invariant distribution on this space is $\widetilde{H}$-invariant. On the open set $y_1 \neq 0$, we have the same argument.

(ii) On the open subset $Q > 0$. We can take the coordinates $\{(x, y) \in X_1 \mid Q > 0\} \ni (x_1, x', y_1, y') \mapsto ((x', y'/(1 - x_1 y_1)), (x_1, y_1)) \in \{(\xi, \eta) \in \mathbf{R}^{2n} \mid {}^t\xi\eta = 1\} \times \{(x_1, y_1) \mid x_1 y_1 < 1\}$. Then both the groups H and $\widetilde{H}$ act on the first factor transitively. So, the H-invariants implies $\widetilde{H}$-invariants. □

Remark 3. (1) The proof shows that the same statement for the theorem holds if we replace 'distribution' by 'hyperfunction'.

(2) We can regards Lemma 7.4 of [4] as a special case of theorem; they have proved that an H-invariant distribution on X supported on $\{(x, y) \in X_1 \mid x' = 0$ or $y' = 0\}$ is supported on $\{(x, y) \in X_1 \mid x' = 0$ and $y' = 0\}$. The proof uses the fact that distributions are of finite order.

In the case of the tangent spaces, such an extension of the symmetry has been observed in [5] with the eigenfunction condition, and is extended in [7] without the eigenfunction condition. These works are inspired by [12]. Compared to the case of the tangent spaces, the geometric setting is slightly more subtle for the case of homogeneous spaces.

4. GENERALIZED GELFAND PAIRS

We define an anti-involution $J: G \to G$ by $J(g) = {}^t g$ for $g \in G$. The map J induces the linear endomorphism J^* on $C^{-\infty}(G)^{H\times H}$ by $f \mapsto f \circ J$. The natural isomorphism $i: G/H_1 \to X_1$ induces the isomorphism

$$i^*: C^{-\infty}(X_1)^{H\times GL^+(1,\mathbf{R})} \to C^{-\infty}(G)^{H\times H}.$$

We now describe the map J^* on $C^{-\infty}(G)^{H\times H} \cong C^{-\infty}(X_1)^{H\times GL^+(1,\mathbf{R})}$. We define $j: X_1 \ni (x_1, x', y_1, y') \mapsto (x_1, -y', y_1, -x') \in X_1$.

Lemma 4. *For any $f \in C^{-\infty}(X_1)^{H\times GL^+(1,\mathbf{R})}$, we have $J^* i^* f = i^* j^* f$.*

Proof. (i) On $Q < 1$. Let $U = \{(x, y) \in X_1 \mid x_1 y_1 > 0\}$,

$$\alpha: U \times H_1 \ni \big((x, y), h\big) \mapsto \begin{pmatrix} x_1 & -y_2 & -{}^t y''/y_1 \\ x_2 & y_1 & 0 \\ x'' & 0 & I_{n-1} \end{pmatrix} \in G,$$

where $x' = {}^t(x_2, x'')$, $y' = {}^t(y_2, {}^t y'')$ and $\varphi: U \times H_1 \to GL^+(n, \mathbf{R}) \subset \widetilde{H}$ by $\varphi((x, y), h) = \mathrm{diag}(y_1, 1, \ldots, 1)/y_1$. The image of α is an open subset $i^{-1}(\{Q > 0\})$ of G. We have $i \circ J \circ \alpha = \varphi \cdot (j \circ i \circ \alpha)$. This proves $(J^* i^* f)(\alpha((x, y), h)) =$

$f(i \circ J \circ \alpha((x, y), h))) = f(\varphi((x, y), h) j \circ i \circ \alpha((x, y), h)) = f(j \circ i \circ \alpha((x, y), h)) = (i^* j^* f)(\alpha((x, y), h))$.

(ii) On $Q > 0$. Let $U = \{(x_1, y_1) \in \mathbf{R}^2 \mid x_1 y_1 < 1\}$,

$$\alpha : U \times H \times H \ni ((x_1, y_1), h, h')$$
$$\mapsto h \begin{pmatrix} x_1 & -1 + x_1 y_1 & 0 \\ 1 & y_1 & 0 \\ 0 & 0 & I_{n-1} \end{pmatrix} h' \in G,$$

and $\varphi : U \times H \times H \to GL^+(n, \mathbf{R}) \subset \widetilde{H}$ by $\varphi(h, h') = \det(hh')^{-1}\, {}^t(hh') \in GL^+(n, \mathbf{R})$. The image of α is $i^{-1}(\{Q < 1\})$. We have $i \circ J \circ \alpha = \varphi \cdot (j \circ i \circ \alpha)$. This proves $(J^* i^* f)(\alpha((x_1, y_1), h, h')) = (i^* j^* f)(\alpha((x_1, y_1), h, h'))$. □

Theorem 5. *For $n \geqslant 3$, J^* is the identity on $C^{-\infty}(G)^{H \times H}$.*

Proof. It is enough to prove that j^* is the identity on $C^{-\infty}(X_1)^{\widetilde{H} \times GL^+(1,\mathbf{R})}$. For any $f \in C^{-\infty}(X_1)^{\widetilde{H} \times GL^+(1,\mathbf{R})}$, the support of $j^* f - f$ is contained in $\{(x, y) \in X_1 \mid x' = y' = 0\}$. Then the distribution $j^* f - f$ can be written as $p(\Box)\delta(x', y')$ with some polynomial p of the indefinite Laplacian $\Box = \sum_{i=2}^{n+1} \partial^2 / \partial x_i \partial y_i$ on each open subset $\{(x, y) \in X_1 \mid x_1 y_1 > 0, \pm x_1 > 0\}$. This means that $(j^* f - f)$ is invariant under the action of j, and that it is zero. □

For the space G/H, the J-criterion that any bi-H-invariant *eigen*distribution on G is invariant under J^* has been proved in [13]. The J-criterion implies that the space G/H is a generalized Gelfand pair [10,13].

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