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## Detailed balance in micro- and macrokinetics and micro-distinguishability of macro-processes



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## ABSTRACT

We develop a general framework for the discussion of detailed balance and analyse its microscopic background. We find that there should be two additions to the well-known  $T$ - or  $PT$ -invariance of the microscopic laws of motion:

1. Equilibrium should not spontaneously break the relevant  $T$ - or  $PT$ -symmetry.
2. The macroscopic processes should be microscopically distinguishable to guarantee persistence of detailed balance in the model reduction from micro- to macrokinetics.

We briefly discuss examples of the violation of these rules and the corresponding violation of detailed balance.

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## 1. The history of detailed balance in brief

VERY deep is the well of the past. . . For the deeper we sound, the further down into the lower world of the past we probe and press, the more do we find that the earliest foundation of humanity, its history and culture, reveal themselves unfathomable.

T. Mann [1]

Detailed balance as a consequence of the reversibility of collisions (*at equilibrium, each collision is equilibrated by the reverse collision*, Fig. 1) was introduced by Boltzmann for the Boltzmann equation and used in the proof of the  $H$ -theorem [2] (Boltzmann's arguments were analysed by Tolman [3]). Five years earlier, Maxwell used the principle of detailed balance for gas kinetics with the reference to the *principle of sufficient reason* [4]. He analysed equilibration in cycles of collisions and in the pairs of mutually reverse collisions and mentioned "Now it is impossible to assign a reason why the successive velocities of a molecule should be arranged in this cycle, rather than in the reverse order."

In 1901, Wegscheider introduced detailed balance for chemical kinetics on the basis of classical thermodynamics [5]. He used the assumption that each elementary reaction is reversible and should respect thermodynamics (i.e. entropy production in this reaction should be always non-negative). Onsager used this work of Wegscheider in his famous paper [6]. Instead of direct citation he

wrote: "Here, however, the chemists are accustomed to impose a very interesting additional restriction, namely: when the equilibrium is reached each individual reaction must balance itself." Einstein used detailed balance as a basic assumption in his theory of radiation [7]. In 1925, Lewis recognized the principle of detailed balance as a new general principle of equilibrium [8]. The limit of the detailed balance for systems which include some irreversible elementary processes (without reverse processes) was recently studied in detail [9,10].

In this paper, we develop a general formal framework for discussion of detailed balance, analyse its microscopic background and persistence in the model reduction from micro- to macrokinetics.

2. Sampling of events,  $T$ -invariance and detailed balance

## 2.1. How detailed balance follows from microreversibility

In the sequel, we omit some technical details assuming that all the operations are possible, all the distributions are regular and finite Borel (Radon) measures, and all the integrals (sums) exist.

The basic notations and notions:

- $\Omega$  – a space of states of a system (a locally compact metric space).
- Ensemble  $\nu$  – a non-negative distribution on  $\Omega$ .
- Elementary process has a form  $\alpha \rightarrow \beta$  (Fig. 2), where  $\alpha, \beta$  are non-negative distributions.

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- Complex – an input or output distribution of an elementary process.
- $\Upsilon$  – the set of all complexes participating in elementary processes. It is equipped with the weak topology and is a closed and locally compact set of distributions.
- The reaction rate  $r$  is a measure defined on  $\Upsilon^2 = \{(\alpha, \beta)\}$ . It describes the rates of all elementary processes  $\alpha \rightarrow \beta$ .
- The support of  $r$ ,  $\text{suppr} \subset \Upsilon^2$ , is the *mechanism* of the process, i.e. it is the set of pairs  $(\alpha, \beta)$ , each pair represents an elementary process  $\alpha \rightarrow \beta$ . (Usually,  $\text{suppr} \subsetneq \Upsilon^2$ .)
- The rate of the whole kinetic process is a distribution  $W$  on  $\Omega$  (the following integral should exist):

$$W = \frac{1}{2} \int_{(\alpha, \beta) \in \Upsilon^2} (\beta - \alpha) d[r(\alpha, \beta) - r(\beta, \alpha)].$$

The distribution  $\nu$  depends on time  $t$ . For systems with continuous time,  $\dot{\nu} = W$ . For systems with discrete time,  $\nu(t + \tau) - \nu(t) = W$ , where  $\tau$  is the time step. To create the closed kinetic equation (the associated *nonlinear Markov process* [11]) we have to define the map  $\nu \mapsto r$  that puts the reaction rate  $r$  (a Radon measure on  $\Upsilon^2$ ) in correspondence with a non-negative distribution  $\nu$  on  $\Omega$  (the *closure problem*). In this definition, some additional restrictions on  $\nu$  may be needed. For example, one can expect that  $\nu$  is absolutely continuous with respect to a special (equilibrium) measure. There are many standard examples of kinetic systems: mass action law for chemical kinetics [12,13], stochastic models of chemical kinetics [18], the Boltzmann equation [14] in quasicheical representation [15] for space-uniform distributions, the lattice Boltzmann models [16], which represent the space motion as elementary discrete jumps (discrete time), and the quasicheical models of diffusion [17].

We consider interrelations between two important properties of the measure  $r(\alpha, \beta)$ :

- (EQ)  $W = 0$  (equilibrium condition);
- (DB)  $r(\alpha, \beta) \equiv r(\beta, \alpha)$  (detailed balance condition).

It is possible to avoid the difficult closure question about the map  $\nu \mapsto r$  in discussion of  $T$ -invariance and relations between EQ and DB conditions.

Obviously,  $\text{DB} \Rightarrow \text{EQ}$ . There exists a trivial case when  $\text{EQ} \Rightarrow \text{DB}$  (a sort of linear independence of the vectors  $\gamma = \beta - \alpha$  for elementary processes joined in pairs with their reverse processes): if

$$\int_{(\alpha, \beta) \in \text{suppr}} (\beta - \alpha) d\mu(\alpha, \beta) = 0 \Rightarrow \mu = 0$$

for every antisymmetric measure  $\mu$  on  $\Upsilon^2$  ( $\mu(\alpha, \beta) = -\mu(\beta, \alpha)$ ), then  $\text{EQ} \Rightarrow \text{DB}$ .

There is a much more general reason for detailed balance,  $T$ -invariance. Assume that the kinetics give a coarse-grained description of an ensemble of interacting microsystems and this interaction of microsystems obeys a reversible in time equation: if we look on the dynamics backward in time (*operation T*) we will observe the solution of the same dynamic equations. For  $T$ -invariant microscopic dynamics,  $T$  maps an equilibrium ensemble into an equilibrium ensemble. Assuming uniqueness of the equilibrium under given values of the conservation laws, one can just postulate the *invariance of equilibria with respect to the time reversal transformation*

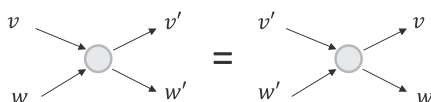


Fig. 1. Schematic representation of detailed balance for collisions: at equilibrium, each collision is equilibrated by the reverse collision.

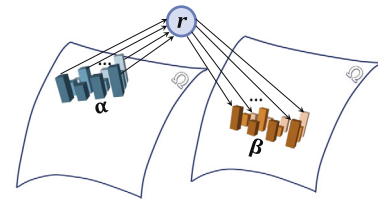


Fig. 2. Schematic representation of an elementary process. Input ( $\alpha$ ) and output ( $\beta$ ) distributions are represented by column histograms.

or  $T$ -invariance of equilibria: if we observe an equilibrium ensemble backward in time, nothing will change.

Let the complexes remain unchanged under the action of  $T$ . In this case, the time reversal transformation for collisions (Fig. 1) leads to the reversal of arrow: the direct collision is transformed into the reverse collision. The same observation is valid for inelastic collisions. Following this hint, we can accept that the reversal of time  $T$  transforms every elementary process  $\alpha \rightarrow \beta$  into its reverse process  $\beta \rightarrow \alpha$ . This can be considered as a restriction on the definition of direct and reverse processes in the modelling (a “model engineering” restriction): the direct process is an ensemble of microscopic events and the reverse process is the ensemble of the time reversed events.

Under this assumption,  $T$  transforms  $r(\alpha, \beta)$  into  $r(\beta, \alpha)$ . If the rates of elementary processes may be observed (for example, by the counting of microscopic events in the ensemble) then  $T$ -invariance of equilibrium gives DB: at equilibrium,  $r(\alpha, \beta) = r(\beta, \alpha)$ , i.e.  $\text{EQ} \Rightarrow \text{DB}$  under the hypothesis of  $T$ -invariance.

The assumption that the complexes are invariant under the action of  $T$  may be violated: for example, in Boltzmann’s collisions (Fig. 2) the input measure is  $\alpha = \delta_v + \delta_w$  and the output measure is  $\beta = \delta_{v'} + \delta_{w'}$ . Under time reversal,  $\delta_v \xrightarrow{T} \delta_{-v}$ . Therefore  $\alpha \xrightarrow{T} \delta_{-v} + \delta_{-w}$  and  $\beta \xrightarrow{T} \delta_{-v'} + \delta_{-w'}$ . We need an additional invariance, the space inversion invariance (transformation  $P$ ) to prove the detailed balance (Fig. 1). Therefore, the detailed balance condition for the Boltzmann equation (Fig. 1) follows not from  $T$ -invariance alone but from  $PT$ -invariance because for Boltzmann’s kinetics

$$\{\alpha \rightarrow \beta\} \xrightarrow{PT} \{\beta \rightarrow \alpha\}.$$

In any case, the microscopic reasons for the detailed balance condition include existence of a symmetry transformation  $\mathfrak{T}$  such that

$$\{\alpha \rightarrow \beta\} \xrightarrow{\mathfrak{T}} \{\beta \rightarrow \alpha\} \tag{1}$$

and the microscopic dynamics is invariant with respect to  $\mathfrak{T}$ . In this case, one can conclude that (i) the equilibrium is transformed by  $\mathfrak{T}$  into the same equilibrium (it is, presumably, unique) and (ii) the reaction rate  $r(\alpha, \beta)$  is transformed into  $r(\beta, \alpha)$  and does not change because nothing observable can change (equilibrium is the same). Finally, at equilibrium  $r(\alpha, \beta) \equiv r(\beta, \alpha)$  and  $\text{EQ} \Rightarrow \text{DB}$ .

There remain two questions:

1. We are sure that  $\mathfrak{T}$  transforms the equilibrium state into an equilibrium state but is it necessarily the same equilibrium? Is it forbidden that the equilibrium is degenerate and  $\mathfrak{T}$  acts non-trivially on the set of equilibria?
2. We assume that the rates of different elementary processes are physical observables and the ensemble with different values of these rates may be distinguished experimentally. Is it always true?

The answer to both questions is “no”. The principle of detailed balance can be violated even if the physical laws are  $T, P$  and  $PT$

symmetric. Let us discuss the possible reasons for these negative answers and the possible violations of detailed balance.

## 2.2. Spontaneous breaking of $\mathfrak{T}$ -symmetry

Spontaneous symmetry breaking is a well known effect in phase transitions and particle physics. It appears when the physical laws are invariant under a transformation, but the equilibrium of the system transforms into another state, which should be also equilibrium. Hence, the equilibrium is degenerated. The best known examples are magnets. They are not rotationally symmetric (there is a continuum of equilibria that differ by the direction of magnetic field). Crystals are not symmetric with respect to translation (there is continuum of equilibria that differ by a shift in space). In these two examples, the multiplicity of equilibria is masked by the fact that all these equilibrium states can be transformed into each other by a proper rigid motion transformation (translation and rotation).

The *nonreciprocal media* violate  $T$  and  $PT$  invariance [19–21]. These media are transformed by  $T$  and  $PT$  into different (*dual*) equilibrium media and cannot be transformed back by a proper rigid motion. The implication  $EQ \Rightarrow DB$  for the nonreciprocal media may be wrong and for its validity some strong additional assumptions are needed, like the linear independence of elementary processes.

Spontaneous breaking of  $\mathfrak{T}$ -symmetry provides us a counterexample to the proof of detailed balance. In this proof, we used the assumption that under transformation  $\mathfrak{T}$  elementary processes transform into their reverse processes (1) and, at the same time, the equilibrium ensemble does not change.

If the equilibrium is transformed by  $\mathfrak{T}$  into another (but obviously also equilibrium) state then our reasoning cannot be applied to reality and the proof is not valid. Nevertheless, the refutation of the proof does not mean that the conclusion (detailed balance) is necessarily wrong. Following the Lakatos terminology [23] we should call the spontaneous breaking of  $\mathfrak{T}$ -symmetry the *local counterexample* to the principle detailed balance. It is an intriguing question whether such a local counterexample may be transformed into a *global* one: does the violation of the Onsager reciprocal relation mean the violation of detailed balance (and not only the refutation of its proof)?

## 2.3. Reciprocal relation and detailed balance

It is known that for many practically important kinetic laws the Onsager reciprocal relations follow from detailed balance. In these cases, violation of the reciprocal relations implies violation of the principle of detailed balance. For example, for the systems in magnetic fields the reciprocal relations may be violated [24], and we can expect that detailed balance for these systems will be also violated.

For master equation (first order kinetics or continuous time Markov chains) the principle of detailed balance is *equivalent* to the reciprocal relations ([24] Ch. 10, Section 4). For the nonlinear mass action law the implication “detailed balance  $\Rightarrow$  reciprocal relations” is also well known (see, for example, [12]) but the equivalence is not correct because the number of nonlinear reactions for a given number of components may be arbitrarily large and it is possible to select such values of reaction rate constants that the reciprocal relations are satisfied but the principle of detailed balance does not hold. For transport processes, the quasichemical models [17] also demonstrate how the reciprocal relations follow from detailed balance for the mass action law kinetics or the generalized mass action law. We confine the discussion of kinetic laws to systems with finite sets of components.

Consider a finite-dimensional system with the set of components (species or states)  $A_1, \dots, A_n$  given. For each  $A_i$  the extensive

variable  $N_i$  (“amount” of  $A_i$ ) is defined. The Massieu-Planck function  $\Phi(N, \dots)$  (*free entropy* [25]) depends on the vector  $N$  with coordinates  $N_i$  and on the variables that are constant under given conditions. For isolated systems instead of  $(\dots)$  in  $\Phi$  we should use internal energy  $U$  and volume  $V$  (and this  $\Phi$  is the entropy), for isothermal isochoric systems these variables are  $1/T$  and  $V$ , where  $T$  is temperature, and for isothermal isobaric systems we should use  $1/T$  and  $P/T$ , where  $P$  is pressure. For all such conditions,

$$\frac{\partial \Phi}{\partial N_i} = -\frac{\mu_i}{T},$$

where  $\mu_i$  is the chemical potential of  $A_i$  or the *generalized chemical potential* for the quasichemical models where interpretation of  $A_i$  is wider than just various atomic particles.

Elementary processes in the finite-dimensional systems are represented by their stoichiometric equations

$$\sum_i \alpha_{\rho i} A_i \rightarrow \sum_i \beta_{\rho i} A_i.$$

This is a particular case of the general picture presented in Fig. 2. The *stoichiometric vector* is  $\gamma_\rho$ :  $\gamma_{\rho i} = \beta_{\rho i} - \alpha_{\rho i}$  (gain minus loss). The generalized mass action law represents the reaction rate in the following form:

$$r_\rho = \phi_\rho \exp\left(\sum_i \alpha_{\rho i} \frac{\mu_i}{RT}\right), \quad (2)$$

where  $\exp(\sum_i \alpha_{\rho i} \mu_i / RT)$  is the Boltzmann factor ( $R$  is the gas constant) and  $\phi_\rho > 0$  is the kinetic factor (this representation is closely related to the transition state theory [26] and its generalizations [27]).

The equilibria and conditional equilibria are described as the maximizers of the free entropy under given conditions. For a system with detailed balance every elementary process has a reverse process and the couple of processes  $\sum_i \alpha_{\rho i} A_i \rightleftharpoons \sum_i \beta_{\rho i} A_i$  should move the system from the initial state to the partial equilibrium, that is the maximizer of the function  $\Phi$  in the direction  $\gamma_\rho$ . Assume that the equilibrium is not a *boundary point* of the state space. For a smooth function  $\Phi$ , the conditional maximizer in the direction  $\gamma_\rho$  should satisfy the necessary condition  $\sum_i \gamma_{\rho i} \mu_i = 0$ . In the generalized mass action form (2) the detailed balance condition has a very simple form:

$$\phi_\rho^+ = \phi_\rho^-, \quad (3)$$

where  $\phi_\rho^+$  is the kinetic factor for the direct reaction and  $\phi_\rho^-$  is the kinetic factor for the reverse reaction.

Assume that the detailed balance condition (3) holds. Let us join the elementary processes in pairs, direct with reverse ones, with the corresponding change in their numeration. The kinetic equation is  $\dot{N} = V \sum_\rho \gamma_\rho (r_\rho^+ - r_\rho^-)$ . The Jacobian matrix at equilibrium is

$$\left. \frac{\partial \dot{N}_i}{\partial N_j} \right|_{\text{eq}} = -\frac{V}{R} \sum_k \left( \sum_\rho r_\rho^{\text{eq}} \gamma_{\rho i} \gamma_{\rho k} \right) \frac{\partial (\mu_k / T)}{\partial N_j} \Big|_{\text{eq}},$$

where  $r_\rho^{\text{eq}} = r_\rho^{+\text{eq}} = r_\rho^{-\text{eq}}$  is the rate at equilibrium of the direct and reverse reactions (they coincide due to detailed balance) and the subscript ‘eq’ corresponds to the derivatives at the equilibrium. The linear approximation to the kinetic equations near the equilibrium is

$$\frac{d\Delta N_i}{dt} = -\frac{V}{R} \sum_k \left( \sum_\rho r_\rho^{\text{eq}} \gamma_{\rho i} \gamma_{\rho k} \right) \Delta \left( \frac{\mu_k}{T} \right),$$

where  $\Delta N_i$  and  $\Delta(\mu_k/T)$  are deviations from the equilibrium values. The variables  $\Delta N_i$  are extensive thermodynamic coordinates and  $\Delta(\mu_k/T)$  are intensive conjugated variables – thermodynamic forces.

Time derivatives  $dN_i/dt$  are thermodynamic fluxes. Symmetry of the matrix of coefficients and, therefore, validity of the reciprocal relations is obvious.

Thus, for a wide class of kinetic laws the reciprocal relations in a vicinity of a regular (non-boundary) equilibrium point follow from detailed balance in the linear approximation. In these cases, the non-reciprocal media give *global counterexamples* to the detailed balance. Without reference to a kinetic law they remain local counterexamples to the proof of detailed balance.

2.4. Sampling of different macro-events from the same micro-events

In kinetics, only the total rate  $W$  is observable (as  $W = \dot{v}$  or  $W = \Delta v = v(t + \tau) - v(t)$ ). In the macroscopic world the observability of the rates of the elementary processes is just a hypothesis.

Imagine a microscopic demon that counts collisions or other microscopic events of various types. If different elementary processes correspond to different types of microscopic events then the rates of elementary processes can be observed. If the equilibrium ensemble is invariant with respect to  $\mathfrak{A}$  then the demon cannot detect the difference between the equilibrium and the transformed equilibrium and the rates of elementary processes should satisfy DB. But it is possible to sample the elementary processes of macroscopic kinetics from the events of microscopic kinetics in a different manner.

For example, in chemical mass action law kinetics we can consider the reaction mechanism  $A \rightleftharpoons B$  (rate constants  $k_{\pm 1}$ ),  $A + B \rightleftharpoons 2B$  (rate constants  $k_{\pm 1}$ ) [22]. We can also create a stochastic model for this system with the states  $(xA, yB)$  ( $x, y$  are nonnegative integers) and the elementary transitions  $(xA, yB) \rightleftharpoons ((x-1)A, (y+1)B)$  (rate constants  $\kappa_+ = k_{+1}x + k_{+2}x^2$ ,  $\kappa_- = k_{-1}(y+1) + k_{-2}(x-1)(y+1)$ ). The elementary transitions in this stochastic model are linearly independent and  $EQ \iff DB$ . In the corresponding mass action law chemical kinetics detailed balance requires additional relation between constants:  $k_{+1}/k_{-1} = k_{+2}/k_{-2}$ .

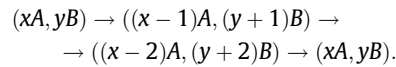
Thus, macroscopic detailed balance may be violated in this example when microscopic detailed balance holds. (For more examples and theoretic consideration of the relations between detailed balance in mass action law chemical kinetics and stochastic models of these systems see [22].) Indeed, both of the macroscopic elementary processes  $A \rightleftharpoons B$  and  $A + B \rightleftharpoons 2B$  correspond to the same set of microscopic elementary processes  $(xA, yB) \rightleftharpoons ((x-1)A, (y+1)B)$ . Each of these elementary events is “shared” between two different macroscopic elementary processes. Therefore, the macroscopic elementary processes in this example are *microscopically indistinguishable*.

The microscopic indistinguishability in this example follows from the coincidence of the stoichiometric vectors for two macroscopic processes  $A \rightleftharpoons B$  and  $A + B \rightleftharpoons 2B$ . If the stoichiometric vectors are just linearly dependent then it does not imply microscopic indistinguishability.

For example, let us take two reactions  $A \rightleftharpoons B$  and  $2A \rightleftharpoons 2B$ . For the first reaction the corresponding microscopic processes have the form  $(xA, yB) \rightleftharpoons ((x-1)A, (y+1)B)$  (if all the coefficients are nonnegative). For the reaction  $2A \rightleftharpoons 2B$  the microscopic processes have the form  $(xA, yB) \rightleftharpoons ((x-2)A, (y+2)B)$  (if all the coefficients are nonnegative). These sets do not intersect, the elementary processes are microscopically distinguishable and macroscopic detailed balance follows from microscopic detailed balance.

Nontrivial Wegscheider identities appear in this example at the microscopic level (in the first example all the microscopic transitions are linearly independent and there exist no additional relations). Let the microscopic reaction rate constants for the reaction  $(xA, yB) \rightleftharpoons ((x-1)A, (y+1)B)$  be  $\kappa_1^+(x, y)$  and  $\kappa_1^-(x, y)$  for the reaction  $(xA, yB) \rightleftharpoons ((x-2)A, (y+2)B)$ . Due to detailed balance, in each cycle of a linear reaction network the product of reaction

rate constants in the clockwise direction coincides with the product in the anticlockwise direction. It is sufficient to consider the basis cycles (and their reversals):



Therefore,

$$\kappa_1^+(x, y)\kappa_1^-(x-1, y+1)\kappa_2^-(x, y) = \kappa_2^+(x, y)\kappa_1^-(x-1, y+1)\kappa_1^-(x, y).$$

In the macroscopic limit these conditions transform into the macroscopic detailed balance conditions.

3. Relations between elementary processes beyond microreversibility and detailed balance

If microreversibility does not exist, is everything permitted? What are the relations between the reaction rates beyond the microreversibility conditions if such universal relations exist? The radical point of view is: beyond the microreversibility we face just the world of kinetic equations with preservation of positivity, various specific restrictions on the coefficients appear in some specific cases and the variety of these cases is unobservable. Development of this point of view leads to the general theory of nonlinear Markov processes [11], i.e. the general theory of kinetic equations with preservation of positivity.

The problem of the relations between elementary processes beyond microreversibility and detailed balance was stated by Lorentz in 1887 [28]. Boltzmann immediately proposed the solution [29] and used it for extension of his  $H$ -theorem beyond microreversibility. These conditions have the form of partially summed conditions of detailed balance (Fig. 3, compare to Fig. 1). This solution was analysed, generalized and proved by several generations of researchers (Heitler, Coester, Watanabe, Stueckelberg [30] and others, see the review in [27]). It was rediscovered in 1972 [31] in the context of chemical kinetics and popularized as the *complex balance condition*.

For finite-dimensional systems which obey the generalized mass action law (2) the complex balance condition is also the summarized detailed balance condition (3). Consider the set  $Y$  of all input and output vectors  $\alpha_\rho$  and  $\beta_\rho$ . The complex balance condition reads: for every  $y \in Y$

$$\sum_{\rho, \alpha_\rho=y} \phi_\rho = \sum_{\rho, \beta_\rho=y} \phi_\rho. \tag{4}$$

Now, the complex balance conditions in combination with generalized mass action law are proven for the finite-dimensional systems in the asymptotic limit proposed first by Michaelis and Menten [32] for fermentative reactions and Stueckelberg [30] for the Boltzmann equation. This limit is constituted by three assumptions (Fig. 4): (i) the elementary processes go through the intermediate compounds, (ii) the compounds are in fast equilibria with the components (therefore, these equilibria can be described by thermodynamics) and (iii) the concentrations of compounds are small with respect

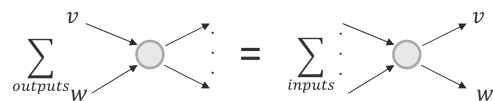
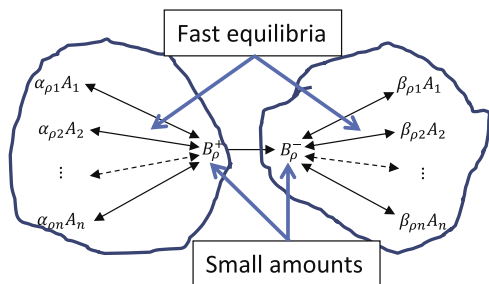


Fig. 3. Boltzmann’s cyclic balance (1887) (or semi-detailed balance or complex balance) is a summarized detailed balance condition: at equilibrium the sum of intensities of collisions with a given input  $v + w \rightarrow \dots$  coincides with the sum of intensities of collisions with the same output  $\dots \rightarrow v + w$  (for general systems see (4)).



**Fig. 4.** Schematic representation of the Michaelis–Menten–Stueckelberg asymptotic assumptions: an elementary process  $\sum \alpha_{\rho i} A_i \rightarrow \sum \beta_{\rho i} A_i$  goes through intermediate compounds  $B_{\rho}^{\pm}$ . The fast equilibria  $\sum \alpha_{\rho i} A_i \rightleftharpoons B_{\rho}^{+}$  and  $\sum \beta_{\rho i} A_i \rightleftharpoons B_{\rho}^{-}$  can be described by conditional maximum of the free entropy. Concentrations of  $B_{\rho}^{\pm}$  are small and reaction between them obeys linear kinetic equation.

to concentrations of components (hence, (iiiA) the quasi steady state assumption is valid for the compound kinetics and (iiiB) the transitions between compounds follow the first order kinetics) [27]. (It is worth mentioning that Michaelis and Menten in 1913 [32] found the asymptotic limit where the fermentative reaction can be described by the mass action law. The so-called Michaelis–Menten kinetics is different and was invented 12 years later by Haldane and Briggs [33]).

Thus, beyond microreversibility, Boltzmann’s cyclic balance (or semi-detailed balance, or complex balance) holds and it is as universal as the idea of intermediate compounds (activated complexes or transition states) which exist in small concentrations and are in fast equilibria with the basic reagents.

#### 4. Conclusion

Thus,  $\text{EQ} \iff \text{DB}$  if:

1. There exists a transformation  $\mathfrak{T}$  that transforms the elementary processes into reverse processes and the microscopic laws of motion are  $\mathfrak{T}$ -invariant.
2. The equilibrium is symmetric with respect to  $\mathfrak{T}$ , that is, there is no spontaneous breaking of  $\mathfrak{T}$ -symmetry.
3. The macroscopic elementary processes are microscopically distinguishable. That is, they represent disjoint sets of microscopic events.

In applications,  $\mathfrak{T}$  is usually either time reversal  $T$  or the combined transform  $PT$ .

For level jumping (reduction of kinetic models [15]), the equivalence  $\text{EQ} \iff \text{DB}$  persists in the reduced (“macroscopic”) model if:

1.  $\text{EQ} \iff \text{DB}$  in the original (“microscopic”) model.
2. Equilibria of the macroscopic model correspond to equilibria of the microscopic model. That is, the reduced kinetic model has no equilibria, which correspond to non-stationary dynamical regimes of the original kinetic model.
3. The macroscopic elementary processes are microscopically distinguishable. That is, they represent disjoint sets of microscopic processes.

In this note, we avoid the discussion of an important part of Boltzmann’s legacy which is very relevant to the topic under consideration. Boltzmann represented kinetic process as an *ensemble of indivisible elementary events – collisions*. In the microscopic world, a collision is continuous in time and infinitely divisible process (and it requires infinite time in most of the models of pair interaction). In the macroscopic world it is instant and indivisible.

The transition from continuous motion of particles to an ensemble of indivisible instant collisions is not digested by modern mathematics up to now, more than 130 years after its invention. The known results [34,35] state that the Boltzmann equation for an ensemble of classical particles with pair interaction and short-range potentials is asymptotically valid starting from a non-correlated state during a fraction of the mean free flight time. That is very far from the area of application. Nevertheless, if we just accept that it is possible to count microscopic events then the reasons of validity and violations of detailed balance in kinetics are clear.

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