Modeling and Optimal Design of 3 Degrees of Freedom Helmholtz Resonator in Hydraulic System

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Abstract

Three degrees of freedom (3-DOF) Helmholtz resonator which consists of three cylindrical necks and cavities connected in series (neck-cavity-neck-cavity-neck-cavity) is suitable to reduce flow pulsation in hydraulic system. A novel lumped parameter model (LPM) of 3-DOF Helmholtz resonator in hydraulic system is developed which considers the viscous friction loss of hydraulic fluid in the necks. Applying the Newton’s second law of motion to the equivalent mechanical model of the resonator, closed-form expression of transmission loss and resonance frequency is presented. Based on the LPM, an optimal design method which employs rotate vector optimization method (RVOM) is proposed. The purpose of the optimal design is to search the resonator’s unknown parameters so that its resonance frequencies can coincide with the pump-induced flow pulsation harmonics respectively. The optimal design method is realized to design 3-DOF Helmholtz resonator for a certain type of aviation piston pump hydraulic system. The optimization result shows the feasibility of this method, and the simulation under optimum parameters reveals that the LPM can get the same precision as transfer matrix method (TMM).

Keywords: structural optimization; vibration analysis; hydraulic resonator; lumped parameter model; rotate vector optimization method; transmission loss; resonance frequency

1. Introduction

Most of the aircraft hydraulic systems are driven by axial piston pumps because axial piston pumps have high output pressure, high efficiency and high reliability. However, axial piston pumps will generate large flow pulsation because of the pumps’ inherent structure and working principle \(^[1]\). Flow pulsation can induce pressure fluctuation and piping vibration, which are very harmful to aircraft hydraulic systems \(^[2]\). According to statistics, almost half of the reported failures of hydraulic systems on aircraft were due to the fracture of hydraulic pipes \(^[3]\). Consequently, the hydraulic flow pulsation suppression has been a research hotspot. One important research aspect on hydraulic flow pulsation suppression is the hydraulic fluid pulsation attenuator \(^[4]\).

Helmholtz resonator, which consists of a cavity communicating with the main duct through a neck, was firstly used to attenuate the narrow-band low-frequency noise. Afterwards, some researchers \(^[5-8]\) also employed Helmholtz resonator to reduce the hydraulic fluid pulsation as reactive type attenuator. But the traditional single Helmholtz resonator has only one resonance frequency so that it can only reduce one frequency flow pulsation harmonic, while the hydraulic flow pulsation has several higher harmonics besides the fundamental frequency \(^[9]\). Kojima and Ichiyanagi \(^[10]\) developed a three degrees of freedom (3-DOF) Helmholtz resonator and pointed out that it can get good attenuation properties at three frequency harmonics of hydraulic flow pulsation. Considering that...
the energy of hydraulic flow pulsation mainly distributes at the fundamental frequency and its first two harmonics, 3-DOF Helmholtz resonator is very suitable to reduce the hydraulic flow pulsation and has important research value for hydraulic flow pulsation suppression.

The existing theoretical researches on 3-DOF Helmholtz resonator in hydraulic system established the mathematical model of 3-DOF Helmholtz resonator by transfer matrix method (TMM). Although TMM which is based on the distributed parameter model has high accuracy, it has two disadvantages to investigate 3-DOF Helmholtz resonator: a) the physical meaning of the model is not obvious so that it is difficult to comprehend the attenuation mechanism of 3-DOF Helmholtz resonator; b) TMM cannot give out the closed-form expression of resonance frequency which is an important parameter to predict the attenuation characteristic of 3-DOF Helmholtz resonator.

The lumped parameter theory, which is the classic method to predict the attenuation characteristic of Helmholtz resonator, can overcome the above disadvantages of TMM. And previous studies all used the lumped parameter theory to investigate the traditional single Helmholtz resonators in hydraulic system. But Refs. [5]-[8] directly employed the lumped parameter model (LPM) of Helmholtz resonator in air medium, which approximates Helmholtz resonator as an equivalent mass (neck) and spring (cavity) system to calculate the resonance frequency of Helmholtz resonator in hydraulic fluid medium. The LPM in Refs. [5]-[8] are not accurate, because the viscous friction loss generated when the hydraulic fluid flows through the neck of Helmholtz resonator is not considered, whereas the viscous friction loss cannot be ignored considering that the density and kinetic viscosity of hydraulic fluid are significantly bigger than those of air.

In this paper, a novel LPM of 3-DOF Helmholtz resonator in hydraulic system is developed, which takes the effect of viscous friction loss in necks into consideration and regards the resonator as 3-DOF mass-spring-damping system. In this LPM, closed-form expression of transmission loss and resonance frequency is given out. Based on the LPM, an optimal design method of 3-DOF Helmholtz resonator in hydraulic system is proposed, which employs rotate vector optimization method (RVOM) as optimizer.

2. LPM of 3-DOF Helmholtz Resonator in Hydraulic System

2.1. Equivalent mechanical system

Figure 1(a) shows the structure of 3-DOF Helmholtz resonator. It consists of three cylindrical necks and cavities connected in series (Neck 1-Cavity 1-Neck 2-Cavity 2-Neck 3-Cavity 3). Where \( l_1, l_2, l_3 \) and \( \frac{d_1}{2}, \frac{d_2}{2}, \frac{d_3}{2} \) are the lengths and diameters of Neck 1, Neck 2 and Neck 3 respectively; \( p_1 \) is the fluid pressure in the main pipe, \( \frac{A_1}{4} \) the diameter of the main pipe, and \( A_1 \) the cross section area of Neck 1.

![Fig. 1 Structure and equivalent mechanical system of 3-DOF Helmholtz resonator in hydraulic system.](image-url)
2.2. Parameter determination of equivalent mechanical system

1) Equivalent spring stiffness
As shown in Fig. 2, the cavity, which is filled with fluid and closed by piston, acts as a fluid spring.

![Diagram of Cavity closed by piston.]

Applying the Hooke’s law to the closed cavity, the stiffness of fluid spring can be given by

\[ k = \frac{\Delta p A}{\Delta x} \]  \hspace{1cm} (1)

where \( A \) is the cross-sectional area of the piston, and \( A = \pi D^2/4, \Delta x \) the displacement of the piston, and \( \Delta p \) the pressure variable quantity in the cavity when the displacement of piston is \( \Delta x \).

According to the definition of hydraulic fluid bulk modulus, the expression of \( \Delta p \) can be represented by

\[ \Delta p = -E \frac{\Delta V}{V_0} \]  \hspace{1cm} (2)

where \( E \) is the bulk modulus of hydraulic fluid, \( \Delta V \) the volume variable quantity of the closed cavity, and \( V_0 \) the initial volume of the closed cavity.

Substituting Eq. (2) into Eq. (1) yields

\[ k = -E \frac{\Delta V}{V_0} \]  \hspace{1cm} (3)

From Eq. (3), it can be concluded that the stiffness of hydraulic cavity is related to \( A \) and \( \Delta x \). That is why both Spring 1 and Spring 2 have different stiffness to the two masses connected to them.

Applying Eq. (3), the stiffness of the three springs in the equivalent mechanical system of 3-DOF Helmholtz resonator in hydraulic system can be calculated.

For Spring 1, its stiffness to the first mass \( m_1 \) is given by

\[ k_{11} = \frac{E(A_1 x_1 - A_2 x_2) A_1}{V_1 x_1} \]  \hspace{1cm} (4)

where \( A_1 = \pi D_1^2/4 \) and \( A_2 = \pi D_2^2/4 \) are the cross-sectional areas of Neck 1 and Neck 2, \( A_1 = \pi D_1^2 L_1/4 \) is the volume of Cavity 1, \( x_1 \) and \( x_2 \) are the displacements of hydraulic fluid in Neck 1 and Neck 2.

Similarly, the stiffness of Spring 1 to the second mass \( m_2 \) is written as

\[ k_{12} = \frac{E(A_1 x_1 - A_2 x_2) A_1}{V_1 x_2} \]  \hspace{1cm} (5)

For Spring 2, its stiffness to the second mass \( m_2 \) is given by

\[ k_{22} = \frac{E(A_2 x_2 - A_3 x_3) A_2}{V_2 x_2} \]  \hspace{1cm} (6)

where \( A_3 = \pi D_3^2/4 \) is the cross-sectional area of Neck 3, \( A_2 = \pi D_2^2 L_2/4 \) the volume of Cavity 2, and \( x_3 \) is the displacement of hydraulic fluid in Neck 3.

The stiffness of Spring 2 to the third mass \( m_3 \) is expressed as

\[ k_{23} = \frac{E(A_2 x_2 - A_3 x_3) A_2}{V_2 x_3} \]  \hspace{1cm} (7)

For Spring 3, its stiffness to the third mass \( m_3 \) is obtained as follows:

\[ k_{33} = \frac{E A_3^2}{V_3} \]  \hspace{1cm} (8)

where \( A_3 = \pi D_3^2 L_3/4 \) is the volume of Cavity 3.

2) Equivalent damping
When hydraulic fluid flows through the necks which are regarded as hydraulic fluid pipes, viscous friction loss will be generated. Firstly assume that

a) The elasticity of the neck walls is negligible compared to the compressibility of hydraulic fluid.

b) The temperature and pressure variations are small, hence the changes of hydraulic fluid viscosity and density are small.

c) The hydraulic fluid moves as steady laminar flow in the necks.

d) The hydraulic fluid velocity in the circumferential direction is negligible.

e) The mean hydraulic fluid velocity is less than the acoustic velocity in hydraulic fluid.

Based on these assumptions, the momentum equation of hydraulic fluid in pipes can be written as \(^{[14]}\)

\[ \frac{\partial u}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial x} = v \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) \right] \]  \hspace{1cm} (9)

where \( \rho \) is the density of hydraulic fluid, \( u \) the hydraulic fluid velocity in axial direction, \( p \) the hydraulic fluid pressure, \( x \) the coordinate in axial direction, \( r \) the coordinate in radial direction, and \( v \) the kinematic viscosity of hydraulic fluid.

The linear friction model of hydraulic fluid pipe is applied when hydraulic fluid moves in the pipes as steady laminar flow. In linear friction model, the viscous friction loss is proportional to the mean fluid velocity and the heat transfer effect is not considered \(^{[15]}\).

Applying the linear friction model, the momentum equation of fluid in pipes can be simplified as \(^{[16]}\)

\[ \frac{\partial p}{\partial x} + \frac{\rho}{A_0} \frac{\partial q}{\partial t} = -\frac{8\pi \nu}{A_0^2} q \]  \hspace{1cm} (10)

where \( q \) is the flow rate at one cross-section of pipe, \( A_0 = \pi r_0^2 \) the cross-sectional area of pipe, and \( r_0 \) the inner radius of pipe. The right-hand side of Eq. (10) represents the pressure drop per unit length of pipe generated when hydraulic fluid flows through the pipe, which can be expressed as

\[ f = -\frac{8\pi \nu}{A_0^2} q \]  \hspace{1cm} (11)

The viscous friction force of hydraulic fluid pipe whose length is \( l \) can be derived by Eq. (11) as
Pipe can be obtained: Neck 1, Neck 2 and Neck 3 can be acquired as follows:

\[
F_i = \frac{8\pi \rho \nu l}{A_0}\]

Replacing \( \rho = \mu \rho_0 \) into Eq. (12) yields

\[
F_i = \frac{8\pi \rho \nu l}{A_0}
\]

where \( \mu \) is the mean velocity of hydraulic fluid.

From Eq. (13), the damping of the hydraulic fluid pipe can be obtained:

\[
c = 8\pi \rho \nu l
\]

According to Eq. (14), the damping coefficients of Neck 1, Neck 2 and Neck 3 can be acquired as follows:

\[
c_1 = 8\pi \rho \nu l_i
\]

\[
c_2 = 8\pi \rho \nu l_2
\]

\[
c_3 = 8\pi \rho \nu l_3
\]

2.3. LPM

According to Fig. 1(b), applying the Newton’s second law of motion to Mass 1 yields

\[
m_1 \frac{d^2 x_1}{dt^2} = \rho_1 A_1 \frac{d x_1}{dt} - c_1 \frac{dx_1}{dt}
\]

where \( m_1 = \rho A_1 l_1 \) is the mass of hydraulic fluid in Neck 1.

Applying the Newton’s second law of motion to Mass 2 gives

\[
m_2 \frac{d^2 x_2}{dt^2} = \rho_2 A_2 \frac{d x_2}{dt} - c_2 \frac{dx_2}{dt}
\]

where \( m_2 = \rho A_2 l_2 \) is the mass of hydraulic fluid in Neck 2.

Applying the Newton’s second law of motion to Mass 3, there is

\[
m_3 \frac{d^2 x_3}{dt^2} = \rho_3 A_3 \frac{d x_3}{dt} - c_3 \frac{dx_3}{dt}
\]

where \( m_3 = \rho A_3 l_3 \) is the mass of hydraulic fluid in Neck 3.

Substituting Eqs. (4)-(8) into Eqs. (18)-(20) yields

\[
\begin{align*}
\left\{ 
\frac{d^2 x_1}{dt^2} + \frac{E A_1}{V_1} \left( \frac{1}{V_1} \right) x_1 + c_1 \frac{dx_1}{dt} + \frac{E A_1}{V_1} x_2 &= \rho_1 A_1 \\
\frac{d^2 x_2}{dt^2} + \frac{E A_2}{V_1} \left( \frac{1}{V_1} \right) x_2 + c_2 \frac{dx_2}{dt} + \frac{E A_2}{V_1} x_3 &= \rho_2 A_2 \\
\frac{d^2 x_3}{dt^2} + \frac{E A_3}{V_1} \left( \frac{1}{V_1} \right) x_3 + c_3 \frac{dx_3}{dt} + \frac{E A_3}{V_1} x_2 &= \rho_3 A_3 \\
\end{align*}
\]

Applying Laplace transform to Eq. (21) yields

\[
\begin{align*}
m_1 X_1 s^2 + \frac{E A_1}{V_1} X_1 + c_1 X_1 s - \frac{E A_1}{V_1} X_2 &= \rho_1 A_1 \\
m_2 X_2 s^2 + \frac{E A_2}{V_1} X_2 + c_2 X_2 s - \frac{E A_2}{V_1} X_3 &= \rho_2 A_2 \\
m_3 X_3 s^2 + \frac{E A_3}{V_1} X_3 + c_3 X_3 s - \frac{E A_3}{V_1} X_2 &= \rho_3 A_3 \\
\end{align*}
\]

Rearranging Eq. (22) in matrix form yields

\[
A \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 \\ P_1 A_1 \\ 0 \end{bmatrix}
\]

where \( A \) is the stiffness matrix, and

\[
A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}
\]

The stiffness matrix elements are expressed as

\[
\begin{align*}
a_{11} &= m_1 s^2 + c_1 s + \frac{EA_1^2}{V_1} \\
a_{21} &= -\frac{E A_1 A_2}{V_1} \\
a_{12} &= a_{21} \\
a_{13} &= a_{31} = 0 \\
a_{22} &= m_2 s^2 + c_2 s + \frac{EA_2^2}{V_2} \left( \frac{1}{V_1} + \frac{1}{V_2} \right) \\
a_{23} &= -\frac{E A_2 A_3}{V_2} \\
a_{32} &= a_{23} \\
a_{33} &= m_3 s^2 + c_3 s + \frac{EA_3^2}{V_3} \left( \frac{1}{V_2} + \frac{1}{V_3} \right)
\end{align*}
\]

2.4. Transmission loss and resonance frequency

1) Transmission loss

Manipulating Eq. (23) yields

\[
\frac{X_1}{P_1 A_1} = \frac{a_{21} a_{33} - a_{31} a_{32}}{a_{11} a_{22} - a_{12} a_{13}}
\]

The normalized specific impedance of 3-DOF Helmholtz resonator can be expressed as

\[
Z_H = \frac{P_1}{\rho_0 c_0 U_1}
\]

where \( c_0 = \sqrt{E / \rho} \) is the speed of sound in hydraulic fluid, and \( U_1 = X_1 s \) is the Laplace transform of the velocity of fluid at Neck 1.

Substituting Eq. (25) to Eq. (26) yields

\[
Z_H = \frac{1}{\rho_0 c_0 U_1} \frac{a_{11} a_{22} a_{33} + a_{12} a_{32} a_{33} - a_{13} a_{22} a_{33}}{a_{23} a_{32} - a_{22} a_{33}}
\]

The transmission loss of 3-DOF Helmholtz resonator can be determined by

\[
TL = 20 \log \left| 1 + \frac{A_p}{2 A_p} \frac{1}{Z_H} \right|
\]

where \( A_p = \pi d_b ^2 / 4 \) is the cross-sectional area of the main pipe.

2) Resonance frequency

In order to calculate the undamped natural frequency
of 3-DOF Helmholtz resonator in hydraulic system, \( c_1, c_2, c_3 \) and \( p_1 \) are set to be zero. So Eq. (22) can be re-written as

\[
\begin{align*}
\frac{m_1 X_1 s^2 + E A_1^2}{V_1} X_1 - \frac{E A_2}{V_1} X_2 &= 0 \\
\frac{m_2 X_2 s^2 - E A_2 A_1}{V_1} X_1 + \frac{E A_2}{V_2} \left( \frac{1}{V_1} + \frac{1}{V_2} \right) X_2 - E A_2^2 X_3 &= 0 \\
\frac{m_3 X_3 s^2 - E A_2 A_3}{V_2} X_2 + \frac{E A_3}{V_3} \left( \frac{1}{V_2} + \frac{1}{V_3} \right) X_3 &= 0
\end{align*}
\]

(29)

In view of \( s = j \omega_0 \), eliminating \( X_1, X_2, X_3 \) in Eq. (29) yields

\[
m_1 m_2 m_3 \omega_0^6 - (m_1 m_2 b_{33} + m_2 m_3 b_{11} + m_3 m_1 b_{21}) \omega_0^4 + (m_1 b_{23} b_{32} + m_2 b_{13} b_{31} + m_3 b_{12} b_{23} - m_2 b_{32} b_{31} - m_3 b_{23} b_{31}) \omega_0^2 - h_{11} b_{23} b_{32} + h_{12} b_{23} b_{32} + h_{13} b_{23} b_{31} = 0
\]

(30)

where

\[
\begin{align*}
h_{11} &= \frac{E A_1^2}{V_1} \\
h_{12} &= \frac{b_{21}}{V_2} \\
h_{22} &= \frac{E A_2^2}{V_2} \left( \frac{1}{V_1} + \frac{1}{V_2} \right) \\
h_{23} &= \frac{b_{31}}{V_2} \\
h_{32} &= \frac{E A_3^2}{V_3} \left( \frac{1}{V_2} + \frac{1}{V_3} \right)
\end{align*}
\]

(31)

The three roots \( \omega_0^6, \omega_0^4, \omega_0^2 \) of Eq. (30) reveal the undamped natural angular frequencies. The damped natural angular frequencies of 3-DOF Helmholtz resonator in hydraulic system are then obtained as follows:

\[
\omega_i = \sqrt{\left( \frac{\omega_i}{2m_i} \right)^2 - \left( \frac{c_i}{2m_i} \right)^2}
\]

(32)

So the resonance frequencies of 3-DOF Helmholtz resonator in hydraulic system can be determined by

\[
f_{1,2,3} = \frac{\omega_{1,2,3}}{2\pi}
\]

(33)

3. Optimal Design Method

3.1. Mathematical description of optimization problem

The attenuation characteristic of 3-DOF Helmholtz resonator in hydraulic system is the best when its three resonance frequencies coincide with the three frequency harmonics of hydraulic fluid pulsation respectively. So the purpose of the optimal design is to search the unknown size parameter values so that the resonance frequencies \( f_1, f_2, f_3 \) can coincide with the fundamental frequency and its first two harmonics \( f_{p1}, f_{p2}, f_{p3} \) respectively. The resonance frequencies of 3-DOF Helmholtz resonator in hydraulic system can be obtained by Eq. (33).

The objective function is defined as follows:

\[
F(X) = \left| f_1 - f_{p1} \right|^2 + \left| f_2 - f_{p2} \right|^2 + \left| f_3 - f_{p3} \right|^2
\]

(34)

For the hydraulic system driven by piston pump, the fundamental frequency and its harmonics are related to the pump rotational speed and the number of pistons. \( f_{p1}, f_{p2}, f_{p3} \) can be expressed as

\[
\begin{align*}
f_{p1} &= \frac{nZ}{60} \\
f_{p2} &= \frac{2nZ}{60} \\
f_{p3} &= \frac{3nZ}{60}
\end{align*}
\]

(35)

where \( n \) is the pump rotational speed, and \( Z \) the number of pistons.

Design variables are given by

\[
X = [l_1, l_2, l_3, d_1, d_2, l_1, l_2, L_3, D_1, D_2, D_3]^T
\]

(36)

Considering the space limitation of hydraulic system, the bound for all variables is provided for efficient search in design space:

\[
\begin{align*}
\text{Lower bound} &= \min(X_i) \\
\text{Upper bound} &= \max(X_i) \quad (i = 1, 2, \cdots, 12)
\end{align*}
\]

(37)

3.2. Optimization method

The RVOM, which was first proposed in 1996 [18], is used to search the optimum size parameters of 3-DOF Helmholtz resonator in hydraulic system. RVOM, which is especially suitable to solve the optimization problem with nonlinearity objective function or procedure with complex constraints, has higher efficient and better global optimization ability compared to genetic algorithms.

Figure 3 presents the flowchart of RVOM algorithm. The following text is the description of RVOM algorithm.

Step 1 Define two initial random vectors \( X_L \) and \( X_U \).
Step 2 If $F(X_H)<F(X_L)$, exchange $X_H$ and $X_L$; else, no exchange.

Step 3 Get the vector $X_D=X_H/X_L$ and apply rotation transform to $X_D$ around $X_L$ and obtain the new vector $X_T=c_rX_D$; if $F(X_T)<F(X_L)$, replace $X_H$ by $X_T$ and replace $X_L$ by $X_T$; else, no replace; where $c_r$ is the rotation coefficient, which is a function of one time rotation angle $\Phi$.

Step 4 If a round of rotation is completed, go to Step 5; else, go to Step 3.

Step 5 Get the vector $X_D=X_H-X_L$ again and apply contraction transform to $X_D$ and get a new vector $X_T=X_T+c_cX_D$, where $c_c$ is the contraction coefficient.

Step 6 Let $\varepsilon$ be the setting calculation precision. If $||X_T-X_L||<\varepsilon$, the global optimum is $F(X_{min})$, $X_{min}=X_L$, and terminate the calculation process. Otherwise, $X_H=X_T$, go to Step 2.

4. Results and Discussion

4.1. Optimal design results

The optimal design of 3-DOF Helmholtz resonator for the hydraulic system driven by a certain type of aviation 9-pistons pump, whose speed is 3,000 r/min, is carried out. The diameter of the main pipe $d_0$ is 20 mm. The parameters of hydraulic oil are listed as follows: $\rho=890$ kg/m$^3$, $E=1$ 000 MPa, $v=50\times10^{-6}$ m$^2$/s.

The parameters of RVOM algorithm are set as $\Phi=120^\circ$, $c_c=0.618$, $\varepsilon=0.001$.

The design variables and respective bounds for 12 variables in the optimization model are shown in Table 1. Applying the proposed optimal design method, the optimum size parameters of 3-DOF Helmholtz resonator in hydraulic system can be obtained, as shown in Table 1.

Figure 4 shows the history record of objective function value of 3-DOF Helmholtz resonator in optimization process. The final value of objective function is 0.747 294 4 Hz, which fulfills the design requirements.

4.2. Discussion of LPM

Substituting the optimum design variables to Eq. (28), the transmission loss of 3-DOF Helmholtz resonator in hydraulic system can be calculated by the LPM. Then the transmission loss is also calculated by TMM proposed in Ref. [10] which is based on distributed parameter model of hydraulic pipe and validated by experiment data. Figure 5 compares the transmission losses with the two methods.
As shown in Fig. 5, the transmission loss from the two methods agrees well, which proves that the precision of the LPM is the same as that of TMM in Ref. [10]. Three transmission loss peaks are observed in Fig. 5 at three frequencies, 450.3, 899.6, 1 350.1 Hz, the same as predicted by Eq. (33), which demonstrates that the closed-form expression of the resonance frequency is correct.

In order to examine the effect of the viscous friction loss on the transmission loss, Fig. 6 compares the transmission losses of 3-DOF Helmholtz resonator with and without viscous friction loss by LPM.

As shown in Fig. 6, the three resonance frequencies of 3-DOF Helmholtz resonator with and without \( c_{1,2,3} \) are the same because the viscous friction loss is not big enough to change the resonance frequencies. However, the peak values of transmission loss at three resonance frequencies with \( c_{1,2,3} \) are much smaller than those without \( c_{1,2,3} \), as shown in Table 2. This reveals that: a) the viscous friction loss makes a great impact on the attenuation characteristic of 3-DOF Helmholtz resonator, thus it cannot be ignored; b) the attenuation characteristic of 3-DOF Helmholtz resonator in hydraulic fluid medium is worse than that of 3-DOF Helmholtz resonator in air medium, because the kinetic viscosity of hydraulic fluid is significantly bigger than that of air which is proportional to the viscous friction loss.

<table>
<thead>
<tr>
<th>Resonance frequency/Hz</th>
<th>TL peak with ( c_{1,2,3} )/dB</th>
<th>TL peak without ( c_{1,2,3} )/dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>450.3</td>
<td>52.36</td>
<td>66.66</td>
</tr>
<tr>
<td>899.6</td>
<td>47.91</td>
<td>63.40</td>
</tr>
<tr>
<td>1350.1</td>
<td>49.04</td>
<td>57.35</td>
</tr>
</tbody>
</table>

5. Conclusions

1) A novel LPM of 3-DOF Helmholtz resonator in hydraulic system is developed in this paper, which can get the same precision as TMM and has obvious physical meaning in modeling process. This model can be employed to predict the resonance frequency and transmission loss of 3-DOF Helmholtz resonator in hydraulic system conveniently, which enriches the theoretical research approach to investigate 3-DOF Helmholtz resonator in hydraulic system.

2) In this LPM, the viscous friction loss of hydraulic fluid in necks is taken into consideration and calculated, which is proved that it can make a great impact on the attenuation characteristic of 3-DOF Helmholtz resonator in hydraulic system. This idea has guiding significance to exactly model other types of hydraulic fluid pulsation attenuator by lumped parameter theory.

3) An optimal design method of 3-DOF Helmholtz resonator in hydraulic system based on the LPM is proposed, which employs RVOM as optimizer. The optimization result and simulation show the feasibility of this method. This optimal design method can be used in engineering practice to design the 3-DOF Helmholtz resonator in hydraulic system.

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References


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