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## Spatio-temporal modeling of avalanche frequencies in the French Alps

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### Abstract

Avalanches threaten mountainous regions, and probabilistic long term hazard evaluation is a useful tool for land use planning and the definition of appropriate mitigation measures. This communication focuses on avalanches counts in the French Alps, and investigates their fluctuations in space and time within a Bayesian hierarchical modeling framework.

We have at our disposal a 60 year data set covering the whole French Alps. The considered time scale is the winter. The elementary spatial scale is the township. It is small enough to allow information transfer between neighboring paths and large enough to avoid errors in paths localization. Data are standardized with a variable integrating the number of surveyed paths.

A hierarchical Poisson-lognormal model appears well-adapted to depict the observation process with such discrete data. The spatial and temporal effects are assumed independent, and they are considered in the latent layer of the model. The temporal trend is modeled with a cubic spline whereas different spatial dependence sub-models are tested. The latter ones work on different types of supports (continuous field and discrete grid), and at different embedded spatial scales. Model inference and predictive sampling are carried out using Markov Chain Monte Carlo simulation methods.

The spatial structure explains the larger part of the relative risks. The spatial dependence is visible at the scale of townships, but with a short range. At the larger scale of the massifs, the spatial dependence is weaker.

The regional coherence of the results with the number of avalanche releases suggests that we may also search for other spatially structured variables implicated in the magnitude of avalanches that could help transfer information from one path to another.

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### 1. Introduction

In mountainous regions, serious damages are caused by snow avalanches releases. In order to protect populations and facilities, land use planning and the definition of appropriate mitigation measures have to be introduced. When local information is poor, probabilistic long term hazard must be evaluated at the regional scale.

Few authors have studied avalanche frequencies at the path scale (Keylock et al. 1999 [1]). In France, the EPA database (Enquête Permanente sur les Avalanches), managed by the Cemagref, is a chronicle of events beginning in the early XX<sup>th</sup> century and concerning roughly 3900 sites in Alps and Pyrenees. However, every path is not surveyed in the Alps, and extrapolation is a necessary step for undocumented or poorly documented paths.

With this database Eckert et al. (2007) [2] have shown a spatial dependence of avalanche occurrences in Savoie. We carry on this work by generalizing it to the whole Alps. We are particularly interested in large structures such as massifs. Could they help bringing more information into the analysis?

Eckert et al. (2009) [3] worked on fluctuations of avalanche occurrences, and showed that the annual effect represents 17% of the avalanche occurrence variability. This suggests that it is important to model the temporal structure, in order to study the avalanches spatial repartition for a standard winter.

In this work, we model avalanche occurrences by winter and by township, with a hierarchical spatio-temporal model, under the Bayesian paradigm. By comparison to previous work, our approach introduces, full symmetry between space and time effects using spline smoothing.

### 2. Model

We call  $Y_{ct}$ ,  $c=1,\dots,N$ , and  $t=1,\dots,T$  the variables representing the number of recorded avalanches during the winter  $t$  in the township  $c$ .

We assume that  $Y_{ct}$  follows a Poisson law with parameter  $\lambda_{ct}$ , and that conditionally to the knowledge of  $\lambda_{ct}$  the variables  $Y_{ct}$  are independent.

As in epidemiological models (Mollié et al. 1991 [4]), we model  $E(Y_{ct})=E_c.RR_{ct}$  where  $E_c$  is the expected occurrence number in the township  $c$  during a mean winter and  $RR_{ct}$  is the relative risk of the township  $c$  the year  $t$ . The variable  $E_c$  is computed from the number of paths in the township  $c$ , whereas  $RR_{ct}$  is unknown.

We model the relative risk, through the link function log with a spatial effect  $S_c$  and a temporal effect  $T_t$  under the assumption of separability. For identifiability and symmetry purposes, we sum the vectors  $S$  and  $T$  to 0, adding a constant  $\mu$ .

$$\log(RR_{ct}) = \mu + T_t + S_c \tag{1}$$

We consider the following temporal model:

$$T_t = g_t + \epsilon_t \quad t=1,\dots,T \tag{2}$$

where  $\epsilon_t$  follow a Gaussian centred independent distribution with variance  $\sigma^2$ . Eckert et al (2009) [3] have used several time series models for the  $g$  parameter. The two best models selected using the DIC criterion are the most flexible ones. They are models with jumps at different levels. We suggest to generalize this approach by modeling  $g$  with a smooth non parametric curve, hence even more flexible, and we choose the second order random walk with variance  $\sigma_1$ . This process can be seen as an approximation of the Wahba's prior (Wahba, 1978 [5]), whom the Bayesian estimate  $E(g|T)$  is a cubic spline.

The spatial structure is composed with a spatially structured variable  $\mathbf{u}=(u_{I\dot{y}}, u_{N\dot{y}})$ , and a residual  $\mathbf{v}=(v_{I\dot{y}}, v_{N\dot{y}})$  modeled by a Gaussian white noise with variance  $\sigma_v^2$ .

$$S_c = u_c + v_c \quad c=I, \dot{y}, N \quad (3)$$

We model the  $\mathbf{u}$  variable with three different competing models. The first one uses the townships spatial repartition, and the second one uses the massifs and the spatial repartition of the townships inside each massif. The last model borrows tools from geostatistics, it supposes that  $u_c$  is the realization of a continuous field at the centroid of the  $c$  township.

- Model 1

We use the intrinsic CAR model defined by Mollié et al. (1991) [4] with variance  $\sigma^2$ . It is adapted to irregular grids because the conditional variance of each component depends on the number of its neighbors. Two townships are neighbors if they share a common boundary.

- Model 2

We split up  $\mathbf{u}$  into a massifs relative part  $\mathbf{u}^M=(u_{I\dot{y}}, u_{M\dot{y}})$  with  $M$  the massif number, and embed the relative contribution of the townships inside each massif  $m$   $\mathbf{u}^{C|m}=(u_{I|m\dot{y}}, u_{nm|m\dot{y}})$ , with  $n_m$  the number of townships inside the massif  $m$ .

$$u_{cm} = u_m + u_{c|m} \quad m=I, \dot{y}, M, \text{ et } c=I, \dot{y}, n_m \quad (4)$$

As in the model 1, we use the intrinsic CAR model on the massifs network  $\mathbf{u}^M$  with variance  $\sigma_M^2$  and on the networks composed by the townships inside each massif  $\mathbf{u}_{C|m}$  with a common variance  $\sigma_C^2$ .

- Model 3

In order to describe the township dependence according to their distance instead of their neighbors, we define the covariance between  $c$  and  $c'$  as a function of the distance  $h_{cc'}$  between their centroids such as  $cov(u_c, u_{c'})=C(h_{cc'})$ . We choose the exponential variogram to describe the covariance structure between sites. This covariance model relies on parameters easy to interpret. Particularly, the effective range is defined as the minimal distance between two points, so that their correlation is less than 0.05.

Priors on the variance parameters  $\sigma_0^2, \sigma_I^2, \sigma_v^2, \sigma_M^2$  and  $\sigma_C^2$  are gamma inverses with parameters 0.1 and 0.1. These priors guarantee that the posteriori joint distribution is proper. The prior on the range is uniform in [0,50000], and the prior on  $\rho$  is constant.

### 3. Results

The full conditional distributions are obtained in closed form for most unknowns, so we use Gibbs sampling for these nodes with Metropolis Hastings or rejection sampling step for the other ones. 40,000 iterations are simulated on two chains with different seeds, and the 10,000 first ones are deleted. Convergence is checked comparing the two chains distributions.

To compare models, the DIC criterion (Deviance Information Criterion) is computed. It balances model fit and complexity through the effective degree of freedom  $p_D$  (Spiegelhalter et al., 2002 [6]).

The variability of the temporal structure represents only a small part of the relative risk variability, 12% for model 1, and 17% for models 2 and 3. The annual frequency average increases from 1946 to 1980, and then decreases, see Fig 1. However, there are strong variations from one winter to another, and the trend  $E(\mathbf{g}/\mathbf{Y})$  only accounts for 12% of the temporal variability.

The fraction of variability explained by the spatially structured term  $u$  is large for the three models; this confirms that avalanche occurrence frequencies are spatially dependent.

Model 1 is more flexible than model 2, its effective degree of freedom is actually larger. Therefore, data are better fitted by model 1. Model 1's DIC criterion is smaller, which means that its complexity is justified by the fitness gain.

Model 2 brings massifs into the analysis, but badly fits the data. We note a spatial dependence between massifs, but the variability between massifs is smaller than the variability within massifs.

Model 3 is more flexible than model 1, however the gain in complexity is not justified by the fit. The model 3's DIC criterion is slightly greater than the model 1's one. Yet, this model provides a posterior distribution of the effective range, whose mean is estimated to 33 km, but the standard deviation is relatively large (10 km).

The map of mean estimates of avalanche occurrence counts by path for model 1, see Fig 2(a), shows regions with a risk excess, in the north, the middle, and the south-east, and regions with a risk deficit in the west, and the south-west of the studied zone. These regions rarely match the massifs boundaries, and we understand therefore why the massif's structure is not really informative.

The map of the unstructured term, see Fig 2(b), does not show any trend: as expected with regard to the model hypotheses, we are facing a residual term, with nothing left to explain. Moreover we note that the range of the  $v$  values is small, especially much smaller than the range of the  $u$  values.

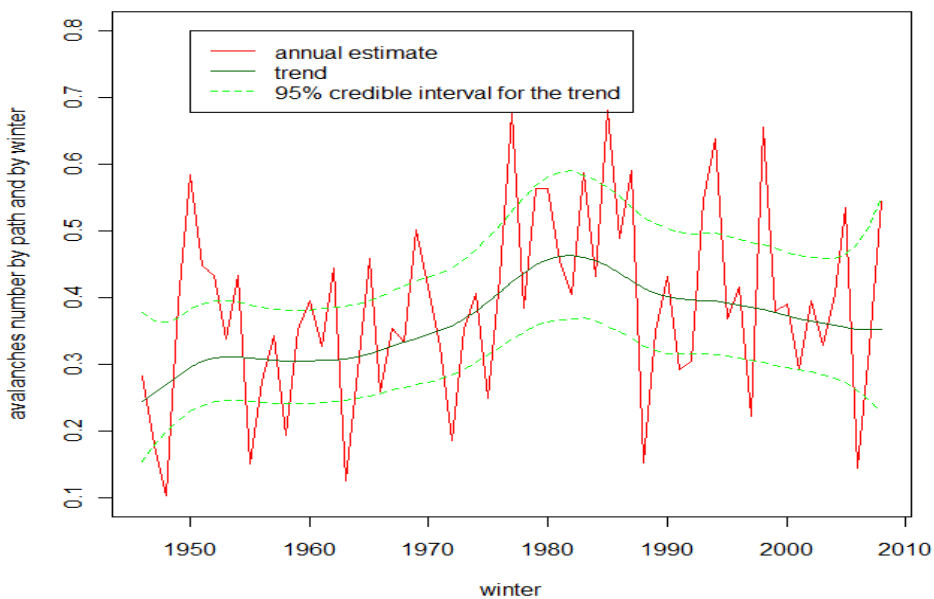
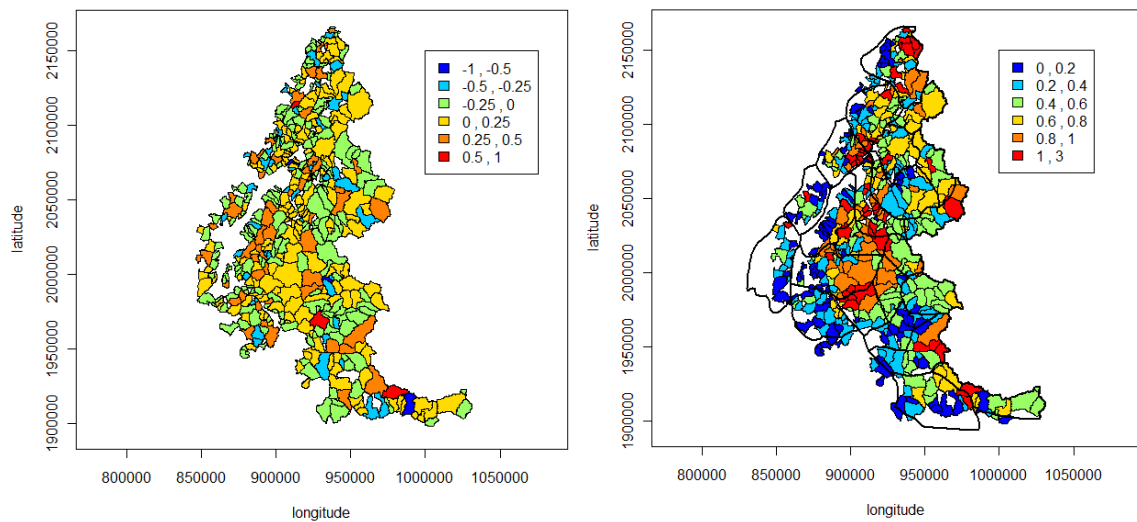


Fig. 1. Mean avalanche number per winter and path, annual estimate and trend from model 1

Table 1. Posterior mean (and standard error) of variance parameters. Deviance Information Criterion DIC, and effective degree of freedom  $p_D$

Table 1. Posterior mean (and standard error) of variance parameters. Deviance Information Criterion DIC, and effective degree of freedom  $p_D$ 

| Parameters | Model 1                                       | Model 2                                       | Model 3                                       |
|------------|---|---|---|
|            | -1.03 (0.06)                                  | -0.15 (0.06)                                  | -1.02 (0.08)                                  |
| $\theta$   | 0.16 (0.04)                                   | 0.18 (0.04)                                   | 0.17 (0.04)                                   |
| $\lambda$  | $9.85 \cdot 10^{-4}$ ( $9.52 \cdot 10^{-4}$ ) | $9.96 \cdot 10^{-4}$ ( $9.43 \cdot 10^{-4}$ ) | $9.88 \cdot 10^{-4}$ ( $9.75 \cdot 10^{-4}$ ) |
| $\sigma^2$ | 2.48 (0.67)                                   | $\sigma_M^2$ 0.62 (0.25)                      | 1.08 (0.31)                                   |
|            |   | $\sigma_C^2$ 1.28 (0.35)                      |   |
| $\nu$      | 0.18 (0.09)                                   | 0.31 (0.08)                                   | 0.24 (0.11)                                   |
| range      |   |   | 33441 (10535)                                 |
| DIC        | 68546   | 70030   | 69771   |
| $p_D$      | 689   | 605   | 884   |

Fig. 2. (a) Mean avalanche occurrence counts by winter and by path:  $\exp(+\mathbf{u})$ , model 1; (b) Posterior mean of the  $\nu$  variable, model 1

#### 4. Discussion

We attempted to predict the last year (not taken into account in the data) with the three models. In all cases, 15% of the observations are outside the 95% confidence interval for predictions. It means that data are somewhat overdispersed. Considering the hypothesis of separability between time and space, this

result is rather encouraging. But we could take into account overdispersion by adding an interaction term between winters and townships.

Some other geographical structure than the massifs could better explain the avalanche frequency. For instance, the excess risk in the middle of the map seems to be structured around massifs delimitations, that means that valleys should be considered as structuring components. However, in order to identify elements precisely, we should work at the path scale.

## 5. Conclusion

With this model, we highlight a spatial dependence of avalanche occurrences at a relatively small scale, in the French Alps.

This result suggests that it is possible to bring information about avalanche events from documented path to undocumented path, by using Cartesian coordinates as surrogate covariables. This step is essential, in order to supply avalanche information at the regional scale.

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