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The g factor of the proton

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Abstract

We consider higher order corrections to the g factor of a bound proton in hydrogen atom and their consequences for a magnetic moment of free and bound proton and deuteron as well as some other objects.

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Investigation of electromagnetic properties of particles and nuclei provides important information on fundamental constants. In addition, one can also learn about interactions of bound particles within atoms and interactions of atomic (molecular) composites with the media where the atom (molecule) is located. Since the magnetic interaction is weak, it can be used as a probe to learn about atomic and molecular composites without destroying the atom or molecule. In particular, an important quantity to study is a magnetic moment for either a bare nucleus or a nucleus surrounded by electrons.

The Hamiltonian for the interaction of a magnetic moment μ with a homogeneous magnetic field \mathbf{B} has a well known form

$$H_{\text{magn}} = -\mu \cdot \mathbf{B}, \quad (1)$$

which corresponds to a spin precession frequency

$$h\nu_{\text{spin}} = \frac{\mu}{I} B, \quad (2)$$

where I is the related spin equal to either $1/2$ or 1 for particles and nuclei under consideration in this Letter. Comparison of the frequencies related to two different objects allows to exclude the magnetic field B from equations and to determine the ratio of their magnetic moments with an accuracy sometimes substantially higher than that in the determination of the applied magnetic field.

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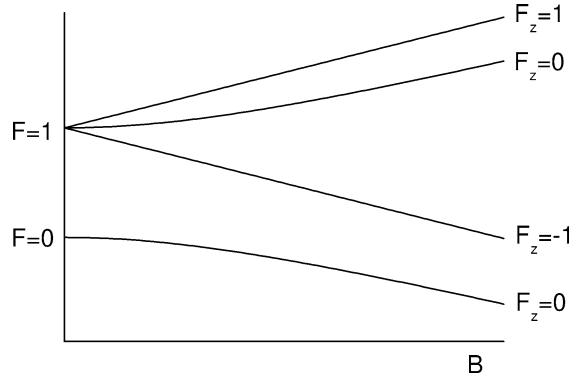


Fig. 1. Sublevels $E_{\text{magn}}(F, F_z)$ of the hyperfine structure in the ground state of hydrogen atom in a homogenous magnetic field B .

To measure the magnetic moment of a given nucleus one has to compare it with a value of some probe magnetic moment, which should be known or determined separately. For a significant number of most accurate measurements the probe value is related to the magnetic moment of a free or bound proton and a crucial experiment on its determination is related to a proton bound in hydrogen atom [1,2]. Nuclear magnetic moments are usually presented in units of nuclear magneton (μ_N) [3,4], which is related to the proton magnetic moment via the relation

$$\mu_p = \frac{1}{2} g_p \mu_N, \quad (3)$$

where g_p is the proton g factor and $\mu_N = e\hbar/2m_p$.

The spin precession frequency was studied not only for a free proton, but also for the one bound in atoms or molecules located in gaseous or liquid media. The magnetic moment and the g factor of a bound proton differ from their free values (see, e.g., [5,6]). The purpose of this Letter is to re-evaluate in part available experimental data for light atoms and in particular to determine the g factor of a free proton (g_p) and a proton bound in the ground state of hydrogen atom ($g_p(\text{H})$) from experiment [2]. We also study the consequences of re-evaluation of g_p and similar experiments for deuterium [7] and muonium [8].

The most accurate determination of the g factor of a free proton was performed studying the hyperfine structure of the hydrogen atom in the homogeneous magnetic field. The dependence of hyperfine sublevels of the ground state in the hydrogen atom on the value of the magnetic field \mathbf{B} directed along the z axis is shown in Fig. 1 (see, e.g., [6]). The energies of hyperfine components $E_{\text{magn}}(F, F_z)$ of the $1s$ state are described by

$$\begin{aligned} E_{\text{magn}}(1, +1) &= \frac{1}{2}(E_e - E_p) + \frac{1}{4}E_{\text{hfs}}, & E_{\text{magn}}(1, 0) &= \frac{1}{2}\sqrt{(E_e + E_p)^2 + E_{\text{hfs}}^2} - \frac{1}{4}E_{\text{hfs}}, \\ E_{\text{magn}}(1, -1) &= -\frac{1}{2}(E_e - E_p) + \frac{1}{4}E_{\text{hfs}}, & E_{\text{magn}}(0, 0) &= -\frac{1}{2}\sqrt{(E_e + E_p)^2 + E_{\text{hfs}}^2} - \frac{1}{4}E_{\text{hfs}}, \end{aligned} \quad (4)$$

where $E_e = g_e(\text{H})\mu_B B$ and $E_p = g_p(\text{H})\mu_N B$ are related to precession frequencies of electron and proton, and $\mu_B = e\hbar/2m_e$ is the Bohr magneton. The energy splitting E_{hfs} is related to the hyperfine interval in the hydrogen ground state known with high accuracy in frequency units.

The experiment [2] devoted to a measurement of splitting and shift of the hyperfine sublevels in hydrogen atom due to the magnetic field led to the following result [1]¹

$$\frac{\mu_e(\text{H})}{\mu_p(\text{H})} = \frac{g_e(\text{H})}{g_p(\text{H})} \frac{\mu_B}{\mu_N} = 658.2107058(66). \quad (5)$$

¹ Here and further we ignore the direction of spin and magnetic moment and thus the sign of some g factors and ratios of magnetic moments.

The result for the related ratio of the free magnetic moments derived in the experiment [2] and quoted in Ref. [1] was based on a theoretical expression which contained relativistic and recoil corrections up to the third order in either of three parameters, such as the free QED parameter α (appearing due to the anomalous magnetic moment of electron), the strength of the Coulomb potential $Z\alpha$ and the recoil parameter m_e/m_p . All these terms are of pure kinematic origin and were derived before (see, e.g., Ref. [9]). The g factors in hydrogen atom in terms of the free g factors of the electron [10]

$$g_e = 2.002\,319\,304\,376(8) \quad (6)$$

and the proton (g_p) are

$$g_e(\text{H}) = g_e \cdot \left\{ 1 - \frac{(Z\alpha)^2}{3} \left[1 - \frac{3m_e}{2m_p} \right] + \frac{\alpha(Z\alpha)^2}{4\pi} \right\}, \quad (7)$$

$$g_p(\text{H}) = g_p \cdot \left\{ 1 - \frac{\alpha(Z\alpha)}{3} \left[1 - \frac{m_e}{2m_p} \frac{3+4a_p}{1+a_p} \right] \right\}, \quad (8)$$

where for the anomalous magnetic moment of the proton we set $a_p \simeq 1.792847\dots$. This value is taken from Ref. [1]. It enters only small corrections (see, e.g., Eq. (8)) and any re-evaluation which can shift the proton g factor on level of a part in 10^8 will not affect those corrections.

These expressions applied to evaluations in Refs. [1,2] include only the terms up to the third order. However, for the determination of the ratio of the magnetic moments at the level of a part in 10^8 the higher order corrections should be taken into account as well. The fourth order corrections are in part nuclear-spin-dependent. E.g., in the case of hydrogen atom ($I = 1/2$) the expression for higher-order terms corrections reads (cf. Ref. [11])

$$\Delta g_e(\text{H}) = g_e \cdot \left\{ -\frac{(Z\alpha)^2(1+Z)}{2} \left(\frac{m_e}{m_p} \right)^2 - \frac{5\alpha(Z\alpha)^2 m_e}{12\pi m_p} - (0.289\dots) \times \frac{\alpha^2(Z\alpha)^2}{\pi^2} - \frac{(Z\alpha)^4}{12} \right\}, \quad (9)$$

$$\Delta g_p(\text{H}) = g_p \cdot \left\{ \alpha(Z\alpha) \left(\frac{m_e}{m_p} \right)^2 \left(-\frac{1}{2} - \frac{Z}{6} \frac{3-4a_p}{1+a_p} \right) - \frac{97}{108} \alpha(Z\alpha)^3 \right\}. \quad (10)$$

After a proper substitution for m_p , g_p and a_p , the results for the leading terms in Eqs. (7) and (8) can be applied to any hydrogen-like atoms, while the higher-order corrections in Eqs. (9) and (10) can be used only in the case of the nuclear spin $1/2$ (e.g., for the tritium atom and a hydrogen-like helium-3 ion). For the deuterium atom ($I = 1$) the results for the higher-order terms differ from Eqs. (7) and (8) and have to be properly corrected. E.g., following Ref. [12], we obtain

$$\Delta g_e(\text{D}) = g_e \cdot \left\{ -\frac{(Z\alpha)^2(11Z+12)}{18} \left(\frac{m_e}{m_d} \right)^2 - \frac{5\alpha(Z\alpha)^2 m_e}{12\pi m_d} - (0.289\dots) \times \frac{\alpha^2(Z\alpha)^2}{\pi^2} - \frac{(Z\alpha)^4}{12} \right\}, \quad (11)$$

$$\Delta g_d(\text{D}) = g_d \cdot \left\{ \alpha(Z\alpha) \left(\frac{m_e}{m_d} \right)^2 \left(-\frac{1}{2} - \frac{Z}{3} \frac{2-2a_d}{1+a_d} \right) - \frac{97}{108} \alpha(Z\alpha)^3 \right\}, \quad (12)$$

where

$$\mu_d = g_d \mu_N \equiv (1+a_d) \frac{e\hbar}{m_d} \quad (13)$$

and $a_d \simeq -0.142987\dots$

There is only a single experiment [13] where the recoil part of the higher-order terms in Eqs. (9) and (11) is important. In this experiment the electron magnetic moments of hydrogen and deuterium [13] were compared. In contrast, the non-recoil higher-order terms in Eqs. (9) and (11) are not important for this isotopic comparison. The accuracy of a similar experiment on hydrogen and tritium [14] was not high enough to be sensitive to the recoil corrections in Eq. (9).

An opposite situation appears in the experimental comparison of the nuclear magnetic moment and the electron magnetic moment while studying, e.g., the hydrogen energy levels in Eq. (4). We note that only one higher-order correction for each g factor can contribute at a level close to a part in 10^8

$$\Delta g_e(\text{H}) = -\frac{(Z\alpha)^4}{12} \cdot g_e, \quad (14)$$

$$\Delta g_p(\text{H}) = -\frac{97}{108}\alpha(Z\alpha)^3 \cdot g_p. \quad (15)$$

The former equation owing a small numerical coefficient $1/12$ is related to a smaller effect ($\Delta g_e/g_e \simeq -2.4 \times 10^{-10}$). It has been known for a while [5] and was an only fourth-order term included into evaluation in Ref. [1], while the latter correction ($\Delta g_p/g_p \simeq -2.6 \times 10^{-9}$) was obtained recently [11,15,16]. Thus, the higher-order recoil effects can be neglected and that is fortunate because the remaining terms in Eqs. (14) and (15) are nuclear-spin-independent.

Combining Eqs. (6), (7) and (9) we find for the hydrogen atom

$$\frac{1}{2}g_e(\text{H}) = \frac{\mu_e(\text{H})}{\mu_B} = 1.001\,141\,926\,3, \quad (16)$$

where the uncertainty is below a part in 10^{10} and can be neglected in further considerations.

Applying the results for the higher-order corrections from Eqs. (15) and (16) to the experimental data in Eq. (5) [1,2], we deduce

$$\frac{\mu_p(\text{H})}{\mu_B} = 0.001\,521\,005\,230(15), \quad (17)$$

$$\frac{\mu_p}{\mu_B} = 0.001\,521\,032\,207(15) \quad (18)$$

and

$$\frac{\mu_p}{\mu_e} = 658.210\,685\,9(66). \quad (19)$$

To interpret the results in units of the nuclear magneton, we have to apply an accurate value of the conversion factor

$$\frac{\mu_B}{\mu_N} = \frac{m_p}{m_e}. \quad (20)$$

The proton-to-electron mass ratio was recently determined from an experiment on the g factor of a bound electron in hydrogen-like carbon [17] and the result [18,19] is more accurate, being slightly different from the one based on comparison of cyclotron frequencies of electron and proton [1,20]. We note that this new approach to the determination of the electron-to-proton mass ratio [18,21] was confirmed by a measurement of the g factor of a bound electron in the hydrogen-like oxygen [22], as suggested in Ref. [23]. The experimental result [22] is in fair agreement with theory [11,19,23]. Other less accurate results on the proton-to-electron mass ratio are overviewed in Ref. [11].

The values of the electron-to-proton mass ratio deduced from experiment [17] are slightly different from evaluation to evaluation, and here we use the one found in Ref. [11] (see also discussion in Ref. [19])

$$\frac{\mu_B}{\mu_N} = \frac{m_p}{m_e} = 1\,836.152\,673\,6(13). \quad (21)$$

Using a value for the magnetic moment of a bound proton from Eq. (17) we arrive to the following results for the proton g factor

$$\frac{g_p(\text{H})}{2} = \frac{\mu_p(\text{H})}{\mu_N} = 2.792\,797\,820(28) \quad (22)$$

and

$$\frac{g_p}{2} = \frac{\mu_p}{\mu_N} = 2.792\,847\,353(28). \quad (23)$$

For further application we also need a value of the electron magnetic moment in hydrogen atom in units of the nuclear magneton. Combining Eqs. (16) and (21), we obtain

$$\mu_e(\text{H}) = 1838.249\,424\,6(13)\mu_N. \quad (24)$$

Similar analysis can be performed for experiments with the deuterium atom. The experimental result for deuterium [7] reads

$$\frac{\mu_e(\text{D})}{\mu_d(\text{D})} = 2143.923\,565(23). \quad (25)$$

Taking into account higher-order corrections in Eqs. (14) and (15), we obtain

$$g_d(\text{D}) = \frac{\mu_d(\text{D})}{\mu_N} = 0.857\,423\,017\,1(94) \quad (26)$$

and

$$g_d = \frac{\mu_d}{\mu_N} = 0.857\,438\,233\,3(94). \quad (27)$$

Let us consider some consequences of correcting the g factor of a free proton and magnetic moments of a proton and electron bound in the hydrogen atom. E.g., the g factor of a shielded proton in water was measured [24] in comparison with the magnetic moment of an electron bound in hydrogen atom (24). The corrected results for the proton magnetic moment are

$$\frac{\mu'_p(\text{H}_2\text{O})}{\mu_e} = 0.001\,520\,993\,127(17) \quad (28)$$

and

$$\frac{g'_p(\text{H}_2\text{O})}{2} = \frac{\mu'_p(\text{H}_2\text{O})}{\mu_N} = 2.792\,775\,600(33), \quad (29)$$

where shielding is denoted by prime. These results are related to a spherical sample of pure water at a temperature $t = 25$ °C. The values for other forms and temperatures of the sample can be recalculated (for detail see Ref. [1] and references therein).

The corrections for a bound proton in water is shifted by approximately 30% from the original result, however, the difference between the results from Ref. [1] and ours has been reduced since the former evaluation included a result from [25] which is ten times less accurate and about two standard deviations off from the more accurate value [24]. Here, we consider only most accurate results while the other data and in particular the result [25] have been dismissed from our consideration.

A determination of the magnetic moment of the proton in water is important because it has been used as a probe in a number of measurements and in particular to determine a value of the magnetic moment of a shielded helion, a nucleus of the ^3He atom [26]

$$\frac{\mu'_h(^3\text{He})}{\mu'_p(\text{H}_2\text{O})} = 0.761\,786\,131\,3(33). \quad (30)$$

With a corrected value for the magnetic moment of the shielded proton in Eq. (29) the helion result now reads

$$\frac{\mu'_h(^3\text{He})}{\mu_N} = 2.127\,497\,720(25). \quad (31)$$

The magnetic moment $\mu'_h(^3\text{He})$ is related to a helion bound in a neutral atom and studied in a low pressure helium-3 gas. To the best of our knowledge, no theoretical calculations are available for the higher-order correction to the g factor $g'_h(^3\text{He})$ similar to the $\alpha(Z\alpha)^3$ in Eq. (15) for hydrogen. The single-electron contribution for helium should be doubled (because of the presence of two electrons) and could receive some enhancement since the effective charge for each electron is somewhat bigger than unity. There should also be some essentially two-electron relativistic effects. We expect that the uncertainty of any theoretical calculation (see, e.g., [27]), ignoring the higher-order relativistic effects in order $\alpha(Z\alpha)^3$, cannot be below a part or even a few parts in 10^8 . Because of the unclear status of the uncertainty of theoretical calculations of the screening effects, we do not consider here a determination of the free nuclear magnetic moment of helion.

We have also considered data related to the muon magnetic moment. The result

$$\frac{\mu_\mu}{\mu_N} = 8.890\,596\,96(42) \quad (32)$$

is a weighted average of two values:

- The first one ($\mu_\mu/\mu_N = 8.890\,597\,05(106)$) is obtained from the measurement [8] of the transitions between hyperfine components of the ground state in muonium (cf. Fig. 1) in the magnetic field calibrated by measuring precession of a free proton. The value was slightly corrected in [11] because of higher-order corrections;²
- The other result ($\mu_\mu/\mu_N = 8.890\,596\,95(46)$) is found from a value of the hyperfine splitting in the muonium ground state measured for zero magnetic field [8] and compared with theory [28]. Note that the fine structure constant used for the calculations here is $\alpha^{-1} = 137.035\,998\,76(52)$ [29].

The less accurate data on the muon magnetic moment have been overviewed in Ref. [11] in terms of a related quantity

$$\frac{m_e}{m_\mu} = \frac{1}{1 + a_\mu} \frac{\mu_\mu}{\mu_N} \frac{\mu_N}{\mu_B}. \quad (33)$$

They are statistically not important and have not been taken into account while calculating the muon result above.

To summarize our consideration, we present the corrected values of the g factors of electron (bound), proton (free and bound), deuteron (free and bound) and helion (bound) in Table 1. We compare our results with those in Ref. [1] which seems to be the only paper where a systematic consideration on theory and experiment in light

Table 1

Magnetic moments ratios of electron, muon, proton, deuteron and helion. The CODATA results are taken from the *CODATA Recommended values—1998* [1] and the *corrected* results are discussed in our Letter. We restore here signs of magnetic moments

Value	CODATA [1]	Corrected
μ_B/μ_N	1836.1526675(39)	1836.1526736(13)
μ_e/μ_N	−1838.2819660(39)	−1838.2819721(13)
$\mu_e(\text{H})/\mu_N$	−1838.2494187(39)	−1838.2494246(13)
μ_μ/μ_N	−8.89059770(27)	−8.89059696(42)
μ_p/μ_N	2.792847337(29)	2.792847353(28)
$\mu_p(\text{H})/\mu_N$	2.792797812(29)	2.792797820(28)
$\mu'_p(\text{H}_2\text{O})/\mu_N$	2.792775597(31)	2.792775600(33)
μ_d/μ_N	0.8574382284(94)	0.8574382333(94)
$\mu_d(\text{D})/\mu_N$	0.8574230144(94)	0.8574230171(94)
$\mu'_h(^3\text{He})/\mu_N$	−2.127497718(25)	−2.127497720(25)

² We note that the $\alpha^2(Z\alpha)m/M$ term in Eq. (9) of Ref. [11] is to be corrected and it now reads $\alpha^2(Z\alpha)/12\pi m/M$.

simple atoms is done. We resume that higher-order corrections are somewhat bigger than it was expected in Ref. [1] but still do not exceed the uncertainty. In particular, the corrections to the proton, deuteron and helion magnetic moments (g factors) are slightly below the uncertainty, which is for all these quantities on level of a part in 10^8 in fractional units. However the corrections are important because they produce a systematic effect on deduced values of all discussed nuclear magnetic moments at a level of an essential part of uncertainty. A shift of the value of the nuclear magnetic moments listed in Table 1 typically varies from 30 to 60% of the value of their uncertainties. Note, that in the case of the magnetic moment of the proton in water and a related value of the helion magnetic moment the shift is still on a level of 30% of the uncertainty, but it corresponds to a result of the most accurate experiment [24], while the CODATA result in Table 1 is related to an average value (see discussions after Eq. (29)).

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