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Physics Letters B 650 (2007) 27–32

PHYSICS LETTERS B

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Joint extraction of m_s and V_{us} from hadronic τ decays

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Received 12 January 2007; received in revised form 21 March 2007; accepted 24 April 2007

Available online 5 May 2007

Editor: B. Grinstein

Abstract

We study the simultaneous determination of m_s and V_{us} from flavor-breaking hadronic τ decay sum rules using weights designed to bring under better control problems associated with the slow convergence of the relevant $D = 2$ OPE series. Much improved stability and consistency is found as compared to the results of conventional analyses based on the "$(k, 0)$ spectral weights". Results for m_s are in excellent agreement with those of recent strange scalar and strange pseudoscalar sum rule analyses, as well as recent lattice analyses, while those for V_{us} agree within errors with the output from recent lattice-based $\Gamma[K_{\mu 2}]/\Gamma[\pi_{\mu 2}]$ and $K_{\ell 3}$-based analyses. Very significant error reductions are shown to be expected, especially for V_{us}, once the improved strange spectral data from the B-factory experiments becomes available.

PACS: 12.15.Ff; 12.15.Hh; 13.35.Dx; 11.55.Hx

1. Background

Measurements of inclusive flavor $ij = ud, us$ vector (V) or axial vector (A) current induced hadronic τ decay distributions yield kinematically weighted linear combinations of the spectral functions, $\rho^{(J)}_{V/A;ij}$, of the spin $J = 0$ and 1 parts, $\Pi^{(J)}_{V/A;ij}$, of the relevant current–current correlators. Explicitly, with $R_{V/A;ij} \equiv \Gamma[\tau^- \to \nu_\tau \,\text{hadrons}_{V/A;ij}(\gamma)]/\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e(\gamma)]$ [1],

$$R_{V/A;ij} = 12\pi^2 |V_{ij}|^2 S_{\rm EW} \int_{\rm th}^{m_\tau^2} \frac{ds}{m_\tau^2}(1-y_\tau)^2 \times \left[(1+2y_\tau)\rho^{(0+1)}_{V/A;ij}(s) - 2y_\tau \rho^{(0)}_{V/A;ij}(s)\right] \quad (1)$$

where $y_\tau = s/m_\tau^2$, V_{ij} is the flavor ij CKM matrix element, $S_{\rm EW} = 1.0201 \pm 0.0003$ [2] is a short-distance electroweak correction, and the superscript $(0+1)$ denotes the sum of $J = 0$ and $J = 1$ contributions. Since the spectral function combinations in Eq. (1) correspond to correlators with no kinematic singularities, each term on the RHS can be rewritten using the basic finite energy sum rule (FESR) relation,

$$\int_{\rm th}^{s_0} ds\, w(s)\rho(s) = \frac{-1}{2\pi i}\oint_{|s|=s_0} ds\, w(s)\Pi(s). \quad (2)$$

Analogous FESRs, corresponding to spectral integrals, $R^{(k,m)}_{V/A;ij}$, obtained by rescaling the kinematic weights in $R_{V/A;ij}$ by $(1-y_\tau)^k y_\tau^m$ before integration, are referred to as the "(k, m) spectral weight sum rules". Similar spectral integrals and FESRs can be constructed for $s_0 < m_\tau^2$, for general non-spectral weights $w(s)$, and for either of the correlator combinations $\Pi^{(0+1)}_{V/A;ij}(s)$ or $s\Pi^{(0)}_{V/A;ij}(s)$. We denote such spectral integrals generically by $R^w_{ij}(s_0)$, and refer to the purely $J = 0$ contribution in "inclusive" FESRs (those having both $J = 0 + 1$ and $J = 0$ contributions) as "longitudinal", in what follows.

V_{us} and/or m_s are extracted using flavor-breaking differences, $\delta R^w(s_0)$, defined by

$$\delta R^w(s_0) = \left[R^w_{ud}(s_0)/|V_{ud}|^2\right] - \left[R^w_{us}(s_0)/|V_{us}|^2\right]. \quad (3)$$

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0370-2693
doi:10.1016/j.physletb.2007.04.053

Since $\delta R^w(s_0)$ vanishes in the $SU(3)_F$ limit, its OPE representation, $\delta R^w_{\rm OPE}(s_0)$, begins with a dimension $D=2$ contribution, proportional to m_s^2. Experimental values for $\delta R^w(s_0)$ over a range of s_0 and $w(s)$ allow m_s and/or V_{us} to be fitted, provided s_0 is large enough that insufficiently-well-known higher D OPE contributions are small [3–9]. As stressed in Refs. [7–9], the smallness of m_s (and hence of the flavor-breaking spectral integral differences) presents a challenge for the determination of m_s but a significant advantage for the determination of V_{us}. Explicitly, one has, from Eq. (3) [7],

$$|V_{us}| = \sqrt{R^w_{us}(s_0)/\big(\big[R^w_{ud}(s_0)/|V_{ud}|^2\big] - \delta R^w_{\rm OPE}(s_0)\big)}. \tag{4}$$

At scales $\sim$ 2–3 GeV^2, and for weights used in the literature, the dominant $D=2$ term in $\delta R^w_{\rm OPE}(s_0)$ is *much* smaller than the separate ud, us $D=0$ OPE contributions, and hence than the separate ud, us spectral integrals (for physical m_s, typically at the few to several percent level). An uncertainty, $\Delta(\delta R^w_{\rm OPE}(s_0))$, in $\delta R^w_{\rm OPE}(s_0)$ thus produces a fractional uncertainty in $|V_{us}|$, $\simeq \Delta(\delta R^w_{\rm OPE}(s_0))/2R^w_{ud}(s_0)$, *much* smaller than that on $\delta R^w_{\rm OPE}(s_0)$ itself. Moderate precision for $\delta R^w_{\rm OPE}(s_0)$ thus suffices for high precision on $|V_{us}|$, provided experimental errors can be brought under control.

In what follows, we perform a combined extraction of m_s and $|V_{us}|$ based on existing spectral data. The ud data [10–12] are already quite precise, but sizeable errors on the us data [13–15] limit currently achievable precision. We thus focus on better controlling uncertainties on the theoretical (OPE) side of the analysis, especially those associated with slower-than-previously-anticipated convergence of the relevant $D=2$ series [16].

2. Technical issues in the hadronic τ decay approach

The first major stumbling block is the very bad behavior of the integrated longitudinal $D=2$ OPE series. Even at the maximum scale $s_0 = m_\tau^2$ allowed by kinematics, the series shows no sign of converging [17]. Even worse, all truncation schemes employed in the literature, *with no exceptions*, badly violate constraints associated with spectral positivity [5]. This is, in fact, the source of a large part of the very strong unphysical k-dependence seen in results for m_s from inclusive $(k,0)$ spectral weight analyses [5]. Inclusive analyses employing the longitudinal OPE representation are thus untenable, and earlier results obtained from such analyses should be discarded in favor of those of more recent non-inclusive treatments [4,7,16].

Fortunately, the severe problems of the longitudinal $D=2$ OPE representation are easily handled phenomenologically, for a combination of chiral and kinematic reasons. Apart from the π and K pole contributions, longitudinal spectral contributions vanish in the $SU(3)_F$ limit and are doubly-chirally suppressed away from it, a suppression preserved in the ratio of non-pole to pole spectral integral contributions as a consequence of the structure of the longitudinal spectral weight [5]. The small residual non-pole us PS and scalar contributions can, moreover, be well-constrained phenomenologically, the former via a sum rule analysis of the us PS channel [4,18], the latter via $K\pi$-scattering-data-based dispersive analyses [4,19,20] (the most reliable being the coupled-channel version discussed in Ref. [20], which incorporates short-distance QCD and chiral constraints). With the very accurately known π and K pole contributions, these results make possible a reliable bin-by-bin subtraction of longitudinal contributions to the experimental distribution,[2] and hence a direct determination of the $(0+1)$ spectral function, allowing us to focus, in what follows, on sum rules involving the flavor-breaking combination $\Delta\Pi(s) \equiv \Pi^{(0+1)}_{V+A;ud}(s) - \Pi^{(0+1)}_{V+A;us}(s)$, which are not afflicted by the longitudinal $D=2$ OPE problem.

The second problem concerns the slow convergence of the $(0+1)$ $D=2$ OPE series. For scales $s_0 \sim$ 2–3 GeV^2, $[\Delta\Pi]_{\rm OPE}$ is dominated by its $D=2$ contribution [16],

$$\big[\Delta\Pi(Q^2)\big]^{\rm OPE}_{D=2} = \frac{3}{2\pi^2}\frac{\bar m_s}{Q^2}\big[1 + 2.333\bar a + 19.933\bar a^2 + 208.746\bar a^3 + (2378 \pm 200)\bar a^4 + \cdots\big], \tag{5}$$

where $\bar a = \alpha_s(Q^2)/\pi$ and $\bar m_s = m_s(Q^2)$, with $\alpha_s(Q^2)$ and $m_s(Q^2)$ the running coupling and strange quark mass in the $\overline{MS}$ scheme. The $O(\bar a^4)$ coefficient has been estimated using approaches previously successful in obtaining accurate predictions for the $O(\bar a^3)$ coefficient in Eq. (5) and n_f-dependent $O(\bar a^3 m_q^2)$ coefficients of the electromagnetic current correlator in advance of the explicit calculations of these values [23].

Since, with 4-loop running, independent high-scale determinations of $\alpha_s(M_Z)$ (see the review section on QCD in Ref. [24]) correspond to $\bar a(m_\tau^2) \simeq 0.10$–$0.11$, Eq. (5) shows that, at the spacelike point on $|s| = s_0$, the convergence of the $(0+1)$ $D=2$ OPE series is marginal at best, even at the highest scales accessible in τ decay. While $|\alpha_s(Q^2)|$ decreases as one moves along the contour away from the spacelike point, allowing the convergence of the integrated series to be improved through judicious weight choices, this observation shows that, for weights not chosen specifically with this constraint in mind, one must expect to find very slow convergence of the integrated $D=2$ series. The $(k,0)$ spectral weights, $w^{(k,0)}(y) = (1+2y)(1-y)^{k+2}$, with $y = s/s_0$, are very much non-optimal in this regard, since $|1-y| = 2|\sin(\phi/2)|$ (where ϕ is the angular position measured counterclockwise from the timelike point), is peaked precisely in the spacelike direction. Slow convergence, deteriorating with increasing k, is thus expected for the integrated $D=2$ series of the $(k,0)$ spectral weights. The results of Table I of Ref. [16] and row 1, Table I of Ref. [8] bear out this expectation.

In evaluating the integrated $(0+1)$ $D=2$ OPE contribution, the level of residual scale dependence, the difference between the direct correlator and Adler function evaluations (both trun-

[2] It is worth noting that scalar and PS sum rule analyses employing the resulting us scalar and PS spectral "models" [21] yield values of m_s in excellent agreement with those of recent $N_f = 2+1$ lattice simulations [22]. The lattice results thus preclude significantly larger non-pole longitudinal spectral strength, ensuring that the small residual longitudinal subtraction is very well under control at the level required for our analysis.

cated at the same order), and the size of the last term kept, have all been used as measures of the truncation uncertainty. The slow convergence of the integrated series, however, can make it hard to be sufficiently conservative. For example, the quadrature sum of the last term size plus residual scale dependence, used previously in the literature to estimate the $O(\bar{a}^3)$ Adler function truncation uncertainty, yields a result ~ 2.5 times smaller than the actual difference between the $O(\bar{a}^3)$-truncated Adler function and $O(\bar{a}^4)$-truncated direct correlator results [8]. Since the growth of α_s with decreasing scale makes higher order terms relatively more important at lower scales, premature truncation of a slowly converging series typically shows up as an unphysical s_0-dependence in extracted, nominally s_0-independent quantities. With polynomial weights, $w(y) = \sum_m c_m y^m$, for which integrated $D = 2N + 2$ OPE contributions not suppressed by additional powers of α_s scale as c_N/s_0^N, such unphysical s_0-dependence can also result if unsuppressed higher D contributions which might in principle be present are incorrectly assumed negligible and omitted from the analysis. Since, typically, not even rough estimates of $D > 6$ condensate combinations are available, such omission is most dangerous for weights, like the higher $(k, 0)$ spectral weights, which have large values of the coefficients c_m, with $m > 2$.[3]

In view of the above discussion, s_0-stability tests are essential components of any FESR determination of V_{us} and/or m_s. The existence of a stability window in s_0 for extracted quantities or, if not a stability window, then a window within which the observed instability is safely smaller than the estimated theoretical uncertainty, is crucial to establishing the reliability of the theoretical error estimate.

3. The spectral and non-spectral weight analyses

The OPE and spectral integral inputs used in our analysis are outlined below. The fact that we restrict our attention to FESRs involving the $V + A$ spectral combination, to weights satisfying $w(s = s_0) = 0$, and to scales $s_0 > 2$ GeV^2, all serve to strongly suppress possible residual OPE breakdown effects [6,25,26].

Integrals of the leading $D = 2$ contribution to $[\Delta\Pi(Q^2)]_{\rm OPE}$, given in Eq. (5), are evaluated using two versions of the $O(\bar{a}^4)$-truncated CIPT prescription [28], one involving the RG-improved $D = 2$ correlator contribution $[\Delta\Pi]^{\rm OPE}_{D=2}$, the other the equivalent Adler function contribution, $[\Delta D(Q^2)]^{\rm OPE}_{D=2} = -Q^2\, d[\Delta\Pi(Q^2)]^{\rm OPE}_{D=2}/dQ^2$. The difference between the two expressions, which would agree to all orders, but differ here by terms of $O(\bar{a}^5)$ and higher, is used as one component of our truncation uncertainty estimate. The full estimate is obtained, in all cases, by taking *twice* the sum in quadrature of the correlator-Adler function difference and the size of the last term kept.[4] For $\bar{a}$ and $\bar{m}_s$ we employ exact solutions corresponding to the 4-loop-truncated β and γ functions [29], with initial condition $\alpha_s(m_\tau^2) = 0.334 \pm 0.022$. The remaining initial condition, $m_s(2\ {\rm GeV})$, is either taken as input or obtained as part of the fit.

Expressions for the $D = 4$ and $D = 6$ contributions are given in Ref. [1]. The dominant $D = 4$ term, proportional to $\langle m_s \bar{s} s\rangle$, is evaluated using ChPT quark mass ratios [27], GMOR for the light quark condensate, and $r_c \equiv \langle m_\ell \bar{\ell}\ell\rangle/\langle m_s \bar{s} s\rangle = 0.8 \pm 0.2$. $D = 6$ contributions are estimated using the vacuum saturation approximation, and assigned an uncertainty of $\pm 500\%$. Contributions with $D > 6$ are assumed negligible, s_0-stability studies being employed to test the self-consistency of this assumption.

For the spectral integrals we work with the ALEPH ud [11] and us [13] data, for which both data and covariance matrices are publicly available. A small global renormalization of the ud data is performed to reflect minor changes in the e, μ and total strange branching fractions since the original ALEPH publication. Following the prescription of Ref. [30], we also perform mode-by-mode rescalings to reflect current (PDG06 [24]) values of the branching fractions of the various strange decay modes.[5] Errors on the K and π pole contributions are reduced by using the more precise expectations based on $\Gamma[\pi_{\mu 2}]$ and $\Gamma[K_{\mu 2}]$. With current data, ud and us spectral integral errors are at the $\sim 0.5\%$ and ~ 3–4% levels, respectively, for the weights to be discussed below. BABAR and BELLE will drastically reduce the size of the us errors in the near future.

All combined fits reported below were performed using MINUIT, and take into account fully all theoretical and experimental correlations.

3.1. The $(k, 0)$ spectral weight analyses

As noted above, slow convergence of the integrated $D = 2$ OPE series is both expected, and observed [8,16], for the $(k, 0)$ spectral weights. Further evidence that the OPE sides of the $(k, 0)$ sum rules are not under good control is provided by the top panel of Fig. 1. The figure shows the 1σ contours for joint fits of m_s and $|V_{us}|$ to the $s_0 = m_\tau^2$ experimental spectral integrals for various pairs amongst those $(k, 0)$ spectral weights employed in previous analyses of $|V_{us}|$ and m_s. It is clear that no sensible common fit region exists, and hence that a reliable joint fit for m_s and $|V_{us}|$ cannot be obtained using this set of $(k, 0)$ spectral weights.

Further consideration is warranted for the $(0, 0)$ analysis, which has been proposed in the literature as a particularly favorable one for the determination of $|V_{us}|$ [7]. Indeed, with m_s from other sources as input, the $s_0 = m_\tau^2$ version of this analysis, would, if reliable, allow an improved determination of $|V_{us}|$ with only improved us branching fractions as input, a feature special to this weight and this s_0 value. Unfortunately, on

[3] The largest of the $c_{m>2}$ for the $(2, 0)$, $(3, 0)$, and $(4, 0)$ spectral weights are $c_3 = 8$, $c_4 = -15$, and $c_4 = -25$, respectively. In contrast, the largest of the $c_{m>2}$ for the non-spectral weights w_{20}, $\hat{w}_{10}$, w_{10}, and w_8 discussed in the text are $c_3 = 2.087$, $c_5 = 1.206$, $c_5 = 2$ and $c_5 = 1.182$, respectively. Since all weights share the common normalization $w(0) = 1$, the non-spectral weights are far less sensitive to possible unknown $D > 6$ contributions.

[4] This estimate is much more conservative than alternatives used previously in the literature. The more conservative approach is required in order to obtain a total theoretical uncertainty compatible with observed s_0-instabilities in $|V_{us}|$.

[5] Thanks to Shaomin Chen for details of the procedure followed in Ref. [30] and the additional information required to perform the rescaling analysis.

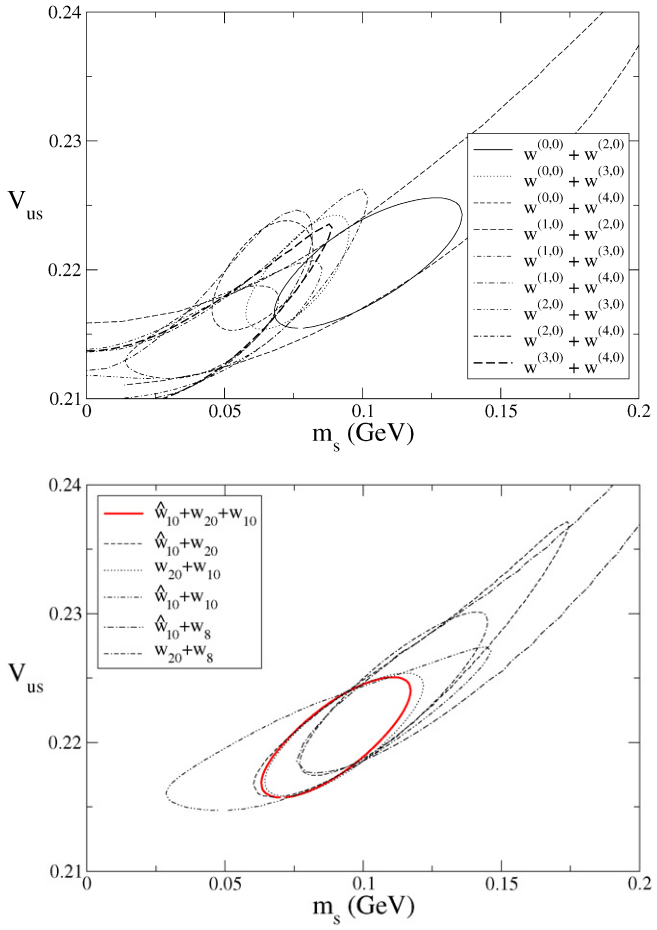

Fig. 1. m_s, $|V_{us}|$ joint fits, using $s_0 = m_\tau^2$, for the $(k, 0)$ spectral weights (top panel) and non-spectral weights (bottom panel).

the OPE side, the $D = 2$ truncation uncertainty appears much larger than previously anticipated [8], as can be seen from (i) the rather poor agreement between the truncated correlator and Adler function evaluations; (ii) the strong deterioration in this situation with increasing truncation order; and (iii) the very significant s_0-instability in the output values of $|V_{us}|$, at least for the PDG04 value of m_s used as input in Ref. [8]. In the left panel of Fig. 2 we demonstrate that this s_0-instability is not an artifact of the particular m_s employed in Ref. [8]. The figure shows the OPE and spectral integrals as a function of s_0 for the $(0, 0)$ FESR, for a range of different fixed input m_s. The value of $|V_{us}|$ needed for this comparison was obtained by matching the OPE and spectral integral versions of $\delta R^{(0,0)}(m_\tau^2)$. Note that very strong correlations exist amongst the OPE integrals for different s_0 and, similarly, amongst the spectral integrals for different s_0. Strongly discrepant s_0-dependences for the OPE and spectral integrals, as seen in the figure *for the entire range of m_s considered*, thus rule out the possibility that an acceptable s_0-stability in $|V_{us}|$ might be obtained from the $(0, 0)$ analysis, in its current form, for any input value of m_s. Even restricting our attention to the rather narrow range of s_0 values within 0.4 GeV^2 of m_τ^2, the level of s_0-instability in $|V_{us}|$ is > 0.0020, more than a factor of 2 larger than previous estimates of the total theoretical uncertainty. With our more conservative estimate for the $D = 2$ OPE truncation uncertainty, the estimated truncation error (total theoretical uncertainty) in $|V_{us}|$ becomes ± 0.0020 (± 0.0022) for $s_0 = m_\tau^2$, compatible at least with the observed s_0-instability in the limited s_0 region noted above. We argue that, given the observed level of s_0-instability, less cautious assessments of the theoretical error can not be justified. Since the $D = 2$ truncation uncertainty is unlikely to be reduced, it appears, unfortunately, that theoretical uncertain-

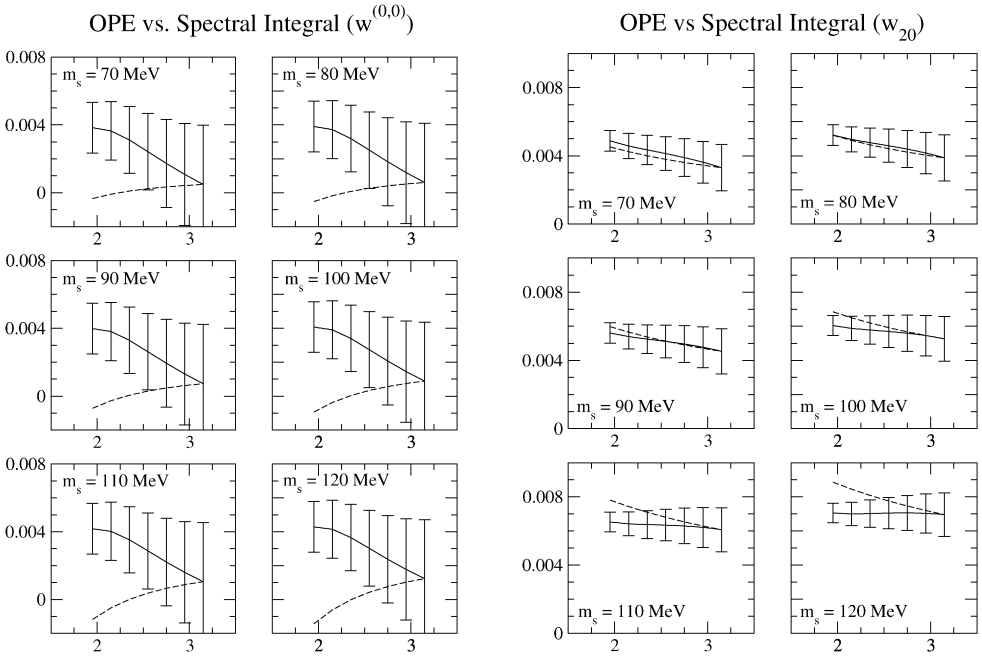

Fig. 2. $w^{(0,0)}$ and w_{20} OPE and spectral integrals vs. s_0 (all in units of GeV^2) for various input m_s (2 GeV). The solid line gives the experimental spectral integrals, the dashed line the corresponding OPE integrals. The $|V_{us}|$ required to make these comparisons are obtained by matching the resulting OPE and spectral integrals at $s_0 = m_\tau^2$.

ties preclude a $(0,0)$ spectral weight determination of $|V_{us}|$ at the sub-± 0.0020 level. Fortunately, alternate weights with significantly improved $D=2$ convergence behavior exist, allowing one to take advantage of the general approach proposed in Ref. [7].

3.2. The non-spectral-weight analyses

In Ref. [4], three non-spectral polynomial weights, $w_{10}(y)$, $\hat{w}_{10}(y)$, and $w_{20}(y)$, designed specifically to produce improved integrated $D=2$ convergence, were constructed. The significantly improved convergence is displayed explicitly in Ref. [4], and Table 1 of Ref. [8]. The weights were also designed to (i) keep higher order coefficients which might enhance $D>6$ contributions small, and, (ii) strongly suppress spectral integral contributions from the region above $s\sim 1\ \text{GeV}^2$, where current us spectral errors are large. Here we consider also a fourth weight, $w_8(y)$, with lower degree, but less strong suppression of the high-s part of the us spectrum.[6] While the latter feature leads to larger us spectral integral errors with present data, the weight will be useful for future analyses based on data with smaller us spectral errors. Having an additional non-spectral weight also allows us to investigate more fully the issue of the mutual consistency of analyses associated with different improved-convergence non-spectral weight choices.

In the right panel of Fig. 2 we display, for illustration purposes, the match between the OPE and spectral integral differences, for a range of input $m_s(2\ \text{GeV})$, for the case of the weight w_{20}.[7] The spectral integral differences are computed as described above for the analogous $(0,0)$ spectral weight results, shown in the left panel of the figure. The contrast with the $(0,0)$ case is immediately evident.

In the bottom panel of Fig. 1 we display the joint fit, $s_0=m_\tau^2$, 1σ contours, for various pairs of the non-spectral weights. Also shown is the 1σ contour for the 3-fold fit used in obtaining our final results below. A good common fit region for m_s and $|V_{us}|$ obviously exists for all of the non-spectral weights, in sharp contrast to the situation for the $(k,0)$ spectral weights.[8]

We first consider results for $|V_{us}|$ obtained, as in previous studies, using single weight analyses with $s_0=m_\tau^2$ and external m_s input. With the average of strange scalar and PS sum rule and $N_f=2+1$ lattice results, $m_s(2\ \text{GeV})=94\pm 6$ MeV, advocated in the last of Ref. [20], one finds $|V_{us}|=0.2209\pm 0.0029_{\text{exp}}\pm 0.0017_{\text{th}}$ for w_{20}, $0.2210\pm 0.0030_{\text{exp}}\pm 0.0010_{\text{th}}$ for $\hat{w}_{10}$, $0.2206\pm 0.0032_{\text{exp}}\pm 0.0007_{\text{th}}$ for w_{10}, and $0.2218\pm 0.0037_{\text{exp}}\pm 0.0009_{\text{th}}$ for w_8, with large experimental errors, dominated by those on the us distribution. The results are consistent, but cannot be directly averaged due to strong correlations.

A cautious approach, given the intrinsically slow convergence of the $(0+1)$ $D=2$ series, is to ignore external information on m_s, perform a combined fit for m_s and $|V_{us}|$, and verify that the results for m_s are consistent with what is known from other sources. The results for the 3-fold, $s_0=m_\tau^2$, w_{20}, $\hat{w}_{10}$, w_{10} fit, including theoretical errors on the same footing as experimental ones in the minimization process, are $m_s(2\ \text{GeV})=89\pm 26$ MeV and $|V_{us}|=0.2202\pm 0.0046$. The analogous full 4-fold non-spectral weight fit yields $m_s(2\ \text{GeV})=96\pm 31$ MeV and $|V_{us}|=0.2208\pm 0.0052$. The larger errors in the second case result from the less strong suppression of the high-s us data region by w_8.

Since the combined joint fit results for m_s are in excellent agreement with the external average noted above, it makes sense now to perform a combined $s_0=m_\tau^2$ fit for $|V_{us}|$ with $m_s(2\ \text{GeV})=94\pm 6$ MeV as input. The lower-error 3-fold fit (without w_8) yields

$$|V_{us}|=0.2209\pm 0.0031, \tag{6}$$

which we take as our main result. Note that, had the fit been performed without including the uncertainty on the central value for m_s, the output central value for $|V_{us}|$ would have been shifted up to 0.2221. The result of Eq. (6), though lower in central value, is compatible, within mutual errors, with both the recent $K_{\ell 3}$ determination, $|V_{us}|=0.2249\pm 0.0019$ [32] (based on the conventional Leutwyler–Roos estimate for $f_+(0)$) and the $\Gamma[K_{\mu 2}]/\Gamma[\pi_{\mu 2}]$ determination, $|V_{us}|=0.2223^{+0.0026}_{-0.0013}$ (based on the updated 2006 MILC evaluation of f_K/f_π [22]). We comment further below on the status/reliability of the current hadronic-τ-decay-based analysis.

4. Discussion, conclusions and prospects

We have shown that it is possible to construct non-spectral weights which improve the convergence behavior of the integrated $D=2$ OPE series and allow a self-consistent joint fit for m_s and $|V_{us}|$. While the results reported above for $|V_{us}|$ are compatible within errors with those from other sources, one should bear in mind that the errors on the present hadronic τ decay data are sufficiently large that non-trivial shifts in the central value are likely when the B-factory data becomes available. To see why this is the case, note that the branching fractions for observed strange decay modes in the current data go down only to the $\sim 3\times 10^{-4}$ level. Missing modes with lower branching fractions could have a non-trivial impact on the extracted value of $|V_{us}|$. Missing a strange mode with branching fraction 1×10^{-4}, for example, would lower $|V_{us}|$ by ~ 0.0004. At the desired level of precision, one thus needs to detect all strange modes with branching fractions at the few-to-several-10^{-5} level. Such determinations are certainly feasible at the B-factory experiments where preliminary results for branching

[6] The explicit form of $w_8(y)$, as well as of other potentially useful weights, will be given elsewhere. Requests for this information, as well as for a numerically more precise form of w_{20} than that given in Ref. [4], can be directed to either of the present authors.

[7] The corresponding matches for the other three non-spectral weights have been omitted for reasons of brevity. For intermediate m_s, the OPE and spectral integral differences are in agreement in all cases. A figure showing the match for the worst of the four cases (w_{10}) can be found in Ref. [31].

[8] It should be stressed that the much improved consistency amongst the different non-spectral weights is non-trivial, and *not* a consequence of using weights with very similar profiles, as can be seen from the fact that the ranges spanned by the ud, us spectral integrals for the four non-spectral weights and for the $(0,0)$, $(1,0)$ and $(2,0)$ spectral weights are in fact comparable to within 10%.

fractions at this level (with errors at the few-to-several-10^{-6} level) have already been reported [33].

Regarding the goal of improved future precision, one should note that the $\sim 1/2\%$ uncertainty on the ud spectral integrals, though negligible in comparison to the us errors at present, contributes ~ 0.0005 to the uncertainty on $|V_{us}|$, assuming the current assessment of the non-strange spectral errors is correct. One should also bear in mind here the disagreement between the isovector vector spectral contributions implied by isospin-breaking-corrected electroproduction data and those measured directly in τ decay experiments [34]. For example, were the electroproduction results for $\pi\pi$ (whose implied $\tau \to \pi\pi\nu_\tau$ branching fraction is 4.5σ below that measured directly in τ decay) to be correct, the value of $|V_{us}|$ obtained from the τ decay analysis above would be raised by ~ 0.0018. Even the slightly lower central value of the preliminary BELLE $\tau \to \pi\pi\nu_\tau$ measurement [35] (which is compatible, within its somewhat larger systematic error, with the earlier τ world average value) would produce an increase of ~ 0.0006 in $|V_{us}|$. Resolving the discrepancy between the $\pi\pi$ electroproduction and τ decay results is thus important not just for clarifying expectations for the hadronic contribution to $(g-2)_\mu$ in the Standard Model, but also for finalizing future τ decay determinations of $|V_{us}|$.

We finally comment that, re-running the analysis with the central values of the us spectral distribution unchanged but with errors reduced by a factor of 5 (covariances by a factor of 25), and assuming $m_s(2\ \text{GeV})$ to be known to ± 5 MeV, a combined total fit error below 0.0010 is achieved. Such an improvement in the us spectral distribution errors should be well within reach of the final B-factory analyses [36]. Further improvement on the theoretical component of the total error is almost certainly also possible through the use of new weights with significantly less strong suppression of the high-s region. Such weights, however, will become useful only once the errors in the us spectrum above the K^* have been significantly reduced. The necessity of self-consistency and stability checks also means that an interaction between theorists and experimentalists will certainly be required for successful development of such weights.

Acknowledgements

Ongoing support from the Natural Sciences and Engineering Research Council of Canada is gratefully acknowledged.

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