



Original Article

Application of Multivariate Adaptive Regression Spline-Assisted Objective Function on Optimization of Heat Transfer Rate Around a Cylinder

Prasenjit Dey* and Ajoy K. Das

Mechanical Engineering Department, National Institute of Technology, Agartala, India

ARTICLE INFO

Article history:

Received 30 November 2015

Received in revised form

3 May 2016

Accepted 8 June 2016

Available online 1 July 2016

Keywords:

Multivariate Adaptive Regression

Splines

Optimized Heat Transfer Rate

Particle Swarm Optimization

Rounded Cornered Square

Cylinder

ABSTRACT

The present study aims to predict the heat transfer characteristics around a square cylinder with different corner radii using multivariate adaptive regression splines (MARS). Further, the MARS-generated objective function is optimized by particle swarm optimization. The data for the prediction are taken from the recently published article by the present authors [P. Dey, A. Sarkar, A.K. Das, Development of GEP and ANN model to predict the unsteady forced convection over a cylinder, *Neural Comput. Appl.* (2015) 1–13]. Further, the MARS model is compared with artificial neural network and gene expression programming. It has been found that the MARS model is very efficient in predicting the heat transfer characteristics. It has also been found that MARS is more efficient than artificial neural network and gene expression programming in predicting the forced convection data, and also particle swarm optimization can efficiently optimize the heat transfer rate. Copyright © 2016, Published by Elsevier Korea LLC on behalf of Korean Nuclear Society. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

1. Introduction

Nowadays, research on fluid flow and heat transfer characteristics over a cylindrical bluff body attracts tremendous attention of researchers, as it has overwhelming engineering significance for nuclear reactors, heat exchangers, natural circulation boilers, solar heating systems, electronic cooling, dry cooling towers, flow dividers, probes, vortex flow meters, sensors, etc. The common geometrical shape of a cylindrical bluff body may be circular, sharp and rounded cornered

square cylindrical, triangular, etc. A square cylinder is the most common sharp-edge body and has widely been investigated in the study of fluid flow and heat transfer. Preceding studies were carried out by numerical, theoretical, and experimental methods. Based on the Reynolds and Prandtl numbers, various flow regimes were recognized in the available studies [1–10]. Also, there are various available studies associated with the circular cylinder that accomplished by the both numerically and experimentally [11–16]. Currently, fluid flow studies have found that the fluid forces

* Corresponding author.

E-mail address: deyprasenjita@gmail.com (P. Dey).
<http://dx.doi.org/10.1016/j.net.2016.06.011>

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acting on a square cylinder can be reduced by rounding the corners [17,18].

Soft computing methods have extensively been used in many areas of mechanical engineering due to their capacity of calculation and data handling. Different strategies are utilized as a part of the forecast, among them artificial neural network (ANN) and gene expression programming (GEP) being the foremost strategies in use. In order to avoid solution methods that are time consuming and need high numbers of iterations, ANN and GEP have increasingly been preferred by researchers. GEP is another framework having the upsides of both genetic programming and genetic algorithm (GA) to assess more mind boggling capacities to display a declaration of the connection between the input and output data [19]. GEP is more useful in predicting the output than ANN, as was recently discovered [20,21]. The GEP model has also been successfully employed to predict various engineering parameters [22–24]. Both models have been effectually applied, even though they have some disadvantages. One of the main disadvantages of ANN is it does not disclose any mathematical relation between the input and output variables of the system. The disadvantage of GEP is that it can generate a highly nonlinear mathematical relation between the input and output data. The multivariate adaptive regression splines (MARS) model has some advantages over ANN and GEP [25,26].

Recently, different optimization techniques, such as GA, particle swarm optimization (PSO), etc., have been applied successfully for optimizing heat transfer [27,28]. GA has been used by different researchers to optimize the convective thermal performance of fin and plate fin heat exchangers. To date, PSO has been applied to thermodynamic optimization of a cross-flow plate fin heat exchanger [29]. A comparison study of GA and PSO has been performed recently [30], and it was found that PSO is more efficient than GA for optimizing the geometry of a longitudinal fin.

Numerical, analytical, and experimental studies require much accomplishment time, and ANN and GEP have some disadvantages, therefore, the MARS model has been used in the present study to predict the heat transfer characteristics around a rounded cornered square cylinder including a square and a circular cylinder. To the best of the authors' knowledge, this is the first study on the application of MARS for the prediction of heat transfer characteristics. A total of 36 data records are collected from the recent paper published by the present authors. Further, the present prediction models are compared with the published ANN and GEP models. The relation between the input and output generated by the MARS model is then used in the PSO tool for optimizing the heat transfer rate.

2. Multivariate adaptive regression splines

MARS, a nonlinear and nonparametric regression organization, was first presented by Friededman [31] as a supple process that replicates interactions between inputs and outputs with fewer variables. This technique creates no ambiguity about the functional connection between the dependent and independent variables; MARS develop this relationship from a

group of coefficients and basis functions that are engaged from the regression data. MARS produce basis functions by examining them in a stepwise method. Each spline function is defined on a given interval and the end points of the interval are called "knots." A MARS model is completed in two steps. In the first step, the model is built and basis functions are added to grow the complexity until extreme complexity is attained. In the next step, a backward calculation is done to remove the minimum substantial basis function from the model.

The principle of the MARS system is built on piecewise linear basis functions of the following forms:

$$|x - t|_+ = \max(0, x - t) = \begin{cases} x - t & x > t \\ 0 & x \leq t \end{cases} \quad (1)$$

$$|t - x|_+ = \max(0, t - x) = \begin{cases} t - x & x < t \\ 0 & x \geq t \end{cases} \quad (2)$$

where t represents the "knots." The above formulations serve as the basis functions for linear or nonlinear development that estimates the function $f(x)$.

If a dependent variable (i.e., the outcome) "y" is dependent on "M" terms, then the MARS model can be summarized in the following equation:

$$y = f(x) = \beta_0 + \sum_{i=1}^M \beta_i H_{ki}(x_{v(k,i)}) \quad (3)$$

where β_0 and β_i are the basis function parameters of the model, and the function H can be defined as follows:

$$H_{ki}(x_{v(k,i)}) = \prod_k -i^k h_{ki} \quad (4)$$

where $x_{v(k,i)}$ is the predictor in the k^{th} of the i^{th} product. For order of interactions $K = 1$ the model is additive, and for $K = 2$ the model is pairwise interactive [31].

2.1. Input and output parameters

It is necessary to have a set of data to train the predictive models, and some portion of that set is further used to test the trained models to verify their accuracy. In the present study, the input parameters are the nondimensional corner radius (r) of the square cylinder and Prandtl number (Pr). Six values of "r" were selected: $r = 0.5$ (circular cylinder), $r = 0.51$, $r = 0.54$, $r = 0.59$, $r = 0.64$, and $r = 0.71$ (square cylinder); the values of Pr varied as 0.01, 0.1, 1, 10, 100, and 1,000 at Reynolds number (Re) = 100. By combining all the inputs, a total of 36 data (6 values of "r" \times 6 values of "Pr") were found. All the 36 data sets were collected from the authors' published article [32], where the data sets were established by solving the heat transfer problem numerically using the finite volume method (FVM) code. The governing equations associated with the heat transfer problem were the Navier–Stokes equations and the energy equation. The problem was solved in a two-dimensional unsteady laminar flow regime. A number of trials were performed to find a quite accurate data set to train the model. After achieving quite satisfactory accuracy, 70% of the total data were selected for training and the remaining 30% for testing the model. The single output of the present study, the heat transfer characteristics around the

cylinder, was calculated using the average Nusselt number (Nu_{avg}).

2.2. Statistical error analysis

The error between the numerical and predicted values is calculated as adjusted R^2 ($Adj.R^2$), mean absolute error (MAE), mean absolute percentage error (MAPE), and root mean squared error (RMSE), which are expressed as follows:

$$Adj. R^2 = 1 - \frac{\sum_i (N_i - P_i)^2 / n - p - 1}{\sum_i (N_i - \bar{N})^2 / n - 1} \quad (5)$$

$$MAE = \frac{1}{n} \sum_{i=1}^n |N_i - P_i| \quad (6)$$

$$MAPE = \frac{1}{n} \sum_{i=1}^n \frac{|N_i - P_i|}{N_i} \times 100 \quad (7)$$

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (N_i - P_i)^2} \quad (8)$$

where

- n = sample size
- p = total number of regressors in the training model
- N_i = actual value
- P_i = predicted value

$$\bar{N} = \frac{1}{n} \sum_{i=1}^n N_i$$

3. PSO model

PSO is an evolutionary computation technique and a population-based swarm intelligence algorithm to solve the global optimization problem that was developed by Kennedy and Eberhart in 1995 [33]. It is an arithmetic calculation technique that starts with a crowd of grain called the swarm and mainly based on social models, such as fish schooling, bird flocking, and swarm theory [29], where the concomitant of the swarm's behavior, i.e., maintaining optimal distances between individual members and their neighbors, are the main factors. The position of each particle is optimized by improving its position as designed for the objective function within the search area.

Therefore, the velocity of a particle is an important factor of PSO, which is optimized in each iteration by comparison with the previous one to lead the particle to its best position. In every iteration, each particle in a swarm achieves the best solution (fitness) possible so far, called $pbest$. Another "best" value that a particle obtained so far in the population tracked by the particle swarm optimizer which is global best, called $gbest$. The velocity of each particle in a swarm is given by the following equation [33]:

$$V_{i+1} = wV_i + c_1r_1(pbest_i - X_i) + c_2r_2(gbest_i - X_i) \quad (9)$$

$$X_{i+1} = X_i + V_{i+1} \quad (10)$$

where

V_{i+1} = new velocity for each particle based on the previous velocity (V_i)

w = inertial coefficient (0.8–1.2)

c_1 and c_2 = cognitive coefficient and social coefficient, respectively (0–2)

r_1 and r_2 = random values for each velocity update (0–1)

X_{i+1} = new position for each particle based on the previous position (X_i)

4. Objective function

In the present work, six cases of corner radii were simulated, with the maximum heat transfer rate being found at $r = 0.51$. The main objective of the present work is to determine the optimal value of the corner radius that maximizes the heat transfer rate around the cylinder. Here, the objective function was generated by the MARS model and reported as follows:

$$\text{Maximize, heat transfer rate} = \text{Max } Nu_{avg} = \text{Min} \left(\frac{1}{Nu_{avg}} \right) \quad (11)$$

Subjected to the following inequality constraints:

$$0.5 \leq r \leq 0.71 \text{ and } 0.01 \leq Pr \leq 1,000$$

5. Results and discussion

5.1. Prediction of heat transfer characteristics with different Pr values

In this part of the prediction, the model was acquired using the corner radius " r " and Prandtl number " Pr " as the inputs, and the average Nusselt number " Nu_{avg} " as the output. Then the obtained model was tested using the testing input and output data. Various parameters affect the MARS model, such as the maximal number of basis functions, generalized cross-validation penalty per knot, maximum degree of self-interactions, threshold, prune, etc. All the parameters were varied within limits and the number of combinations was found to build the model. Then every model's efficiency was checked by calculating the $Adj.R^2$, MAE, MAPE, and RMSE. The different parameters that were used to train the MARS model are given in Table 1. After having quite satisfactory accuracy of the predicted heat transfer coefficient, the estimated coefficients and basis functions are summarized in Table 2. The variation between the Nu_{avg} predicted by the MARS model and the actual value is depicted in Fig. 1. Fig. 1A clearly depicts that the predicted values are nearly equal to the actual values, and Fig. 1B shows the efficiency of the MARS model to predict the Nu_{avg} .

Table 1 – Different parameters of the MARS model.

| Parameters | Values |
|---|------------|
| Max functions | 10–40 |
| Generalized cross-validation penalty per knot | 0, 2–4 |
| Self-interactions | No |
| Max interactions | 2–4 |
| Threshold | 1.0000e–04 |
| Prune | Yes |
| MARS, multivariate adaptive regression splines. | |

Table 2 – Parameters of the MARS model for Nu_{avg} with different Pr values.

| BF | Coefficients |
|--|--------------|
| Intercept | 170.03 |
| BF1 = $\max(0, Pr - 10)$ | 14.917 |
| BF2 = $\max(0, 10 - Pr)$ | –15.735 |
| BF3 = $BF1^* \max(0, r - 0.52)$ | 1.5606 |
| BF4 = $\max(0, 100 - Pr)$ | –0.11845 |
| BF5 = $\max(0, Pr - 100)^* \max(0, r - 0.54)$ | 0.64371 |
| BF6 = $\max(0, Pr - 100)^* \max(0, 0.54 - r)$ | –0.43587 |
| BF7 = $\max(0, Pr - 1)$ | –3.321 |
| BF8 = $BF7^* \max(0, r - 0.52)$ | 10.02 |
| BF9 = $BF7^* \max(0, 0.52 - r)$ | –11.848 |
| BF10 = $\max(0, Pr - 0.1)$ | –11.82 |
| BF11 = $BF10^* \max(0, r - 0.54)$ | –12.267 |
| BF12 = $BF10^* \max(0, 0.54 - r)$ | 12.172 |
| BF, basis function; MARS, multivariate adaptive regression splines; Nu_{avg} , average Nusselt number; Pr, Prandtl number. | |

The regression equation of Nu_{avg} , which is a function of “ r ” and “Pr”, can be generated easily using Table 2 and Eq. (3) as follows:

$$\begin{aligned}
 Nu_{avg} = & 170.03 + 14.917 \times BF1 - 15.735 \times BF2 + 1.5606 \times BF3 \\
 & - 0.11845 \times BF4 + 0.64371 \times BF5 - 0.43587 \times BF6 \\
 & - 3.321 \times BF7 + 10.02 \times BF8 - 11.848 \times BF9 \\
 & - 11.82 \times BF10 - 12.267 \times BF11 + 12.172 \times BF12
 \end{aligned}
 \quad (12)$$

Using Eq. (12), Nu_{avg} can easily be predicted; the variation between the predicted data and the actual data is presented in Fig. 1.

5.2. Comparing MARS-based Nu_{avg} with those of GEP and ANN models

Further, to evaluate the capability of the MARS model to estimate the heat transfer characteristics around a cylinder, its outcomes are compared with those of ANN and GEP models [32]. The ANN and GEP models in the authors' previous work [32] were trained and tested for different Re and Pr values, but in Table 3 of that work, the models were tested for different Re values at Pr = 0.7 and for different Pr values at Re = 100. The MAPE values of the ANN and GEP models, as given in that table, have been considered for comparison with the corresponding values of the present MARS model. The authors

have also checked the ANN and GEP models only with different Pr values at Re = 100 values, and the deviation of the Adj.R² values between the two replicas is very negligible (<0.3%). The error between the prediction efficiencies of different models is examined by the statistical data Adj.R² and MAPE, as presented in Table 4.

It clearly shows that the MARS model is a more efficient prediction tool than either of the two remaining tools, having an Adj.R² value of 0.99999. The prediction accuracy of the MARS model is higher than that of GEP and ANN for different Pr conditions. The MARS model also has the least MAPE for the present model.

5.3. Optimization of heat transfer

Once the objective function is generated by MARS, it is optimized using the PSO algorithm. The PSO algorithm is written in MATLAB to maximize the objective function. A number of trials were performed to obtain the least numbers of populations, generations, and various parameters adequate for PSO. The algorithm and different parameters used in PSO are encapsulated in Table 5. The optimized result provided by the PSO model is $Nu_{avg} = 36.93294$ at $r = 0.52$ and Pr = 1,000. Then, a confirmatory test was accomplished by the FVM-based code on the geometry of $r = 0.52$ at Pr = 1,000, and the value of Nu_{avg} was found to be 36.59598. However, the maximum heat transfer rate was already achieved at $r = 0.51$ and Pr = 1,000, which is $Nu_{avg} = 37.68012$. Therefore, the PSO model was slightly modified to skip the data of $r = 0.52$, and then the model was rebuilt to achieve the next optimized result. The best and mean fitness of Nu_{avg} optimization calculated by PSO are presented in Fig. 2. The optimized value of the corner radius calculated by the PSO code is compared with the FVM-calculated data [32] and tabulated in Table 3. It is noticeable that PSO can optimize the objective function minutely to give the optimized parameters, but totally relies on the efficiency of the predicted objective function and appropriateness of the MARS model. As a MARS model is just a regression model so it does not take the physics into account in itself. If the data set is not enough and not fully representative of the phenomenon, the prediction of the MARS model is associated with more uncertainty.

6. Conclusion

In this study, the efficiency of MARS in predicting heat transfer characteristics has been investigated. Further, present models were also examined with existing GEP and ANN models. All the prediction models were developed to predict the average Nusselt number “ Nu_{avg} ” for different corner radii of a square cylinder (“ r ”) and Prandtl number “Pr.” The models were trained and tested with the data collected from the authors' recently published article. The MARS model is more efficient than ANN and GEP having an Adj.R² of 0.99999 for different Pr models at a constant Re value of 100.

Further, this predicted model is used in the PSO tool as the objective function for maximizing the heat transfer rate. The

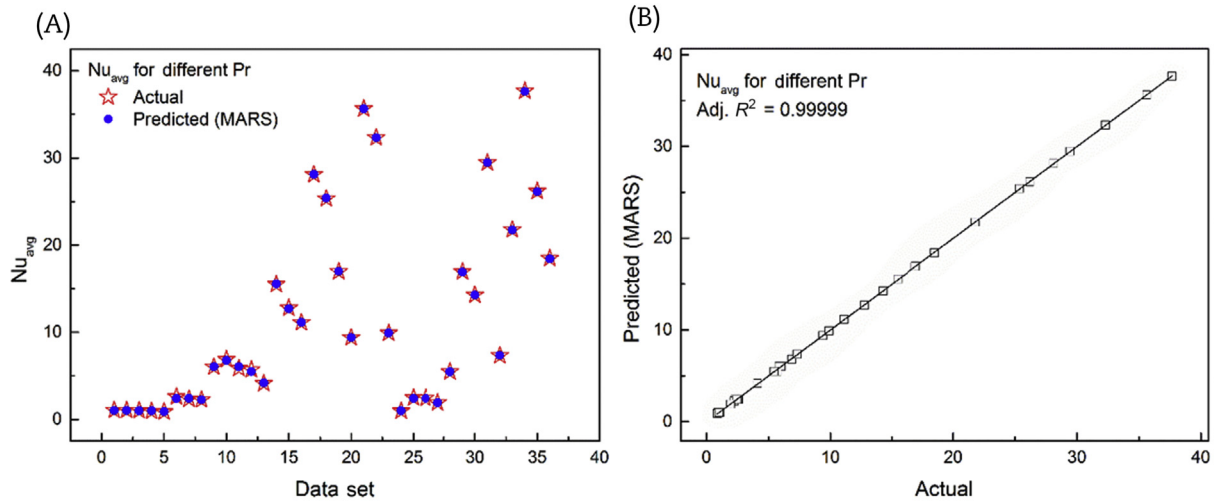


Fig. 1 – Variation between the predicted and actual data for different Pr values by the MARS model. (A) The discrepancy between the actual and predicted data. (B) Fitting line plot between the actual and predicted data. Adj.R², adjusted R²; MARS, multivariate adaptive regression splines; Nu_{avg}, average Nusselt number; Pr, Prandtl number.

| Table 3 – Optimization results. | | | |
|---------------------------------|-------------------|-------------------------------|------|
| Corner radius (r) at Pr = 1,000 | Nu _{avg} | Percentage variation with FVM | |
| FVM [32] | 0.51 | 37.68012 | |
| PSO | 0.51 | 36.35647 | 3.51 |

FVM, finite volume method; Nu_{avg}, average Nusselt number; PSO, particle swarm optimization.

| Table 4 – Calculated values of Adj.R ² and MAPE for different models. | | | | | |
|--|-----|------------------------------|----------|--------------------|-------|
| r | Re | Pr | Model | Adj.R ² | MAPE |
| 0.50, 0.51, 0.54, 0.59, 0.64, 0.71 | 100 | 0.01, 0.1, 1, 10, 100, 1,000 | ANN [32] | 0.98263 | 3.354 |
| | | | GEP [32] | 0.99997 | 1.248 |
| | | | MARS | 0.99999 | 1.159 |

Adj.R², adjusted R²; ANN, artificial neural network; GEP, gene expression programming; MAPE, mean absolute percentage error; MARS, multivariate adaptive regression splines; Pr, Prandtl number; Re, Reynolds number.

| Table 5 – Parameters used in PSO. | |
|-----------------------------------|-------|
| Parameter | Value |
| Population size | 40 |
| Generation | 500 |
| W | 0.9 |
| c ₁ | 1.25 |
| c ₂ | 0.5 |
| Swarm velocity | 1 |

PSO, particle swarm optimization.

PSO-generated optimized cylinder corner radius is 0.51 at Pr = 1,000; comparing this value with FVM, it has been found that the PSO tool can efficiently maximize the heat transfer rate.

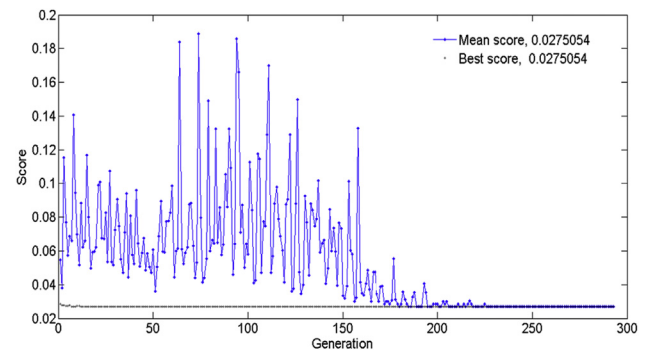


Fig. 2 – Optimization of the objective function using PSO. PSO, particle swarm optimization.

Thus, for a simple and easily understandable regression relation between a small amount of input and output data, MARS can be utilized for predicting the heat transfer characteristics where ANN has failed to achieve marked accuracy. Therefore, the MARS and PSO codes can be applied to different heat transfer problems in different nuclear engineering areas for efficient prediction and optimization, and can also be useful for various scientific applications where energy management plays a vital role in improving the energy economy.

Conflicts of interest

The authors announced that there are no conflicts of interest.

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