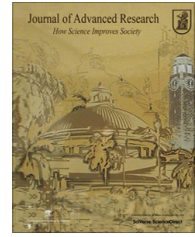




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## ORIGINAL ARTICLE

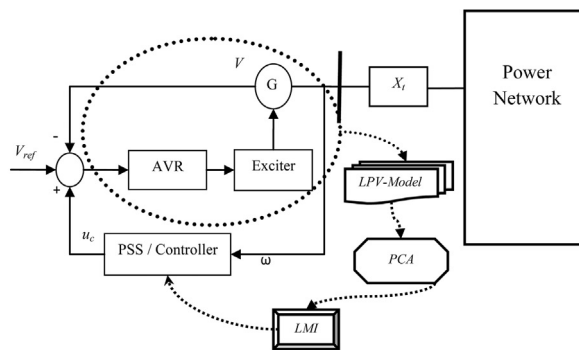
# A new LPV modeling approach using PCA-based parameter set mapping to design a PSS



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## GRAPHICAL ABSTRACT



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## ABSTRACT

This paper presents a new methodology for the modeling and control of power systems based on an uncertain polytopic linear parameter-varying (LPV) approach using parameter set mapping with principle component analysis (PCA). An LPV representation of the power system dynamics is generated by linearization of its differential-algebraic equations about the transient operating points for some given specific faults containing the system nonlinear properties. The time response of the output signal in the transient state plays the role of the scheduling signal that is used to construct the LPV model. A set of sample points of the dynamic response is formed to

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**Keywords:**

Power system stabilizer  
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generate an initial LPV model. PCA-based parameter set mapping is used to reduce the number of models and generate a reduced LPV model. This model is used to design a robust pole placement controller to assign the poles of the power system in a linear matrix inequality (LMI) region, such that the response of the power system has a proper damping ratio for all of the different oscillation modes. The proposed scheme is applied to controller synthesis of a power system stabilizer, and its performance is compared with a tuned standard conventional PSS using nonlinear simulation of a multi-machine power network. The results under various conditions show the robust performance of the proposed controller.

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**Introduction**

The current electric power systems have been operated close to their capacity limits, thus increasing the instability risk. Small-signal stability can be defined as the ability of the system to maintain synchronism when subjected to small disturbances. With this stability concept, the probable instability can be of two forms: a steady increase in the generator rotor angle caused by the lack of synchronizing torque and an increase in the amplitude of rotor oscillations caused by the lack of sufficient damping torque [1]. Currently, small-signal instability occurs more frequently because of the latter form of instability. Dynamic stability can be defined as the behavior of the power system when subjected to small disturbances. It usually involves insufficient or poor damping of system oscillations. These oscillations are undesirable, even at low frequencies, because they reduce power transfer in transmission lines. The most important types of these oscillations are local-mode (which occurs between one machine and the rest of the system) and inter-area mode oscillations (which occurs between interconnected machines) [2]. Thus, our main objective in this paper was to propose a suitable methodology for overcoming the undesired oscillations.

A power system stabilizer (PSS) is used to provide positive damping of the power system oscillations. The conventional PSS design involves producing a component of electrical torque in phase with rotor speed deviations. In the literature [3], the effects of some PSS schemes on improving power system dynamic performance have been analyzed. Generally, it is possible to categorize PSS design methodologies as follows: (a) classical methods, (b) adaptive and variable structure methods, (c) robust control approaches, (d) artificial intelligent techniques and (e) digital control schemes [4].

It is probable that conventional PSS (CPSS) fails to dampen system oscillations over a wide range of operating conditions or at least leads to a dishonored performance. Consequently, a priority of robust PSSs is to address a variety of uncertainties imposed by plausible variation in operating points, and it is important to have proper performance for different load conditions while ensuring stability [5]. Therefore, the robustness of the PSS is a major issue [6], and synthesis of robust PSSs has been one of the most notable research topics in power and control engineering. In many research studies, such as literature reports [7–10], robust performance of a controller in various operating points has been studied and investigated. Over the past years, several methods and approaches have been presented regarding robust control in power systems, especially for oscillation damping [11–13].

Various robust control techniques can be used in the design stage, for example,  $H_\infty$  optimal control [14] and Linear Matrix

Inequality (LMI) [15]. The basic theories and some applicable techniques of robust control in power systems can be found in the literature [16]. Most conventional techniques for the design of a PSS are based on linearized models. The robustness of the designed PSSs is limited because of operating point variations resulting from the linearized model being valid only in the neighborhood of the operating point used for linearization. A polytopic model is an effective solution to this problem [17].

One special issue to address with nonlinear dynamical systems, which has received significant attention, is the issue of linear systems, where the dynamics are described by some combination of linear subsystems. The main reason for this interest may be the efficiency of linear systems in developing the control concepts in an uncomplicated fashion. This matter led to the tendency to form hybrid, linear parameter-varying (LPV) and polytopic linear models. The stability problem for polytopic linear systems still remains a challenging research topic [18–21]. Many research studies have focused on facilitating the implementation of the fundamental results obtained previously regarding the asymptotic stability of a certain class of interconnected systems via switched linear systems [22–26].

To overcome all of the perturbation from parameter uncertainties and nonlinearity effects due to operating point variation of the power system, construction of polytopic linear models based on the LPV framework is proposed in our paper. In Hoffmann and Werner [27], a complete survey of the experimental results in LPV control was provided. It briefly reviewed and compared some of the different LPV controller synthesis techniques. The methods are categorized as polytopic, linear fractional transformation and gridding based techniques; in each of these approaches, synthesis was found to be achieved via LMIs. LPV models are known as linear state-space models with time-varying parameter-dependent matrices. Their dynamics are linear, but non-stationary [28]. In fact, LPV models of a nonlinear system describe its nonlinearities by parameter variations. This point of view is relatively straightforward for system descriptions, especially when the system variations are state-dependent, e.g., power system dynamics.

In this paper, the nonlinearity of the power system dynamics is considered in the control designing process via the LPV method [29–32]. A common approach for LPV modeling of nonlinear systems is using a set of simulation data obtained from the original nonlinear model [33]. It is assumed that this set of data sufficiently captures the transient behavior of the system. Thus, the main concept is the construction of a polytopic model of a power system using transient response samples that contain the nonlinear properties of the power system. Next, parameter set mapping based on the PCA proposed in the literature [32] is used to obtain LPV models with

a tighter parameter set. In addition, the less significant directions in the parameter space are detected and neglected without losing much information regarding the plant.

This note is organized as follows. In the next section, an LPV model for a power system is introduced. Parameter set mapping and the problem statement are presented in Section ‘Parameter set mapping and problem formulation’. In Section ‘Controller design’, the proposed algorithm to verify the stability conditions is described for the family of systems considered in the polytopic based model. In Section ‘Simulation’, a discussion is provided on the applicability of the proposed controller and a comparison is made between the robustness of the proposed controller with a tuned conventional standard PSS in a simple model and a multi-machine power system. Finally, conclusions are presented in Section ‘Conclusion’.

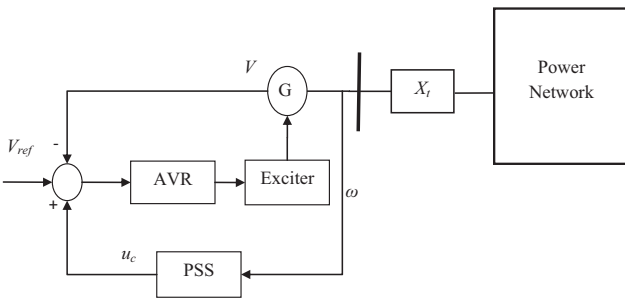
### LPV modeling

The dynamic behavior of a power system is affected by its complex components (generators, exciters, transformers, etc.), which are coupled with the network model. The linear behavior of the system can be expected in steady-state operational points. However, the nonlinearity of the system is very obvious whenever a fault or a disturbance specifically occurs in transient behavior. The objective in this section was to introduce an LPV-model based on transient operating points of the system to account for nonlinearity effects and uncertainties. The mathematical model of the power system can be represented by two sets of equations [34]: one set of differential equations (consisting of state variables) and one set of algebraic equations (for the other variables), as

$$\begin{aligned} \dot{x} &= f(x, \xi, u) \\ 0 &= g(x, \xi), \end{aligned} \quad (1)$$

where  $x \in \mathfrak{R}^n$  is the vector of the state variables,  $\xi \in \mathfrak{R}^q$  is the vector of the (non-state) network variables (such as load flow variables) and  $u \in \mathfrak{R}^p$  is the vector of control inputs (such as the reference signal of Automatic Voltage Regulator (AVR) called  $V_{ref}$ ). In particular, the vector  $x$  contains the state variables of generators and controllers (AVR, PSS, etc.). Fig. 1 shows the configuration of a power system for which the state variables of the generator are as follows: excitation flux  $\psi_e$ , flux in D-Damper winding  $\psi_D$ , flux in Q-Damper winding  $\psi_Q$ , rotor speed  $\omega$  in p.u. and rotor position angle  $\delta$  in rad.

It is assumed that the functions  $f(x, \xi, u)$  and  $g(x, \xi)$  are continuously differentiable for a sufficient number of times.



**Fig. 1** A power system configuration with detailed connections of a generator.

Solution of (1) for a specific control input  $\bar{u}(t)$  is presented by vectors of  $\bar{x}$  and  $\bar{\xi}$ , and  $\rho(t)$  is defined below as the power system transient trajectory:

$$\rho(t) := \begin{bmatrix} \bar{x}(t) \\ \bar{\xi}(t) \\ \bar{u}(t) \end{bmatrix}. \quad (2)$$

If it is considered that

$$\begin{aligned} x &= \bar{x} + \delta x \\ \xi &= \bar{\xi} + \delta y \\ u &= \bar{u} + \delta u, \end{aligned} \quad (3)$$

then function  $f(x, \xi, u)$  can be approximated by linear Taylor expansion with respect to its components. In fact, the power system dynamics in the immediate proximity of the transient trajectory  $(\bar{x}, \bar{\xi}, \bar{u})$  are approximated by the first terms of the Taylor series. Thus, the following LPV model  $P(\theta)$  can be introduced for the power system about the transient trajectory,

$$\begin{aligned} \delta \dot{x} &= A(\theta(t))\delta x + B(\theta(t))\delta u \\ \delta y &= C(\theta(t))\delta x + D(\theta(t))\delta u \end{aligned} \quad (4)$$

where

$$A(\theta(t)) := \left[ \frac{\partial f}{\partial x} - \frac{\partial f}{\partial \xi} \left( \frac{\partial g}{\partial \xi} \right)^{-1} \frac{\partial g}{\partial x} \right]_{\substack{x=\bar{x} \\ \xi=\bar{\xi} \\ u=\bar{u}}}, \quad (5)$$

$$B(\theta(t)) := \left[ \frac{\partial f}{\partial u} \right]_{\substack{x=\bar{x} \\ \xi=\bar{\xi} \\ u=\bar{u}}}, \quad (6)$$

and  $\delta y \in \mathfrak{R}^m$  is the deviation vector of defined output variables about its transient trajectory  $\bar{y}$ . The time-dependent parameter vector  $\theta(t) \in \mathfrak{R}^l$  depends on the vector of measurable signals  $\rho(t) \in \mathfrak{R}^k$ , where  $k := n + p + q$  is referred to as scheduling signals, according to

$$\theta(t) = h(\rho(t)), \quad h: \mathfrak{R}^k \rightarrow \mathfrak{R}^l, \quad (7)$$

where the parameter function  $h$  is continuous mapping. Without a loss of generality, it can be assumed that  $D(\cdot) = 0$ . However, this assumption is not implausible in power systems. The matrix  $C(\cdot)$  can be computed when the desired output variables are defined. In fact, the power system transient trajectory  $\rho(t)$  may be interpreted as a time-varying scheduling signal vector for the mappings  $A(\cdot)$  and  $B(\cdot)$ . The compact set  $P_\theta \subset \mathfrak{R}^l: \theta \in P_\theta, \forall t > 0$  is considered to be a polytopic set defined by the convex hull

$$P_\theta := Co\{\theta_{v1}, \theta_{v2}, \dots, \theta_{vN}\} \quad (8)$$

where  $N$  is the number of vertices. It follows that the system can be represented by a linear combination of LTI models at the vertices; this is called a polytopic LPV system

$$P(\theta) \in Co\{P(\theta_{v1}), P(\theta_{v2}), \dots, P(\theta_{vN})\} = \sum_{i=1}^N \alpha_i P(\theta_{vi}) \quad (9)$$

where  $\sum_{i=1}^N \alpha_i = 1$  and  $\alpha_i \geq 0$  are the convex coordinates. The  $i$ th vertex of this convex polytope is defined by  $P_i := (A_i, B_i, C_i)$  for  $i = 1, 2, \dots, N$ , where each of these matrices is constant.

Each model is computed at some transient operating points that are assigned at predefined time intervals in system transient trajectory. The number of points is chosen relative to the system operating range, transient response and nonlinearity effects.

### Parameter set mapping and problem formulation

In this section, parameter set mapping based on the PCA algorithm is used to find tighter regions in the space of the scheduling parameters. By neglecting insignificant directions in the mapped parameter space, approximations of LPV models are achieved that will lead to a less conservative controller synthesis [32]. For the given LPV system (4) and a set of trajectories of typical scheduling signals  $\rho(t)$ , the problem of parameter set mapping can be summarized to find a mapping

$$\phi(t) = r(\rho(t)), \quad r: \mathfrak{R}^k \rightarrow \mathfrak{R}^s \quad (10)$$

where  $s \leq l$ , such that the model

$$\begin{aligned} \delta \dot{x} &= \widehat{A}(\phi(t))\delta x + \widehat{B}(\phi(t))\delta u \\ \delta y &= \widehat{C}(\phi(t))\delta x + \widehat{D}(\phi(t))\delta u \end{aligned} \quad (11)$$

provides a sufficient approximation of (4). The basic details of PCA can be found in [33]. The sampling data at time instants  $t = 1, 2, \dots, N$  can be used to generate a  $l \times N$  data matrix

$$\Xi = [\theta_1, \theta_2, \dots, \theta_N]. \quad (12)$$

The rows  $\Xi_i$  are normalized by an affine law  $\Pi_i$  to generate scaled data with a zero mean and unit standard deviation

$$\Xi_i^n = \Pi_i(\Xi_i), \quad \Xi_i = \Pi_i^{-1}(\Xi_i^n), \quad (13)$$

and normalized data matrix  $\Xi^n = \Pi(\Xi)$ . Next, the following singular value decomposition

$$\Xi^n = [\widehat{U}^T \quad U^T] = \begin{bmatrix} \widehat{\Sigma} & 0 & 0 \\ 0 & \Sigma & 0 \end{bmatrix} \begin{bmatrix} \widehat{V} \\ V \end{bmatrix} \quad (14)$$

yields  $s$  significant singular values corresponding to  $\widehat{U}$ ,  $\widehat{\Sigma}$ , and  $\widehat{V}$ . Neglecting less significant singular values leads to

$$\widehat{\Xi}^n = \widehat{U}\widehat{\Sigma}\widehat{V}^T \approx \Xi^n \quad (15)$$

such that  $\widehat{\Xi}^n$  is an approximation of the given data, and the matrix  $\widehat{U}$  as a basis of the significant column space of the data matrix  $\Xi^n$  can be used to obtain the reduced mapping  $r$  from  $\rho(t)$  to  $\phi(t)$  by computing

$$\phi(t) = r(\rho(t)) = \widehat{U}^T \Pi(h(\rho(t))) = \widehat{U}^T \Pi(\theta(t)) \quad (16)$$

In other words, the approximate mapping of  $\widehat{A}(\cdot)$ ,  $\widehat{B}(\cdot)$ ,  $\widehat{C}(\cdot)$ ,  $\widehat{D}(\cdot)$  in (11) is related to (4) by

$$\widehat{P}(\phi) = \begin{bmatrix} \widehat{A}(\phi(t)) & \widehat{B}(\phi(t)) \\ \widehat{C}(\phi(t)) & \widehat{D}(\phi(t)) \end{bmatrix} = \begin{bmatrix} A(\hat{\theta}(t)) & B(\hat{\theta}(t)) \\ C(\hat{\theta}(t)) & D(\hat{\theta}(t)) \end{bmatrix} \quad (17)$$

where

$$\hat{\theta}(t) = \Pi^{-1}(\widehat{U}\phi(t)) = \Pi^{-1}(\widehat{U}\widehat{U}^T \Pi(\theta(t))) \quad (18)$$

and  $\Pi^{-1}$  denotes row-wise rescaling. Thus, the polytopic LPV system (9) is reduced to the following polytopic LPV system with  $S = 2^S$  vertices.

$$\widehat{P}(\hat{\theta}) \in \text{Co}\{\widehat{P}(\hat{\theta}_{v_1}), \widehat{P}(\hat{\theta}_{v_2}), \dots, \widehat{P}(\hat{\theta}_{v_S})\} = \sum_{i=1}^S \alpha_i \widehat{P}(\hat{\theta}_{v_i}) \quad (19)$$

The quality of the approximation can be measured by the fraction of the total variation  $v_s$ , which is determined by the singular values in (14) as

$$v_s = \frac{\sum_{i=1}^s \sigma_i^2}{\sum_{i=1}^l \sigma_i^2} \quad (20)$$

**Definition 3.1** (*LMI Region* [35]). A subset  $D$  of the complex plane is called an LMI region if there is a symmetric matrix  $L = L^T \in \mathfrak{R}^{m \times m}$  and matrix  $M \in \mathfrak{R}^{m \times m}$  such that

$$D = \{z \in C : f_D(z) < 0\} \quad (21)$$

with

$$f_D(z) = L + zM + \bar{z}M^T. \quad (22)$$

Subset  $D$  defines a region in the complex plane that has certain geometric shapes, such as disks, vertical strips, and conic sectors. A ‘conic sector’ with inner angle  $\alpha$  and an apex at the origin is an appropriate region for power system applications as it ensures a minimum damping ratio  $\zeta_{\min} = \cos \frac{\alpha}{2}$  for the closed-loop poles [36]. This LMI region has a characteristic function given by

$$f_z(z) = \begin{bmatrix} \sin \frac{\alpha}{2}(z + \bar{z}) & \cos \frac{\alpha}{2}(z - \bar{z}) \\ \cos \frac{\alpha}{2}(\bar{z} - z) & \sin \frac{\alpha}{2}(z + \bar{z}) \end{bmatrix}. \quad (23)$$

**Theorem 3.2** (*(D-stability)* [35]). The matrix  $A$  is  $D$ -stable if and only if there is a symmetric matrix  $X$  such that

$$M_D(A, X) < 0, \quad X > 0, \quad (24)$$

where  $M_D(A, X)$  is an  $m \times m$  block matrix defined as

$$M_D(A, X) := L \otimes X + M \otimes (AX) + M^T \otimes (AX)^T. \quad (25)$$

and  $\otimes$  denotes the Kronecker product.

From this theorem, matrix  $A$  has its poles in an LMI region with characteristic function (23) if and only if  $X > 0$  such that

$$\begin{bmatrix} \sin \frac{\alpha}{2}(AX + XA^T) & \cos \frac{\alpha}{2}(AX - XA^T) \\ \cos \frac{\alpha}{2}(XA^T - AX) & \sin \frac{\alpha}{2}(AX + XA^T) \end{bmatrix} < 0. \quad (26)$$

Here, the objective was to find a control law

$$\delta u = K(s)\delta y, \quad (27)$$

for the LPV model (11) as a robust PSS such that the closed-loop poles lie in region  $D$ .

### Controller design

In this section, the design procedure of the control law (27) for the LPV model (11) is described and a sufficient condition to ensure the asymptotic stability for system (11) is given by using the proposed controller. The linear time varying system (4) describes the nonlinear dynamic of the power system (1) about the system transient trajectory  $\rho(t)$ . Applying parameter set

mapping based on a PCA algorithm to (4), the reduced LPV model (11) will be achieved. Thus, a polytopic model with vertices  $(\hat{A}_i, \hat{B}_i, \hat{C}_i)$  is obtained that is computed by implementing the PCA algorithm on the initial LPV model after evaluating the power system transient trajectory  $\rho(t)$  in  $N$  distinct transient operating points. The objective was to find the controller  $K(s)$ , as a robust PSS, such that the poles of a closed-loop system given by (11) and (27) lie in defined region  $D$ . Suppose that the state-space representation of the LTI controller  $K(s)$  is given by

$$\begin{aligned}\dot{x}_k(t) &= A_k x_k(t) + B_k \delta y \\ u(t) &= C_k x_k(t) + D_k \delta y.\end{aligned}\quad (28)$$

Implementing controller (28) to the LPV model (11), the following closed-loop state-space equation is obtained:

$$\dot{x}_{cl} = A_{cl}(\hat{\theta}(t)) x_{cl} \quad (29)$$

where  $x_{cl} := [\delta x^T \ x_k^T]^T$  is the vector of closed loop system state variables and

$$A_{cl}(\hat{\theta}(t)) := \begin{bmatrix} A(\hat{\theta}(t)) + B(\hat{\theta}(t))D_k C(\hat{\theta}(t)) & B(\hat{\theta}(t))C_k \\ B_k C(\hat{\theta}(t)) & A_k \end{bmatrix}. \quad (30)$$

Using polytopic representation (19), the closed loop system (29) can be rewritten as

$$\dot{x}_{cl} = \sum_{i=1}^S \alpha_i \hat{A}_{cli} x_{cl} \quad (31)$$

where  $\hat{A}_{cli}$  is the closed loop system matrix of the  $i$ th model  $\hat{P}_i := (\hat{A}_i, \hat{B}_i, \hat{C}_i)$  in the form of

$$\hat{A}_{cli} := \begin{bmatrix} \hat{A}_i + \hat{B}_i D_k \hat{C}_i & \hat{B}_i C_k \\ B_k \hat{C}_i & A_k \end{bmatrix}. \quad (32)$$

Here, the problem is to find  $X > 0$  and a controller  $K(s)$ , as described in (28), that satisfy

$$M_D(\hat{A}_{cli}, X) < 0. \quad (33)$$

This is a regular pole placement problem for which the solution can be followed from [35]. A change of controller variables is necessary to convert the problem into a set of LMIs. Partition  $X$  and its inverse are given by

$$X = \begin{bmatrix} R & T \\ T^T & U \end{bmatrix}, \quad X^{-1} = \begin{bmatrix} S & N \\ N^T & V \end{bmatrix}. \quad (34)$$

Thus, the new controller variables for each vertex are defined as

$$\begin{aligned}\hat{A}_k &= N A_k T^T + N B_k \hat{C}_i R + S \hat{B}_i C_k T^T + S(\hat{A}_i + \hat{B}_i D_k \hat{C}_i) R, \\ \hat{B}_k &= N B_k + S \hat{B}_i D_k, \\ \hat{C}_k &= C_k T^T + D_k \hat{C}_i R, \\ \hat{D}_k &= D_k.\end{aligned}\quad (35)$$

Note that, in this study,  $\hat{D}_i = 0$  for all  $i = 1, \dots, S$ ; thus,  $\hat{D}_k = D_k = 0$ . If  $T$  and  $N$  have a full row rank, then the controller variables  $(A_k, B_k, C_k)$  can always be computed from (35). Moreover, the controller variables can be determined uniquely if the controller order is chosen to be equal to the order of the plant, that is, when  $T$  and  $N$  are square invertible.

A challenging point is the uniqueness of the solution of (35) if the objective was to have an unique controller for all vertices. There are no difficulties in determining  $B_k$  and  $C_k$  because, according to (35), they do not depend on the parameters of the vertices, whereas the computation of  $A_k$  is dependent on these parameters and may explicitly cause different solutions at each vertex. As will be shown in the simulation results, in spite of the difference in solutions, because of the LMI region restriction for each vertex, the poles of the closed loop system with the resulting controllers all lie in the desired region. Therefore, the matrix  $A_k$  can be achieved by solving (35) at any arbitrary vertex. However, for taking an optimal solution with a minimum error norm, the use of the average values of matrices in all of the vertices is recommended, that is, using  $\frac{1}{S} \sum_{i=1}^S (\hat{A}_i, \hat{B}_i, \hat{C}_i)$  instead of  $(\hat{A}_i, \hat{B}_i, \hat{C}_i)$  in (35).

Next, using (35) and some matrix algebraic manipulations, the following set of LMIs is obtained to find a solution for (33).

Find  $R = R^T$ ,  $S = S^T$ , and matrices  $(\hat{A}_k, \hat{B}_k, \hat{C}_k)$  such that

$$\begin{bmatrix} R & I \\ I & S \end{bmatrix} > 0, \quad (36)$$

$$\begin{bmatrix} \sin \frac{\alpha}{2} (\Phi_i + \Phi_i^T) & \cos \frac{\alpha}{2} (\Phi_i - \Phi_i^T) \\ \cos \frac{\alpha}{2} (\Phi_i^T - \Phi_i) & \sin \frac{\alpha}{2} (\Phi_i + \Phi_i^T) \end{bmatrix} < 0, \quad (37)$$

for  $i = 1, \dots, S$ , where

$$\Phi_i = \begin{bmatrix} \hat{A}_i R + B_i \hat{C}_k & \hat{A}_i \\ \hat{A}_k & S \hat{A}_i + \hat{B}_k \hat{C}_i \end{bmatrix}. \quad (38)$$

## Simulation

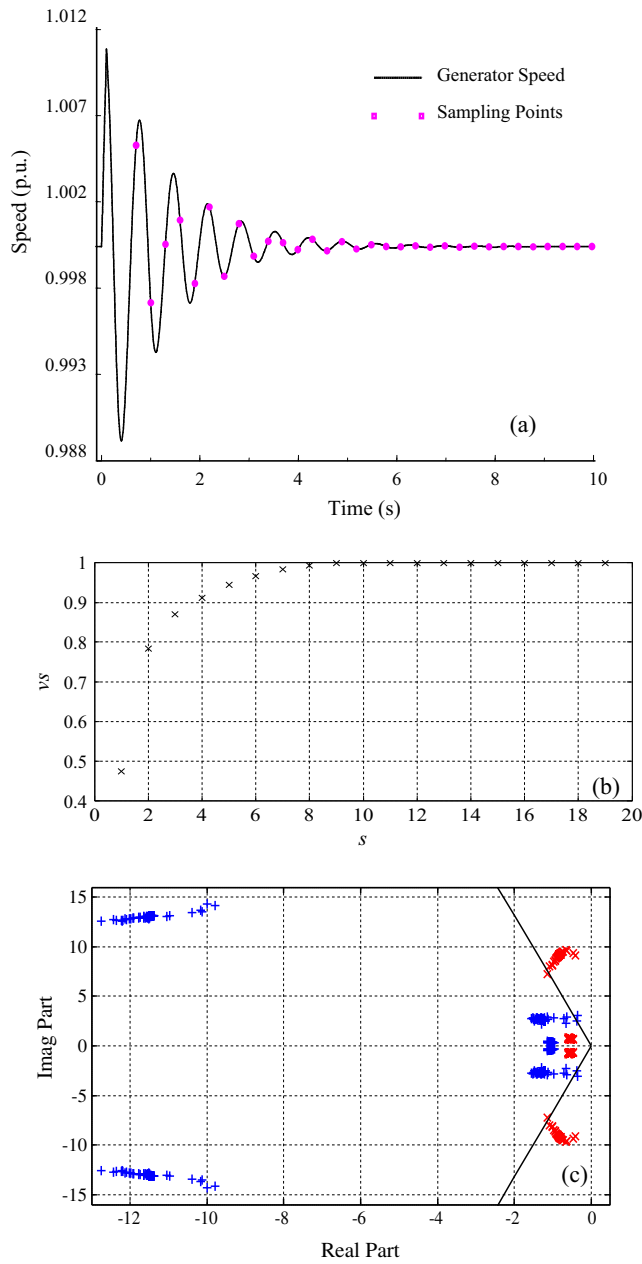
In this part, two case study problems are considered. First, the proposed design method is simulated and applied to a simple model of a single machine connected to an infinite busbar, and then, the resultant controller is evaluated using a multi-machine power system model. The simulation results are compared with a tuned conventional power system stabilizer.

### Simulation of a simplified power system model

In the following, a simplified power system is considered for implementing the proposed scheme and investigating the stability behavior and performance of the closed loop system subjected to nonlinearity, disturbance and operation condition variation.

To show the procedure of proposed controller design and evaluate the efficiency of the results, particularly through the use of nonlinear simulations, a practical and simple model of a 612 MVA power system from [37] is studied. The system contains a generator connected to an infinite busbar and equipped with a standard excitation system EXST3 and standard PSS structure IEEEEST [37]. Simulations are performed using DIG-SILENT PowerFactory software.

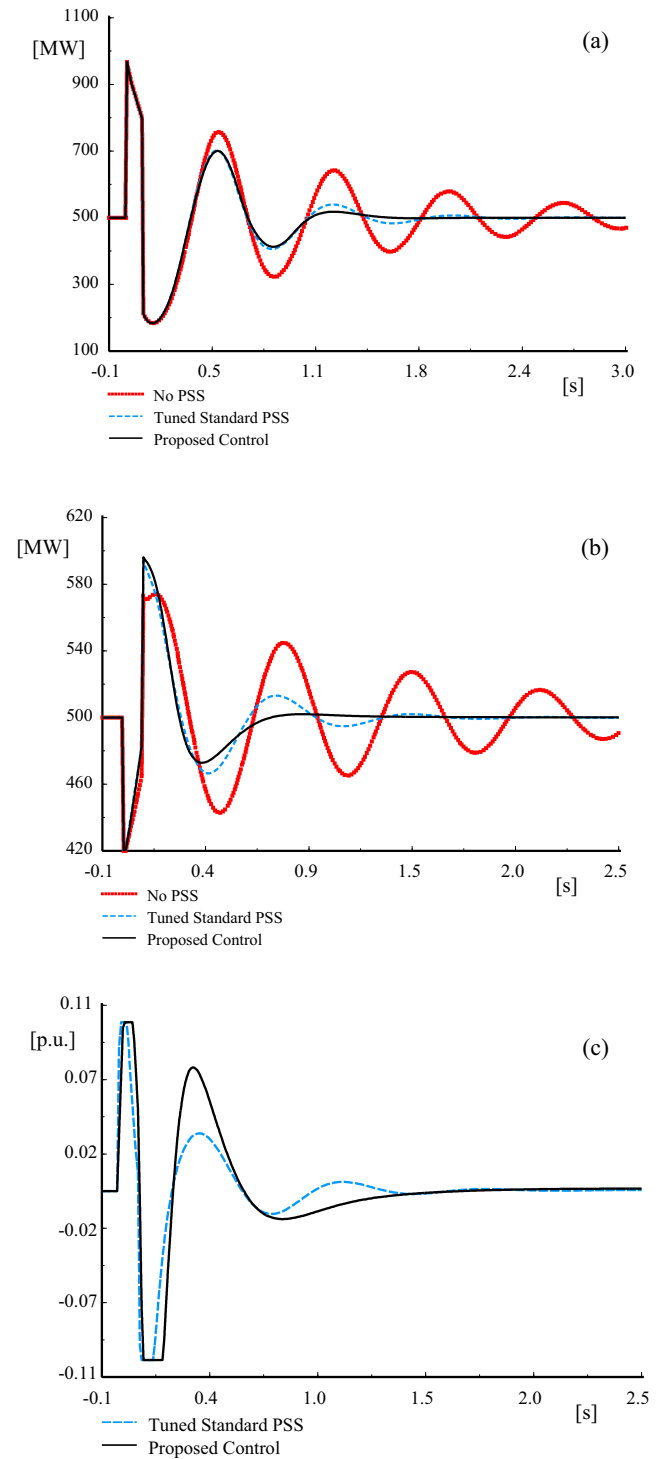
The objective was to design the proposed controller for damping control of oscillations in the power system, which is shown in Fig. 1 as the PSS block. Thus,  $u_c$ , the output of the



**Fig. 2** (a) Sample transient points of the power system; (b) fraction of total variation and (c) open and closed loop system poles for all sample models (red: no control, blue: with proposed control).

controller, and  $\omega$ , the speed of generator, are used as the input and output, respectively, of the system under study for constructing the LPV model. For extracting the initial LPV model (4), it is possible to use the response of the power system without PSS after a 3-phase short-circuit fault at the generator busbar (at  $t = 0$  sec with 100 ms clearing time).

In the nominal steady state operating point similar to the base condition of Shin et al. [37], the unit is assumed to have loading conditions of 500 MW and 0.0 MVAR. Linearization is performed at each transient operating point for duration of 10 s after fault with 300 ms intervals, that is, a sampling rate



**Fig. 3** (a) Generator active power deviations after a 3-phase fault, in the base condition; (b) generator active power deviation after 1-phase switching in another operation condition and (c) control output after 1-phase switching in another operation condition.

of 3.33 Hz. Fig. 2a shows the samples on the time-domain simulation, where the initial polytopic models are generated in those transient operating points. The parameters of the generated LPV model (4) are reshaped in the form of data matrix  $\Xi$

in (12). Next, after data normalization, the explained PCA algorithm is used to construct the reduced LPV models. The singular value decomposition of the normalized data is computed as (14). To determine the number of required principal components, the fractions of the total variation  $v_s$  are plotted for 20 first singular values in Fig. 2b. As indicated in this figure, choosing  $s = 3$  implies that 87% of the information is captured. Thus, the resulting LPV model can be formulated as (19), which only has eight vertices in a parameter space with three dimensions. It has much less over-bounding than the original one, leading to a less conservative controller.

The reduced LPV model is used for the proposed controller synthesis described in Section ‘Controller design’. The objective of controller design was to improve the damping ratio  $\zeta$  of the oscillation modes to 15%. In other words, a conic sector of inner angle  $2 \cos^{-1}(0.15)$  with an apex at the origin is chosen as the desired pole region. For the open loop LPV models, the locations of the poles are shown in Fig. 2c. The LMIs (36) and (37) can be solved by choosing the controller order equal to the order of the plant. The resultant changed controller variables  $(\hat{A}, \hat{B}, \hat{C})$  are

$$\hat{A} = \begin{bmatrix} 97.86 & 2720.07 & 281.81 & 138.90 & -258.16 & 40.89 & 11.02 \\ 15.51 & 250.36 & 18.77 & 20.05 & -33.61 & 6.19 & 1.32 \\ 30.83 & 917.81 & 161.97 & -15.18 & 22.48 & -8.05 & 2.00 \\ 450.23 & -318.89 & -90.51 & 80.23 & 98.35 & 17.63 & 5.24 \\ -299.45 & 364.95 & 81.51 & -48.27 & -70.37 & -10.06 & -2.05 \\ -274.58 & 2117.46 & 348.22 & -273.96 & -136.66 & -72.56 & -17.90 \\ 437.23 & -2335.01 & -453.86 & 286.86 & 80.58 & 78.55 & 13.90 \end{bmatrix}$$

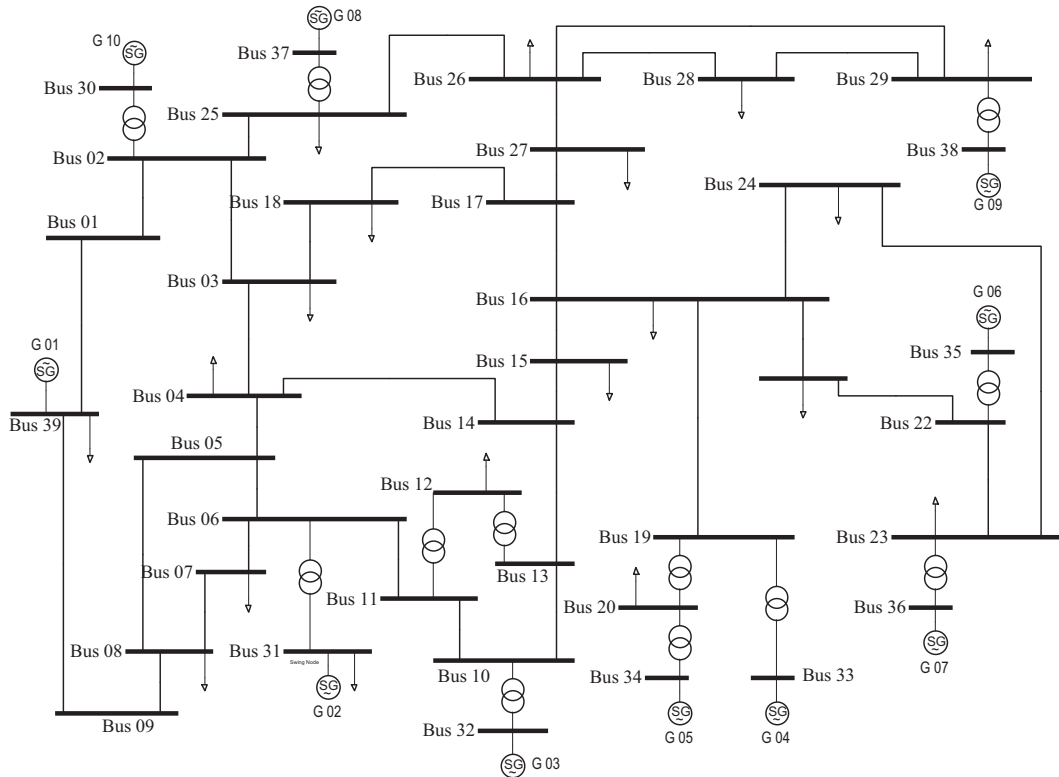


Fig. 4 39-Bus multi-machine power system configuration.

$$\hat{B} = \begin{bmatrix} -59128.44 \\ -35670.64 \\ -89321.55 \\ -665213.89 \\ -268022.81 \\ -95612.92 \\ -1695571.09 \end{bmatrix}$$

$$\hat{C} = \begin{bmatrix} -11957.80 & 14984.62 & 1019.93 & 165.59 \\ -30.23 & 11.02 & 9.74 \end{bmatrix}$$

The main controller variables ( $A_k, B_k, C_k$ ) can be found by solving (35). As stated before, the obtained matrix  $A_k$  may be different when using different vertices for solving (35); however, the resultant closed loop poles locations are not varied and laid in the desired region. This can be seen in Fig. 2c, where the desired damping ratio restriction is satisfied with all of the closed loop poles for all of the sample models.

The designed controller is applied to the power system. Next, its effectiveness is compared with a tuned standard conventional PSS (CPSS) proposed in the literature [37] and with the case of no PSS in the nominal condition (500 MW and 0.0 MVAR generation as the base of LPV model construction mentioned before). The cases are simulated in the time-domain using DIgSILENT PowerFactory software.

#### Results and analysis of simplified power system study

In this study, the limiters for proposed control are considered to be similar to CPSS in the literature [37]. Fig. 3a shows the generator response (active power) after a 3-phase fault on the connected busbar. In this condition, as shown in Fig. 3a, there is no significant difference between the effects of the proposed controller and the tuned standard CPSS because the design and tuning of CPSS were both performed under the same conditions.

To study the robustness of the proposed controller, especially in different situations, an asymmetrical event with a new initial condition is simulated. In this event, 500 MW and  $-180$  MVAR generation is considered, and phase "a" of the grid substation (infinite bus) is opened at  $t = 0$  sec and then closed at  $t = 0.1$  s. Fig. 3b shows the generator response (active power) for all of the predefined control conditions. The system with the proposed control clearly has a powerful robust performance against system variation and perturbation. For further comparison, the control signals of the controllers are also shown in Fig. 3c. The proposed controller with the same limits is clearly more effective for damping the oscillations, even under nonconventional operation conditions.

#### Simulation of a multi-machine 39-bus power system

In this part, a multi-machine power system is studied to illustrate the efficiency of the proposed controller and its robustness under different network conditions. The model consists of 39 buses (nodes), 10 generators, 19 loads, 34 lines and 12 transformers. Fig. 4 shows the single line diagram, which is a simplified model of the transmission system in the New England area in the northeast of the U.S.A. The simulation model, as represented in the Ref. [38], is used and modified slightly to test the proposed controller in comparison with the tuned standard PSS proposed in the literature [37].

Considering a nominal capacity approximation, generator G08 in the original model can be replaced by the 612 MVA generator studied in the previous section without a loss of generality and without any steady-state problems for system performance. This replacement is performed for using the LPV model extracted in the previous section. The excitation system for G08 is similar to the previous case. The proposed controller and the tuned standard CPSS are separately implemented on the generator and the performances are studied using DIgSILENT PowerFactory software. To prevent any interference, other generators are considered with no PSS.

#### Results and analysis of the multi-machine power system study

To evaluate the multi-machine system response, the events represented in Table 1 are investigated. Each event contains a 3-phase short-circuit fault, but the fault locations and pre- and post-fault conditions are different.

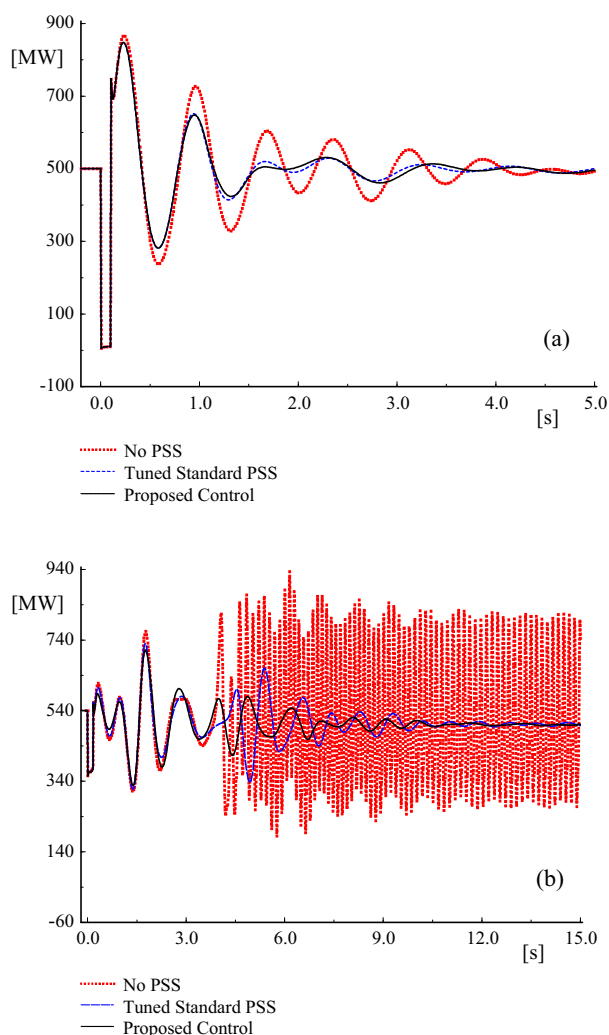
The generated active power of G08 is considered to be the system response after each event. In Fig. 5, all cases (without PSS, with tuned standard CPSS and with proposed controller) are studied and compared. Fig. 5a shows that the standard CPSS and proposed controller have satisfactory behaviors in the conditions of Event 1. Note that, in this event, because the system conditions are approximately similar to the proposed design and CPSS cases, the responses are found to be close to each other. Alternatively, to study the robustness of controllers, Event 2 is considered because it has different conditions. As shown in Fig. 5b, the system response is unstable in the case of no PSS and has undesired oscillations with tuned standard CPSS, while it has satisfactory damped oscillations with the proposed controller.

Therefore, the simulation results show that although the CPSS and proposed controller have the same behavior under basic conditions (where the CPSS is tuned), by altering the system conditions, the CPSS weakened, while the proposed controller had a suitable damped response and showed its robust properties against the system uncertainties.

**Table 1** Descriptions of events.

| Event no. | Pre-fault generation of G08 | Description  |
|-----------|-----------------------------|--|
| 1         | 500 MW & $-19$ MVAR         | Fault at Line 25–26 near Bus 25 at $t = 0$ and switching and outage of the line at $t = 0.100$ s |
| 2         | 540 MW & $-19$ MVAR         | Fault at Bus 17 at $t = 0$ and switching the Lines 17–18, 17–27 and 16–1 at $t = 0.167$ s        |





**Fig. 5** Multi-machine power system responses: (a) after Event 1 and (b) after Event 2.

## Conclusions

In this paper, an output feedback control synthesis was presented based on the LPV representation using parameter set mapping with principle component analysis (PCA) in power systems, where the stabilization and damping of oscillations were the main objectives. Transient response sample points were used to produce an initial LPV model, and then, PCA-based parameter set mapping was applied to reduce the number of models. The proposed output feedback controller was designed by solving a set of linear matrix inequalities (LMIs). Although the calculations appear to be burdensome because of the large number of LMIs, especially for large scale power systems, the method proposed in this paper is very convenient for real-time implementation. Because all of the control computations are based on power system information, they may be conducted offline once the probable faults have been defined, and hence, there is no restriction for online implementation of the proposed control. In other words, it is unnecessary to solve the LMIs in real time. A sufficient condition is also extracted such that the asymptotic stability is guaranteed against the uncertainties that may have occurred on the ver-

tices. The proposed scheme was applied to controller synthesis of a power system as a PSS for damping control of the oscillations. As stated in the paper, one challenging point that may be considered in future studies is to find a new method of changing the controller variables, such as in (35), independent of vertices variables, although it was shown that the change of variables in (35) had different solutions for vertices, but the same properties.

After constructing the LPV model and designing the corresponding controller (as a new PSS) based on the proposed method, the effectiveness of the proposed controller was assessed through nonlinear simulations for nominal and other operation conditions and perturbations in comparison with the case of no PSS and tuned standard PSS. The simulation results, especially for a multi-machine power system, confirmed the robust performance properties of the considered power system equipped with the proposed controller.

## Conflict of Interest

*The authors have declared no conflict of interest.*

## Compliance with Ethics Requirements

*This article does not contain any studies with human or animal subjects.*

## References

- [1] Kundur P, Paserba J, Ajarapu V, Andersson G, Bose A, Canizares C, et al. Definition and classification of power system stability. *IEEE Trans Power Syst* 2004;19:1387–401.
- [2] Kundur P. *Power system stability and control*. New York: McGraw-Hill; 1994.
- [3] He P, Wen F, Ledwich G, Xue Y, Wang K. Effects of various power system stabilizers on improving power system dynamic performance. *Int J Electr Power Energy Syst* 2013;46:175–83.
- [4] Shahgholian G. Review of power system stabilizer: application, modeling, analysis and control strategy. *Int J Tech Phys Probl Eng* 2013;5:41–52.
- [5] Jabr RA, Pal BC, Martins N. A sequential conic programming approach for the coordinated and robust design of power system stabilizers. *IEEE Trans Power Syst* 2010;25:1627–37.
- [6] Abdel-Magid YL, Abido MA, Mantaway AH. Robust tuning of power system stabilizers in multimachine power systems. *IEEE Trans Power Syst* 2000;15:735–40.
- [7] de Campos VAF, da Cruz JJ, Zanetta LC. Robust and optimal adjustment of power system stabilizers through linear matrix inequalities. *Int J Electr Power Energy Syst* 2012;42:478–86.
- [8] Ataei M, Hooshmand R-A, Parastegari M. A wide range robust PSS design based on power system pole-placement using linear matrix inequality. *J Electr Eng* 2012;63:233–41.
- [9] Ellithy K, Said S, Kahlout O. Design of power system stabilizers based on  $\mu$ -controller for power system stability enhancement. *Int J Electr Power Energy Syst* 2014;63:933–9.
- [10] El-Razaz ZS, Mandor ME-D, Salim Ali E. Damping controller design for power systems using LMI and GA techniques. In: *IEEE Elev int Middle East power syst conf (MEPCON 2006)*, El-Minia; 2006, p. 500–6.
- [11] Soliman M. Robust non-fragile power system stabilizer. *Int J Electr Power Energy Syst* 2015;64:626–34.
- [12] Abd-Elazim SM, Ali ES. Power system stability enhancement via bacteria foraging optimization algorithm. *Arab J Sci Eng* 2013;38:599–611.

- [13] Ali ES. Optimization of power system stabilizers using BAT search algorithm. *Int J Electr Power Energy Syst* 2014;61:683–90.
- [14] Simfukwe DD, Pal BC. Robust and low order power oscillation damper design through polynomial control. *IEEE Trans Power Syst* 2013;28:1599–608.
- [15] Rao PS, Sen I. Robust pole placement stabilizer design using linear matrix inequalities. *IEEE Trans Power Syst* 2000;15:313–9.
- [16] Pal B, Chaudhuri B. Robust control in power systems. London (UK): Springer; 2005.
- [17] Soliman M, Elshafei AL, Bendary F, Mansour W. Robust decentralized PID-based power system stabilizer design using an ILMI approach. *Electr Power Syst Res* 2010;80:1488–97.
- [18] Amato F, Garofalo F, Glielmo L, Pironti A. Robust and quadratic stability via polytopic set. *Int J Robust Nonlin Control* 1995;5:745–56.
- [19] Ordóñez-Hurtado RH, Duarte-Mermoud Ma. Finding common quadratic Lyapunov functions for switched linear systems using particle swarm optimisation. *Int J Control* 2012;85:12–25.
- [20] Tong Y, Zhang L, Shi P, Wang C. A common linear copositive Lyapunov function for switched positive linear systems with commutable subsystems. *Int J Syst Sci* 2013;44:1994–2003.
- [21] Xiang W, Xiao J. Finite-time stability and stabilisation for switched linear systems. *Int J Syst Sci* 2011:1–17.
- [22] Ramos SD, Domingos ACJ, Vazquez Silva E. An algorithm to verify asymptotic stability conditions of a certain family of systems of differential equations. *Appl Math Sci* 2014;8:1509–20.
- [23] Lin H, Antsaklis PJ. Stability and stabilizability of switched linear systems: a survey of recent results. *IEEE Trans Automat Contr* 2009;54:308–22.
- [24] Xiong J, Lam J, Shu Z, Mao X. Stability analysis of continuous-time switched systems with a random switching signal. *IEEE Trans Automat Contr* 2014;59:180–6.
- [25] She Z, Xue B. Discovering multiple Lyapunov functions for switched hybrid systems. *SIAM J Control Optim* 2014;52:3312–40.
- [26] De Souza WA, Teixeira MCM, Santim MPA, Cardim R, Assuncao E. On switched control design of linear time-invariant systems with polytopic uncertainties. *Math Probl Eng* 2013:2013.
- [27] Hoffmann C, Werner H. A survey of linear parameter-varying control applications validated by experiments or high-fidelity simulations. *IEEE Trans Control Syst Technol* 2014;23:416–33.
- [28] Shamma JS. An overview of LPV systems. In: Mohammadpour J, Scherer CW, editors. *Control of linear parameter varying systems with applications*. US: Springer; 2012. p. 3–26.
- [29] Pal A, Thorp JS, Veda SS, Centeno VA. Applying a robust control technique to damp low frequency oscillations in the WECC. *Int J Electr Power Energy Syst* 2013;44:638–45.
- [30] Soliman HM, Soliman MH, Hassan MF. Resilient guaranteed cost control of a power system. *J Adv Res* 2014;5:377–85.
- [31] Bos R, Bombois X, Van den Hof PMJ. Accelerating simulations of computationally intensive first principle models using accurate quasi-linear parameter varying models. *J Process Control* 2009;19:1601–9.
- [32] Kwiatkowski A, Werner H. PCA-based parameter set mappings for LPV models with fewer parameters and less overbounding. *IEEE Trans Control Syst Technol* 2008;16:781–8.
- [33] Jolliffe IT. *Principal component analysis*. 2nd ed. New York: Springer; 2002.
- [34] Tang L. *Dynamic security assessment processing system*. Iowa State University; 2014.
- [35] Chilali M, Gahinet P.  $H_\infty$  design with pole placement constraints: an LMI approach. *IEEE Trans Automat Contr* 1996;41:358–67.
- [36] Bikash Pal BC. *Robust control in power systems*. Springer; 2005.
- [37] Shin J, Nam S, Lee J, Baek S, Choy Y, Kim T. A practical power system stabilizer tuning method and its verification in field test. *J Electr Eng Technol* 2010;5:400–6.
- [38] DIGSILENT. 39 Bus New England system; 2015.