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## Reducing resonant vibration of a rotor by tuning the gap between a superconducting bulk and a permanent magnet

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### Abstract

This study investigated passing through a critical speed of a rotor supported by a superconductor with an electromagnet. Here we adopted the idea that the gap between the superconductor and the rotor can be tuned variably by using electromagnetic force of the electromagnet so that the natural frequency or the stiffness can be changed. By using this method, it can be expected that resonant vibration be reduced. We developed an analytical model and then carried out numerical simulation. Numerical results show that considerable reduction of the resonant amplitude can be achieved by proper tuning of switching the electromagnet.

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### 1. Introduction

Superconducting magnetic bearings (SMBs) can achieve maintenance-free and high-speed rotation, because of the feature of non-contact support. Recently, they have been studied for applications such as flywheel energy storage [1]–[3]. However, because rotor axes are supported by magnetic force, restoring force of SMBs is relatively smaller than that of mechanical bearings and a critical speed is lower. Because of their low damping, an increase in the amplitude causes various problems like a disruptive accident and noise. Furthermore, complicated vibration can be caused by nonlinearity of the magnetic force. Because there are many opportunities to operate it exceeding the main critical speed in the application, reducing the amplitude and considering the nonlinearity of the system is necessary.

Concerning the nonlinearity of a high-Tc superconducting magnetic levitation system, Kordyuku developed the advanced mirror image method for evaluation of a magnetic force of a superconductor [4]. As the past research on reducing a resonance peak, Ishida investigated effect of a nonlinearity of a magnetic force on a vibration reduction of a rotor [5]. The authors previously investigated vibration reduction of a rotor system with an electromagnet, utilizing a feature of a nonlinearity of an SMB and a permanent magnetic bearing [6].

This study investigates reducing a resonant vibration of a rotor supported by a SMB. Here we adopt the idea that the gap between the superconducting bulk and a permanent magnet of the rotor can be tuned variably by using an electromagnetic force of an electromagnet so that the natural frequency or the stiffness of the system can be changed. By using this method, it can be expected that resonant vibration be reduced during passing through a critical speed. We first developed an analytical model composed of a superconducting bulk, a permanent magnet as a rotor and an electromagnet. Next by using the Runge-Kutta method, numerical calculation of the system is performed and effect of reducing vibration by the above method is investigated.

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### 2. Analytical model and governing equations

Fig.1 shows our analytical model. In this model, a permanent magnet as a rotor is levitated above a superconducting bulk and an electromagnet is located above the permanent magnet. The horizontal axes of the permanent magnet are defined as in Fig. 1.  $z_0$  is the initial levitation height of the rotor and  $B$  is a magnetic flux density of the electromagnet. Magnetic forces exerted by the superconductor are evaluated by the advanced mirror image method [4]. Electromagnetic force exerted by the electromagnet is assumed that it works only in the vertical direction. The equations of motion of the permanent magnet rotating at frequency  $\nu$  are derived as follows.

$$\ddot{x} + 2\gamma\dot{x} + K_x x = \alpha_{xzz}xz + \alpha_{xxx}x^3 + \alpha_{xyy}xy^2 - \alpha_{xzz}xz^2 + \epsilon\nu^2\cos\nu t \tag{1}$$

$$\ddot{y} + 2\gamma\dot{y} + K_y y = \alpha_{yyz}yz + \alpha_{yyy}y^3 + \alpha_{xxy}x^2y - \alpha_{yzz}yz^2 + \epsilon\nu^2\sin\nu t \tag{2}$$

Here the forces are expanded into a Taylor series around the equilibrium position up to the 3<sup>rd</sup> order terms. The above equations are nondimensionalized by  $t = t^*/\Omega_x$ ,  $x = z_m x^*$  and  $y = z_m y^*$ .  $\Omega_x = (k_x/M)^{1/2}$  is the natural frequency under  $B = 0.00T$  and  $z_m$  is 10mm. The asterisks (\*) denoting nondimensional variables are omitted and nondimensional parameters are denoted as follows.  $K_x$  and  $K_y$  are coefficients of the linear terms.  $\alpha_{xz}$ ,  $\alpha_{yz}$ ,  $\alpha_{xxx}$ ,  $\alpha_{yyy}$ ,  $\alpha_{xyy}$ ,  $\alpha_{xxy}$ ,  $\alpha_{xzz}$  and  $\alpha_{yzz}$  are coefficients of the nonlinear terms.  $\epsilon$  is the eccentricity of the rotor and  $\gamma$  is the damping coefficient.

The equation describing balances of the vertical force due to the superconducting bulks and the electromagnet and the gravity of the rotor can be expressed as follows.

$$(k_{scz} + k_{emz})z + (k_{sczz} + k_{emzz})z^2 + (k_{sczzz} + k_{emzzz})z^3 + K_z = 0 \tag{3}$$

Here the force due to the electromagnet is evaluated by a method used in [6].  $k_{scz}$  and  $k_{emz}$  are coefficients of the linear terms of the force due to the superconductor and the electromagnet, respectively.  $k_{sczz}$ ,  $k_{sczzz}$ ,  $k_{emzz}$  and  $k_{emzzz}$  are coefficients of the nonlinear terms of the force, as well.  $K_z$  is a constant of a term of gravity.  $k_{emz}$ ,  $k_{emzz}$  and  $k_{emzzz}$  are variable by switching the electromagnet. By using this influence,  $z$  of (3) is altered, resulting in change of the coefficients of the linear terms and nonlinear terms in (1) and (2), and thus leading to change of the natural frequency or the stiffness.

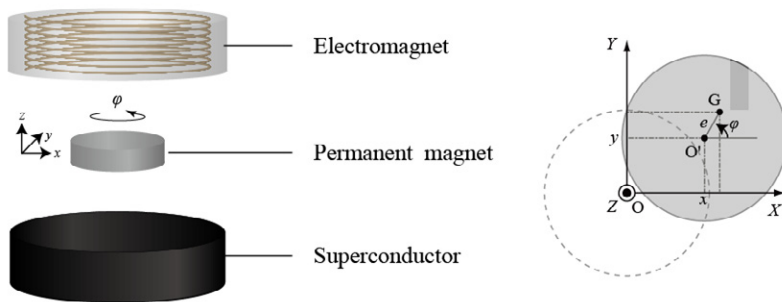


Fig. 1. Analytical model

### 3. Principle of reducing a resonant vibration

The natural frequency of the permanent magnet (rotor) levitated above the superconducting bulk depends on the gap between the superconducting bulk and the permanent magnet. The electromagnetic force of the electromagnet is variable so that the natural frequencies or the stiffness  $K_x$  and  $K_y$  can be changed. Using this change, it can be expected that resonant vibration will be reduced during passing through the critical speed.

Fig. 2 shows an idea of reducing the resonant peaks by switching the electromagnet. If the system is linear, a frequency response curve of the system can be a broken line in Fig. 2. When the electromagnet exerts repulsive force to the permanent magnet, the stiffness coefficients  $K_x$  and  $K_y$  are increased and the frequency response curve is changed into a fine solid line. The principle of passing through a critical speed is based on the following facts. The rotational speed is raised first with the electromagnet on. Before the rotational speed reaches the critical speed, the

electromagnet gets switched off. In consequence, the frequency response of the rotor is first on the fine solid line, but changed to be on the broken line before reaching the resonant peak, and then the peak amplitude can be reduced.

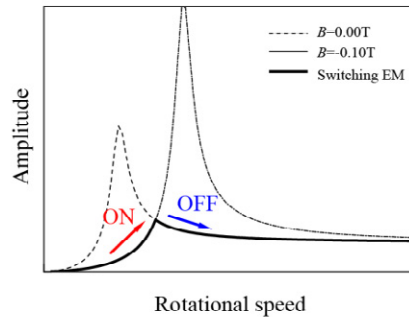


Fig. 2. Principle of reducing resonant vibration

#### 4. Numerical simulation

Numerical simulation of (1) and (2) was carried out by using the Runge-Kutta method. We investigated rotor dynamics with several values of the initial levitation height. In these cases, the gap between the superconducting bulk and the permanent magnet is changed by applying the repulsive or attractive force. Table 1 shows the numerical results. When the magnetic flux density  $B$  is positive, the electromagnetic force acts on the permanent magnet as repulsive force and vice versa.

With the repulsion, the stiffness of the system can be changed to a small degree because the length of vertical translation of the rotor is limited to a small value. Therefore, the resonance peak slides little to higher frequencies and the amplitude can be reduced only in small quantities. With the attraction, however, the stiffness of the system can be changed more because the length of vertical translation can be large. Therefore, the resonance peak slides considerably to lower frequencies and the amplitude can be reduced significantly. Thus, the attractive force of the electromagnet is used below.

##### 4.1. Switching during raise of the rotational speed

Table 1 shows the numerical results of the reduction rate of the amplitude and the translation length and the proper switching value with  $z_0/z_m = 1.00 \sim 1.50$ . It was not able to take balance of vertical force with  $z_0/z_m = 1.30 \sim 1.50$  when the electromagnet gives attractive force, because the attractive force is too large. Fig. 3 (1) shows the frequency response curves with  $z_0/z_m = 1.20$  while the rotational speed is raised. Numerically obtained frequency responses for  $B = 0.00\text{T}$  and  $B = -0.10\text{T}$  are plotted by black circles and blue circles, respectively. The peak amplitude can be reduced by changing  $B$  from  $0.00\text{T}$  to  $-0.10\text{T}$  at  $v = 0.610$ . Black plots indicate passing through the critical speed with switching the electromagnet. It can be found that transition of the response between the two frequency response curves are achieved by avoiding the two resonance peaks. The peak amplitude decreases 41.9% compared with the peak amplitude of  $B = 0.00\text{T}$ . When the electromagnet is switched on at a rotational speed at which the two frequency response curves intersect as seen with Fig. 2, the amplitude moves to a higher stable value and the reduction effect is lower than switching at  $v = 0.610$  with Fig. 3. This is supposed to be due to the influence of the nonlinearity.

##### 4.2. Switching during lowering of the rotational speed

Fig. 3 (2) shows the frequency response curves with  $z_0/z_m = 1.20$  while the rotational speed is lowered. Numerically obtained frequency responses for  $B = 0.00\text{T}$  and  $B = -0.10\text{T}$  are plotted by black circles and blue circles, respectively. The peak amplitude can be reduced by changing  $B$  from  $-0.10\text{T}$  to  $0.00\text{T}$  at  $v = 0.450$ . Black plots indicate passing through the critical speed with switching the electromagnet. It can be found that transition of the response between the two frequency response curves are achieved by avoiding the two resonance peaks. The peak amplitude decreases 50.4% compared with the peak amplitude of  $B = 0.00\text{T}$ . If the electromagnet is switched off at a rotational speed at which the two frequency response curves intersect as seen with Fig. 2, the amplitude moves to a higher stable value and the reduction effect is lower than switching at  $v = 0.450$  with Fig. 3. This is supposed to be due to the influence of the nonlinearity.

According to the above results, the attractive force is more effective for reducing vibration than the repulsive force. Considering the nonlinearity, it is necessary to avoid jump to a higher amplitude by choosing proper tuning of switching the electromagnet.

Table 1. Numerical results of reducing resonant vibration by switching the electromagnet

$(z_0/z_m)$	Translation length (-)		Reduction rate during speed up				Reduction rate during speed down			
	Repulsive	Attractive	Repulsive (%)	Switching at $v(-)$	Attractive (%)	Switching at $v(-)$	Repulsive (%)	Switching at $v(-)$	Attractive (%)	Switching at $v(-)$
1.00	-0.0395	0.0783	1.27	0.870	4.50	0.830	0.00	0.870	2.50	0.540
1.10	-0.0541	0.236	1.62	0.740	19.6	0.700	0.00	0.740	12.5	0.510
1.20	-0.0695	0.567	2.81	0.650	41.9	0.610	1.99	0.500	50.4	0.450
1.30	-0.0859	-	2.62	0.570	-	-	-2.17	0.480	-	-
1.40	-0.101	-	3.46	0.510	-	-	-3.07	0.450	-	-
1.50	-0.113	-	8.62	0.470	-	-	2.83	0.440	-	-

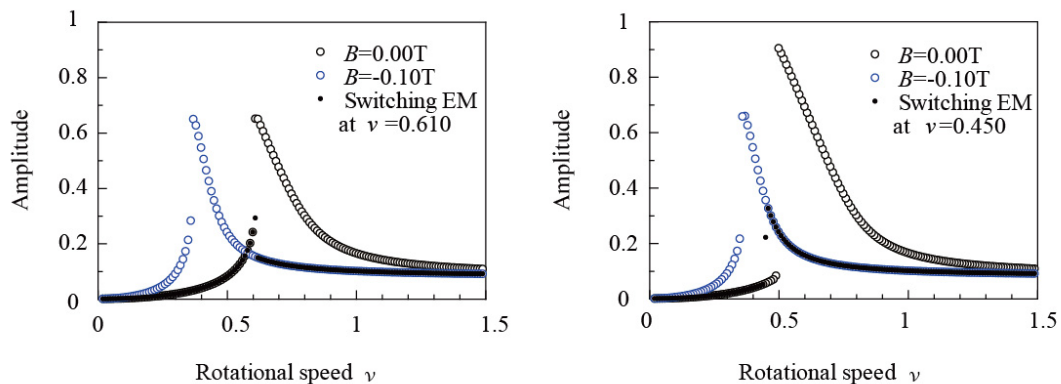


Fig. 3. Frequency responses with  $z_0/z_m = 1.20$  obtained while the rotational speed is raised ((a)) and lowered ((b))

## 5. Conclusion

This study numerically confirmed that resonant vibration of a rotor supported by a superconducting bulk and an electromagnet can be reduced during passing through a critical speed by tuning the gap between the superconducting bulk and the permanent magnet. It is more effective to use attractive force by the electromagnet than to use repulsive force. Because the amplitude may move to a higher stable value due to the influence of the nonlinearity, it is necessary to consider the nonlinearity of the system.

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