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Master integrals for the two-loop light fermion contributions to $gg \to H$ and $H \to \gamma\gamma$

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Abstract

We give the analytic expressions of the eight master integrals entering our previous computation of two-loop light fermion contributions to $gg \to H$ and $H \to \gamma\gamma$. The results are expressed in terms of generalized harmonic polylogarithms with maximum weight four included.

Keywords: Feynman diagrams; Multi-loop calculations

In this Letter we give the analytic expressions of the eight master integrals (MIs) entering our previous computation [1] of two-loop light fermion contributions to

$$g + g \to H \tag{1}$$

and

$$H \to \gamma + \gamma. \tag{2}$$

As the MIs enter, in general, in various processes, we believe that their publication is of general utility. The Feynman diagrams for processes (1) and (2) are shown in Figs. 1 and 2, respectively. As is clearly seen, the amplitudes related to (1) are a subset of those for (2), so it is sufficient to consider only the latter process.

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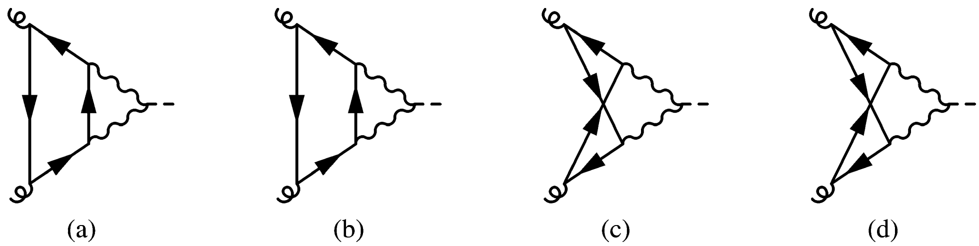


Fig. 1. Feynman diagrams for electro-weak light fermion contributions to $gg \to H$. The straight lines represent light fermions, while the wavy lines stand for the W or Z bosons. The curly lines denote the initial gluons, the dashed line the final Higgs boson.

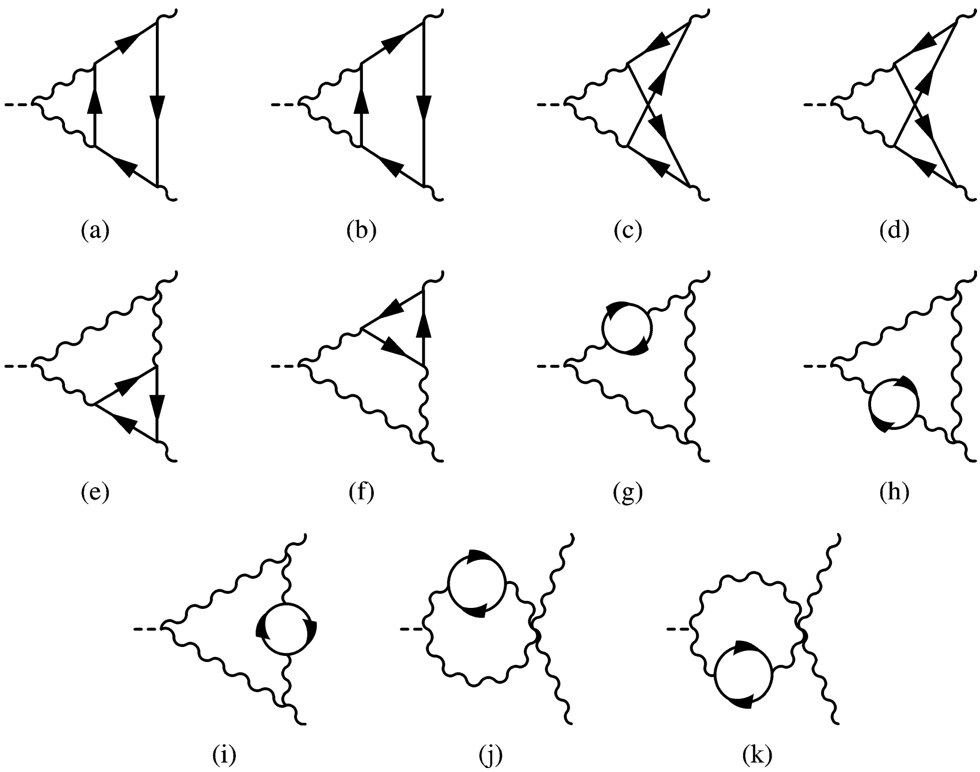


Fig. 2. Feynman diagrams for electro-weak light fermion contributions to the decay $H \to \gamma\gamma$.

The reduction chain leading from the Feynman diagrams to the MIs has been discussed briefly in [1] and in greater detail in [2], so we do not need to repeat the generalities but only the peculiarities to this case. There are three independent topologies with six denominators shown in Fig. 3. By shrinking one internal line, we obtain the five denominator topologies listed in Fig. 4. We do not include the topologies already encountered in the computation of the electro-weak form factor [2,3]. By shrinking a second internal line, we obtain the four denominator topologies shown in Fig. 5. There are no new three denominator topologies. By using the integration-by-parts identities, the topologies shown in Figs. 3–5 are reduced to the eight MIs shown in Fig. 6. All the irreducible topologies have only one MI, with the exception of the topology (a) in Fig. 5, which has two MIs. As is often the case [2], the non-planar topology is the only irreducible one among the six-denominator amplitudes.

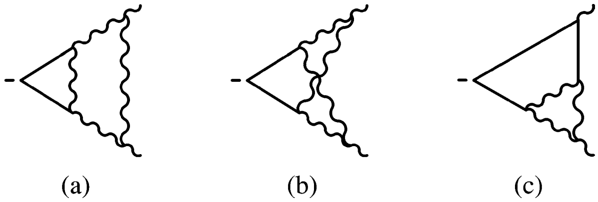


Fig. 3. The set of three independent topologies with six denominators, related to the Feynman diagrams shown in Figs. 1 and 2. The graphic conventions for the topologies are the same as in [2,3]: plain lines represent massive particles, while wavy lines represent massless particles.

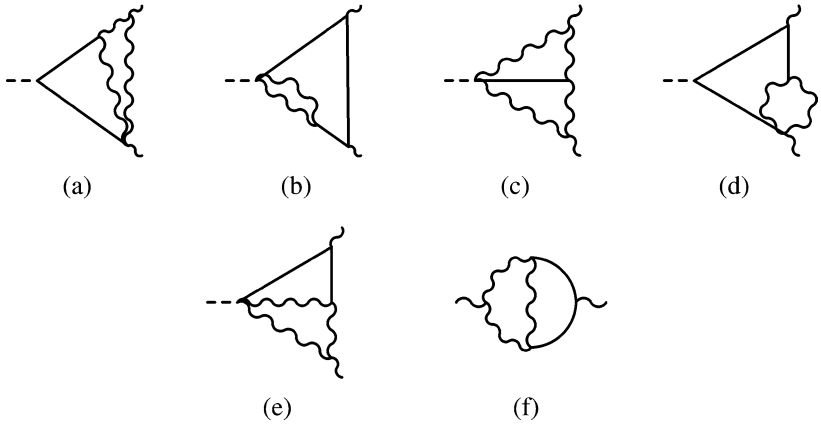


Fig. 4. The set of six independent five-denominator topologies.

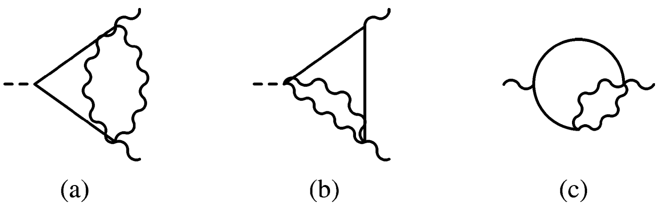


Fig. 5. The set of three independent four-denominator topologies.

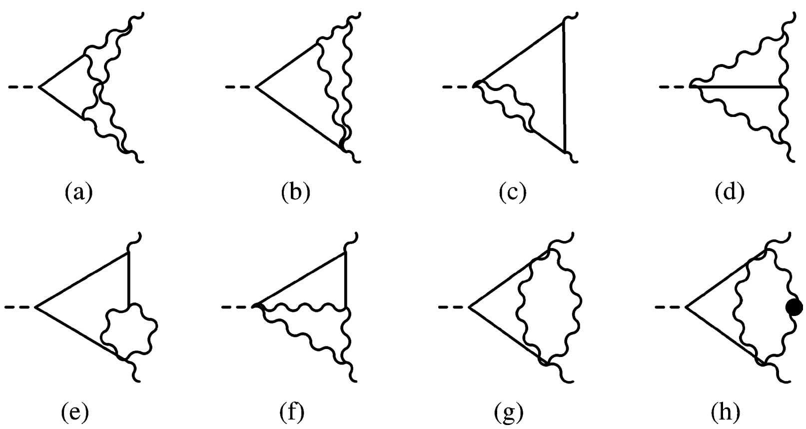


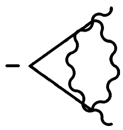
Fig. 6. The set of eight MIs. The dot on a line indicates a square of the corresponding denominator.

All the MIs are computed with the differential equation technique introduced in [4] and applied to similar cases as the present one in [2,3]. For the topology (a) in Fig. 5, we choose the MIs consisting of the scalar amplitude and the amplitude with a massless denominator squared, given in (g) and (h) of Fig. 6, respectively. With this choice, the system of two differential equations is triangular in four dimensions, allowing for an elementary solution.

The MIs that we present, are regularized within the dimensional regularization scheme [7]. They are expanded in a Laurent series of $\epsilon = 2 - D/2$, where D is the space–time dimension. We work in Minkowski space and the loop measure is normalized as: $\mathfrak{D}^D k = d^D k/[i\pi^{D/2}\Gamma(3 - D/2)]$. We have defined $x = -s/a$, where $a = m^2_{W,Z}$ and $s = -(p_1 + p_2)^2$ [2] is the c.m. energy squared, with p_1 and p_2 the light-cone momenta of the final photons. The scale μ is, as usual, the unit of mass of dimensional regularization. The results are naturally expressed as a linear combination of generalized harmonic polylogarithms (GHPLs) [3,5,6] of the variable x, with maximum weight four included. The definition of the generalized harmonic polylogarithms and all the relevant conventions have been given in [2,3], to which we refer for details.

In the following we list the MIs in order of increasing number t of denominators. We provide also a small appendix with the expressions of the one-loop results entering the renormalization of the two-loop corrections to $H \to \gamma\gamma$.

1. Topology $t = 4$



$$= \left(\frac{\mu^2}{a}\right)^{2\epsilon} \sum_{i=-2}^{2} \epsilon^i F_i^{(1)} + \mathcal{O}(\epsilon^3), \tag{3}$$

where:

$$F_{-2}^{(1)} = \frac{1}{2}, \tag{4}$$

$$F_{-1}^{(1)} = \frac{5}{2} - \frac{x+4}{\sqrt{x(x+4)}} H(-r; x), \tag{5}$$

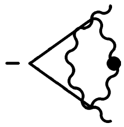
$$F_0^{(1)} = \frac{19}{2} + \zeta(2) + H(-r, -r; x) + \frac{2}{x} H(-r, -r; x) - \frac{x+4}{\sqrt{x(x+4)}} \big[5H(-r; x) - H(-4, -r; x)\big], \tag{6}$$

$$\begin{aligned} F_1^{(1)} = {} & \frac{65}{2} + 5\zeta(2) - \zeta(3) + 5H(-r, -r, x) - H(-r, -4, -r, x) + H(0, -r, -r, x) \\ & + \frac{1}{x}\big[10H(-r, -r, x) - 2H(-r, -4, -r, x) + 2H(0, -r, -r, x)\big] \\ & - \frac{x+4}{\sqrt{x(x+4)}}\big[19H(-r, x) + 2\zeta(2)H(-r, x) - 5H(-4, -r, x) + 3H(-r, -r, -r, x) \\ & \quad + H(-4, -4, -r, x)\big], \end{aligned} \tag{7}$$

$$\begin{aligned} F_2^{(1)} = {} & \frac{211}{2} + 19\zeta(2) + \frac{9}{5}\zeta^2(2) - 5\zeta(3) + 3H(-r, -r, -r, -r, x) + 19H(-r, -r, x) \\ & + 2\zeta(2)H(-r, -r, x) - 5H(-r, -4, -r, x) + H(-r, -4, -4, -r, x) \\ & + 5H(0, -r, -r, x) - H(0, -r, -4, -r, x) + H(0, 0, -r, -r, x) \end{aligned}$$

[2] Scalar products are defined as: $a \cdot b = -a_0 b_0 + \vec{a} \cdot \vec{b}$.

$$
\begin{aligned}
&+\frac{1}{x}\big[6H(-r,-r,-r,-r,x)+38H(-r,-r,x)+4\zeta(2)H(-r,-r,x)\\
&\qquad -10H(-r,-4,-r,x)+2H(-r,-4,-4,-r,x)+10H(0,-r,-r,x)\\
&\qquad -2H(0,-r,-4,-r,x)+2H(0,0,-r,-r,x)\big]\\
&-\frac{x+4}{\sqrt{x(x+4)}}\big[65H(-r,x)+10\zeta(2)H(-r,x)-2\zeta(3)H(-r,x)\\
&\qquad -19H(-4,-r,x)-2\zeta(2)H(-4,-r,x)+15H(-r,-r,-r,x)\\
&\qquad -3H(-r,-r,-4,-r,x)+3H(-r,0,-r,-r,x)-3H(-4,-r,-r,-r,x)\\
&\qquad +5H(-4,-4,-r,x)-H(-4,-4,-4,-r,x)\big].
\end{aligned}
\tag{8}
$$



$$
= \left(\frac{\mu^2}{a}\right)^{2\epsilon}\sum_{i=-1}^{1}\epsilon^i F_i^{(2)}+\mathcal{O}(\epsilon^2),
\tag{9}
$$

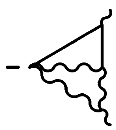
where:

$$
aF_{-1}^{(2)} = -\frac{2}{x}H(-r,-r;x),
\tag{10}
$$

$$
aF_{0}^{(2)} = \frac{2}{x}\big[H(-r,-4,-r;x)-H(0,-r,-r;x)\big],
\tag{11}
$$

$$
\begin{aligned}
aF_{1}^{(2)} = -\frac{2}{x}\big[&2\zeta(2)H(-r,-r;x)+H(0,0,-r,-r;x)+H(-r,-4,-4,-r;x)\\
&-H(0,-r,-4,-r;x)+3H(-r,-r,-r,-r;x)\big].
\end{aligned}
\tag{12}
$$

2. Topology $t=5$

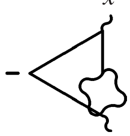


$$
= \left(\frac{\mu^2}{a}\right)^{2\epsilon}\sum_{i=-1}^{0}\epsilon^i F_i^{(3)}+\mathcal{O}(\epsilon),
\tag{13}
$$

where:

$$
aF_{-1}^{(3)} = -\frac{1}{x}H(0,0,-1,x),
\tag{14}
$$

$$
aF_{0}^{(3)} = \frac{1}{x}\big[4H(0,0,-1,-1,x)-H(0,0,0,-1,x)\big].
\tag{15}
$$



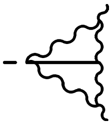
$$
= \left(\frac{\mu^2}{a}\right)^{2\epsilon}\sum_{i=-1}^{1}\epsilon^i F_i^{(4)}+\mathcal{O}(\epsilon^2),
\tag{16}
$$

where:

$$
aF_{-1}^{(4)} = \frac{1}{x}H(-r,-r,x),
\tag{17}
$$

$$
aF_{0}^{(4)} = \frac{1}{x}\big[2H(-r,-r,x)-H(-r,-4,-r,x)-H(0,-r,-r,x)\big],
\tag{18}
$$

$$aF_1^{(4)} = \frac{1}{x}\big[2\big(\zeta(2)+2\big)H(-r,-r,x) - 2H(-r,-4,-r,x) - 2H(0,-r,-r,x) + 3H(-r,-r,-r,-r,x) + H(-r,-4,-4,-r,x) + H(0,-r,-4,-r,x) - H(0,0,-r,-r,x)\big]. \tag{19}$$



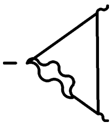
$$= \left(\frac{\mu^2}{a}\right)^{2\epsilon} \sum_{i=-2}^{0} \epsilon^i F_i^{(5)} + \mathcal{O}(\epsilon), \tag{20}$$

where:

$$aF_{-2}^{(5)} = \frac{1}{x}H(0,-1,x), \tag{21}$$

$$aF_{-1}^{(5)} = \frac{2}{x}\big[H(0,0,-1,x) - 2H(0,-1,-1,x)\big], \tag{22}$$

$$aF_0^{(5)} = \frac{1}{x}\big[2\zeta(2)H(0,-1,x) + 16H(0,-1,-1,-1,x) - 6H(0,-1,0,-1,x) - 8H(0,0,-1,-1,x) + 2H(0,0,0,-1,x)\big]. \tag{23}$$



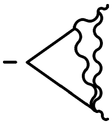
$$= \left(\frac{\mu^2}{a}\right)^{2\epsilon} \sum_{i=-1}^{1} \epsilon^i F_i^{(6)} + \mathcal{O}(\epsilon^2), \tag{24}$$

where:

$$aF_{-1}^{(6)} = \frac{1}{x}H(-r,-r,x), \tag{25}$$

$$aF_0^{(6)} = \frac{1}{x}\left[2H(-r,-r,x) - \frac{3}{2}H(-r,-r,-1,x) - H(-r,-4,-r,x) - \frac{1}{2}H(0,0,-1,x)\right], \tag{26}$$

$$aF_1^{(6)} = \frac{1}{x}\left[2\big(\zeta(2)+2\big)H(-r,-r,x) - 3H(-r,-r,-1,x) - 2H(-r,-4,-r,x) + 6H(-r,-r,-1,-1,x) - 3H(-r,-r,0,-1,x) + H(-r,-4,-4,-r,x) + \frac{3}{2}H(-r,-4,-r,-1,x) - H(0,0,-1,x) + 2H(0,0,-1,-1,x) - \frac{1}{2}H(0,0,0,-1,x)\right]. \tag{27}$$



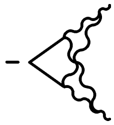
$$= \left(\frac{\mu^2}{a}\right)^{2\epsilon} \sum_{i=-1}^{0} \epsilon^i F_i^{(7)} + \mathcal{O}(\epsilon), \tag{28}$$

where:

$$aF_{-1}^{(7)} = \frac{1}{2x}\big[3H(-r,-r,-1;x) - 2H(0,-r,-r;x) - H(0,0,-1;x)\big], \tag{29}$$

$$aF_0^{(7)} = \frac{1}{2x}\big[6H(-r,-r,-r,-r;x) - 12H(-r,-r,-1,-1;x) + 6H(-r,-r,0,-1;x) - 3H(-r,-4,-r,-1;x) + 2H(0,-r,-4,-r;x) - 2H(0,0,-r,-r;x) + 4H(0,0,-1,-1;x) - H(0,0,0,-1;x)\big]. \tag{30}$$

3. Topology $t = 6$



$$= \left(\frac{\mu^2}{a}\right)^{2\epsilon} \sum_{i=-2}^{0} \epsilon^i F_i^{(8)} + \mathcal{O}(\epsilon), \tag{31}$$

where:

$$a^2 F_{-2}^{(8)} = \frac{6}{x\sqrt{x(x+4)}} H(-r,-1;x), \tag{32}$$

$$a^2 F_{-1}^{(8)} = \frac{1}{x\sqrt{x(x+4)}} \big[16H(-r,-r,-r;x) - 24H(-r,-1,-1;x) + 16H(-r,0,-1;x) - 12H(-4,-r,-1;x)\big], \tag{33}$$

$$\begin{aligned} a^2 F_{0}^{(8)} = \frac{1}{x\sqrt{x(x+4)}} \big[& -12H(-r,-r,-r,-1;x) - 16H(-r,-r,-4,-r;x) \\ & + 12\zeta(2)H(-r,-1;x) + 96H(-r,-1,-1,-1;x) - 36H(-r,-1,0,-1;x) \\ & + 24H(-r,0,-r,-r;x) - 64H(-r,0,-1,-1;x) + 24H(-r,0,0,-1;x) \\ & - 32H(-4,-r,-r,-r;x) + 48H(-4,-r,-1,-1;x) \\ & - 32H(-4,-r,0,-1;x) + 24H(-4,-4,-r,-1;x)\big]. \end{aligned} \tag{34}$$

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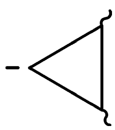
We wish to thank E. Remiddi for discussions and for the C program SOLVE [9], used to solve the linear systems generated with the ibp identities.

Appendix A. A one-loop vertex

The renormalization of the two-loop corrections to process (2) requires the knowledge of the one-loop vertex with three equal masses and of its derivative with respect to the squared mass a, which are given below:



$$= \left(\frac{\mu^2}{a}\right)^{\epsilon} \sum_{i=0}^{2} \epsilon^i F_i^{(9)} + \mathcal{O}\big(\epsilon^3\big), \tag{A.1}$$

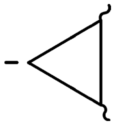
where:

$$aF_0^{(9)} = \frac{1}{x} H(-r,-r;x), \tag{A.2}$$

$$aF_1^{(9)} = -\frac{1}{x} H(-r,-4,-r;x), \tag{A.3}$$

$$aF_2^{(9)} = \frac{1}{x} H(-r,-4,-4,-r;x). \tag{A.4}$$

$$\frac{\partial}{\partial a}$$



$$= \left(\frac{\mu^2}{a}\right)^{\epsilon} \sum_{i=0}^{2} \epsilon^i F_i^{(10)} + \mathcal{O}(\epsilon^3), \tag{A.5}$$

where:

$$a^2 F_0^{(10)} = -\frac{1}{\sqrt{x(x+4)}} H(-r; x), \tag{A.6}$$

$$a^2 F_1^{(10)} = -\frac{1}{x} H(-r, -r; x) + \frac{1}{\sqrt{x(x+4)}} H(-4, -r; x), \tag{A.7}$$

$$a^2 F_2^{(10)} = \frac{1}{x} H(-r, -4, -r; x) - \frac{1}{\sqrt{x(x+4)}} H(-4, , -4, -r; x). \tag{A.8}$$

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