



Restoring holographic dark energy in brane cosmology

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Abstract

We present a generalized version of holographic dark energy arguing that it must be considered in the maximally subspace of a cosmological model. In the context of brane cosmology it leads to a bulk holographic dark energy which transfers its holographic nature to the effective 4D dark energy. As an application we use a single-brane model and we show that in the low energy limit the behavior of the effective holographic dark energy coincides with that predicted by conventional 4D calculations. However, a finite bulk can lead to radically different results.

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1. Introduction

Holographic dark energy [1–9] is an interesting and ingenious idea of explaining the recently observed Universe acceleration [10]. Arising from the cosmological application [11] of the more fundamental holographic principle [12,13], and despite some objections on this approach [14], holographic dark energy reveals the dynamical nature of the vacuum energy by relating it to cosmological volumes. Its framework is the black hole thermodynamics [15,16] and the connection (known from AdS/CFT correspondence) of the UV cut-off of a quantum field theory, which gives rise to the vacuum energy, with the largest distance of the theory [17]. Such a connection is necessary for the applicability of quantum field theory in large distances and results form the argument that the total energy of a system (which entropy is in general proportional to its volume) should not exceed the mass of a black hole of the same size (which entropy is proportional to its area), since in this case the system would collapse to a black hole violating the second law of thermodynamics. When this approach is applied to the Universe, the resulting vacuum energy is identified as holographic dark energy.

Almost all works on the subject remain in the standard 4D framework. However, brane cosmology, according which our Universe is a brane embedded in a higher-dimensional space-time [18,19], apart from being closer to a higher-dimensional fundamental theory of nature, it has also great phenomenological successes and a large amount of current research heads towards this direction [20]. It is therefore desirable to extend holographic dark energy in the braneworld context. Although there is a recent work dedicated to this goal [9] it is based on unstable and non-physical arguments. The main contradiction of the present holographic dark energy foundation and braneworld models is that although holographic principle is itself applicable to arbitrary dimensions [12,21,22] its cosmological application concerning dark energy should be considered in the maximal uncompactified space. The reason is that it is the higher-dimensional black hole formation and the higher-dimensional cut-off's which determine the vacuum energy. Therefore, one should first lay the foundations of bulk holographic dark energy, and then find the corresponding effective 4D one which appears in traditional Friedmann equation.

In this work we present this restored holographic dark energy in brane cosmology. For a specific application we use a general single-brane model in $4 + 1$ dimensions, although the calculations can be extended in arbitrary bulk dimensionality. In this single-brane framework, where the extra dimension is not restricted, in the low energy limit we recover the results of

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the 4D holographic dark energy of the literature in flat, open and closed Universes. We point out that although the obtained effective 4D behavior coincides with previous works, the conceptual framework is radically different. This difference manifests itself in braneworld models with two branes. In the case of a static bulk with constant interbrane distance, which bounds the extra dimension [19,23], the 5D holographic dark energy and therefore the effective 4D one are constants, losing their dynamical nature, leaving the case of moving branes as the only possibility [24]. The rest of the Letter is organized as follows: In Section 2 we formulate the holographic dark energy in the bulk of general and arbitrary spacetimes and in Section 3 we apply it to a general braneworld model in $4 + 1$ dimensions. Finally, in Section 4 we discuss the physical implications of our analysis and we summarize the obtained results.

2. Holographic dark energy in the bulk

We consider a general braneworld model where the bulk is D -dimensional. The restored holographic dark energy states that the vacuum energy in a volume should not exceed the energy of a black hole of the same size, both in the maximal subspace, i.e., in the bulk. The mass M_{BH} of a spherical and uncharged D -dimensional black hole is related to its Schwarzschild radius r_s through [16,25]:

$$M_{\text{BH}} = r_s^{D-3} (\sqrt{\pi} M_D)^{D-3} M_D \frac{D-2}{8\Gamma(\frac{D-1}{2})}. \quad (1)$$

The D -dimensional Planck mass M_D is related to the D -dimensional gravitational constant G_D and the usual 4-dimensional Planck mass M_p through:

$$M_D = G_D^{-\frac{1}{D-2}}, \quad (2)$$

$$M_p^2 = M_D^{D-2} V_{D-4},$$

where V_{D-4} is the volume of the extra-dimensional space [16].

Now, if $\rho_{\Lambda D}$ is the bulk vacuum energy, then application of the restored holographic dark energy gives:

$$\rho_{\Lambda D} \text{Vol}(S^{D-2}) \leq r^{D-3} (\sqrt{\pi} M_D)^{D-3} M_D \frac{D-2}{8\Gamma(\frac{D-1}{2})}, \quad (3)$$

where $\text{Vol}(S^{D-2})$ is the volume of the maximal hypersphere in a D -dimensional spacetime, given from:

$$\text{Vol}(S^{D-2}) = A_D r^{D-1}, \quad (4)$$

with

$$A_D = \frac{\pi^{\frac{D-1}{2}}}{(\frac{D-1}{2})!}, \quad (5)$$

$$A_D = \frac{(\frac{D-2}{2})!}{(D-1)!} 2^{D-1} \pi^{\frac{D-2}{2}},$$

for $D - 1$ being even or odd respectively. Therefore, by saturating inequality (3) introducing L as the largest distance (IR

cut-off) and c^2 as a numerical factor, the corresponding vacuum energy is, as usual, viewed as holographic dark energy:

$$\rho_{\Lambda D} = c^2 (\sqrt{\pi} M_D)^{D-3} M_D A_D^{-1} \frac{D-2}{8\Gamma(\frac{D-1}{2})} L^{-2}. \quad (6)$$

Let us make some comments here. Firstly, in general one can obtain rotational, i.e., non-spherical, or/and charged black holes [16,26] and furthermore in higher-dimensionality more exotic solutions such as black rings and black ‘‘cigars’’ are also possible [27]. Secondly, the volume on the left-hand side of inequality (3) depends on the specific bulk geometry. However, in order to maintain the simplicity and universality which lies in the basis of holographic dark energy, we remain in the aforementioned framework. Equivalently, the presence of c^2 could be regarded as a way of absorbing all extra numerical factors, thus increasing the generality of (6). Thirdly, let us specify the term ‘‘largest distance’’ which was used in the definition of L in (6). Similarly to the usual definition of 4D holographic dark energy, L could be the Hubble radius, the event horizon, the square root of the event horizon or the future event horizon [1,4–6,28]. For a flat Universe the last ansatz is the most suitable and furthermore, in this case, it is the only one that fits holographic statistical physics, namely the exclusion of those degrees of freedom of a system that will never be observed by the effective field theory [29]. In the majority of braneworld models of the literature, which are complex and not maximally isotropic in general, the definition and especially the calculation of the future event horizon is a hard or impossible task. If the bulk is finite then the application is retrieved by the use of the volume of the extra dimensions to define L . However, if the extra dimensions are arbitrary one has to make additional assumptions for the form of L . Lastly, note that a behavior proportional to L^{-2} was also found in [3] through a different approach for a special bulk case with a special action term [30] produced by quantum gravity.

A final comment must be made, concerning the sign of bulk holographic dark energy. In the original Randall–Sundrum model [19] the bulk cosmological constant should be negative in order to acquire the correct localization of low-energy gravity on the brane. Such a negativity is not a fundamental requirement and is not necessary in more complex, non-static models, especially when an induced gravity term is imposed explicitly [31]. This is the reason why we include such a term in the application of the next section, in order to be completely consistent. However, generally speaking, holographic dark energy is a simple idea of bounding the vacuum energy from above. It would be a pity if, despite this effort, one could still have a negative vacuum energy unbounded from below, because then holographic dark energy would lose its meaning. If holography is robust then one should reconsider the case of a negative bulk cosmological constant (although subspaces, such as branes, could still have negative tensions). Another possibility is to try to generalize holographic dark energy to negative values, in order to impose a negative bound. The subject is under investigation.

3. Holographic dark energy in general 5D braneworld models

In this section we apply the bulk holographic dark energy in general 5D braneworld models, which constitute the most investigated case in the field of brane cosmology. We consider an action of the form [32,33]:

$$S = \int d^5x \sqrt{-g} (M_5^3 R - \rho_{\Lambda 5}) + \int d^4x \sqrt{-\gamma} (\mathcal{L}_{\text{br}}^{\text{mat}} - V + r_c M_5^3 R_4). \quad (7)$$

In the first integral M_5 is the 5D Planck mass, $\rho_{\Lambda 5}$ is the bulk cosmological constant which is identified as the bulk holographic dark energy, and R is the curvature scalar of the 5-dimensional bulk spacetime with metric g_{AB} . In the second integral γ is the determinant of the induced 4-dimensional metric $\gamma_{\alpha\beta}$ on the brane, V is the brane tension and $\mathcal{L}_{\text{br}}^{\text{mat}}$ is an arbitrary brane matter content. Lastly, we have allowed for an induced gravity term on the brane, arising from radiative corrections, with r_c its characteristic length scale and R_4 the 4-dimensional curvature scalar [31,34].

In order to acquire the cosmological evolution on the brane we use the Gaussian normal coordinates with the following metric form [33,35]:

$$ds^2 = -m^2(\tau, y) d\tau^2 + a^2(\tau, y) d\Omega_k^2 + dy^2. \quad (8)$$

The brane is located at $y = 0$, we impose a Z_2 -symmetry around it, $m(\tau, y = 0) = 1$ and $d\Omega_k^2$ stands for the metric in a maximally symmetric 3-dimensional space with $k = -1, 0, +1$ parametrizing its spacial curvature. Although we could assume a general matter-field content [36], we consider a brane Universe containing a perfect fluid with equation of state $p = w\rho$. In this case the low-energy ($\rho \ll V$) brane cosmological evolution is governed by the following equation [31,32]:

$$H^2 + \frac{k}{a^2} = (72M_5^6 + 6r_c V M_5^3)^{-1} V \rho + \frac{V^2}{144M_5^6} + \frac{\rho_{\Lambda 5}}{12M_5^3}. \quad (9)$$

In order for Eq. (9) to coincide with the traditional Friedmann equation we have to impose:

$$V = \frac{72M_5^6}{\frac{3M_p^2}{8\pi} - 6r_c M_5^3}. \quad (10)$$

Thus, the evolution of the brane is determined by:

$$H^2 + \frac{k}{a^2} = \frac{8\pi\rho}{3M_p^2} + \frac{8\pi\rho_{\Lambda}}{3M_p^2}, \quad (11)$$

where the (effective in this higher-dimensional model) 4D dark energy is:

$$\rho_{\Lambda} \equiv \rho_{\Lambda 4} = \frac{M_p^2}{32\pi M_5^3} \rho_{\Lambda 5} + \frac{3M_p^2}{2\pi \left(\frac{M_p^2}{8\pi M_5^3} - 2r_c \right)^2}. \quad (12)$$

In the equations above $\rho_{\Lambda 5}$ is the 5D bulk holographic dark energy, which according to (6) is given by:

$$\rho_{\Lambda 5} = c^2 \frac{3}{4\pi} M_5^3 L^{-2}. \quad (13)$$

The holographic nature of $\rho_{\Lambda 5}$ is the cause of the holographic nature of ρ_{Λ} . Having in mind that the 5D Planck mass M_5 is related to the standard 4D M_p through $M_5^3 = M_p^2/L_5$ (according to (2)), with L_5 the volume (size) of the extra dimension, we finally acquire the following form for the effective 4D holographic dark energy:

$$\rho_{\Lambda} = 3c^2 \frac{1}{128\pi^2} M_p^2 L^{-2} + \frac{3M_p^2}{2\pi \left(\frac{L_5}{8\pi} - 2r_c \right)^2}. \quad (14)$$

The first term in relation (14) is a usual holographic term, with a suitable ansatz for the cosmological length L . The presence of the size L_5 of the extra dimension in the second term accounts for the effects of the restrictions to the holographic principle by the bulk boundaries. It is obvious that in general it could radically affect the calculations. In this work we are interested in investigating the restored holographic dark energy, without bothering about bulk boundaries, and this is the justification of the single brane formulation of this section. Therefore, in the following we assume that L_5 is arbitrary large, much larger than any possible L definition and any other length scale, thus we omit the second term in relation (14). Its effect will be considered in a separate publication [24].

The final step before the insertion of the 4D effective holographic dark energy to Friedmann equation (11) is an assumption for the cosmological length L . As we have already mentioned, in this work we are interested in presenting the general bulk holographic dark energy. For the simple application of this section we will consider a flat Universe, in order to safely use the future event horizon to define L , without entering into the relevant discussion of the literature concerning the IR cut-off in non-flat cases [1,4–6,28]. However, we stress that bulk holographic dark energy holds in these cases too, with a suitable L definition.

The analytical calculation of the future event horizon for the complete 5D spacetime with metric (8) and dynamics governed by action (7) is impossible. In this anisotropic model we could alternatively use the 4D future event horizon, without losing the qualitative behavior of the observables. Fortunately, the calculations reveal that also the quantitative results agree with observations and coincide with those obtained within the traditional holographic dark energy [1–8].

The 4D future event horizon for the FRW brane of our model is as usual:

$$R_h = a \int_a^{\infty} \frac{da'}{H a'^2}. \quad (15)$$

Inserting this expression to (14) we find:

$$\rho_{\Lambda} = 3c^2 \frac{1}{128\pi^2} M_p^2 R_h^{-2}. \quad (16)$$

Substituting (16) to Friedmann equation (11), in the flat-Universe case we obtain:

$$\int_a^\infty \frac{da'}{Ha'^2} = \frac{c}{4\sqrt{\pi}} \left(H^2 a^2 - \frac{8\pi\rho_0}{3M_p^2 a} \right)^{-1/2}, \quad (17)$$

where we have also introduced the known matter density dependence on a , namely $\rho = \rho_0 a^{-3}$, with ρ_0 its present value. The above integral equation determines the brane evolution and it incorporates the full 5D spacetime effects in low energy limit, including the bulk holographic dark energy. Seen as 4D equation in conventional holographic dark energy context it has been investigated in [1,7].

In the case of a flat brane-Universe, one finds the H -behavior, then that of R_h and finally, through (16), that of ρ_Λ itself. Inserting the known relation of ρ_Λ evolution, i.e., $\rho_\Lambda \sim a^{-3(1+w_\Lambda)}$, and following the steps of [1] we find:

$$w_\Lambda = -\frac{1}{3} - \frac{8\sqrt{\pi}}{3c} \sqrt{\Omega_\Lambda^0} + \frac{2\sqrt{\pi}}{3c} (1 - \Omega_\Lambda^0) \left(\sqrt{\Omega_\Lambda^0} + \frac{8\sqrt{\pi}}{c} \Omega_\Lambda^0 \right) z, \quad (18)$$

where we used $\ln a = -\ln(1+z) \simeq -z$, and Ω_Λ^0 is the present value of $\Omega_\Lambda = 8\pi\rho_\Lambda/(3M_p^2 H^2)$. Therefore, according to the value of the constant c , one can obtain a 4D holographic dark energy behaving as phantom [37], quintessence or quintom [38], i.e., crossing the phantom divide $w = -1$ [34,39] during the evolution. Furthermore, one can use observational results in order to estimate the bounds of the constant [40,41], having in mind that the constants of every work differ. Finally, one can explore w_Λ behavior in the context of Chaplygin gas or tachyon holographic models [8].

4. Discussion–conclusions

In this work we present holographic dark energy, restored to its natural foundations, and for a consistency test we apply it to a general braneworld model well studied in the literature. Our main motivation is to match the successes of brane cosmology in both theoretical and phenomenological-observational levels, with the successful, simple, and inspired by first principles, notion of holographic dark energy in conventional 4D cosmology. Our basic argument for this generalization is that in a higher-dimensional spacetime, it is the bulk space which is the natural framework for the cosmological application, concerning dark energy, of holographic principle, and not the lower-dimensional brane-Universe. This is obvious since it is the maximally-dimensional subspace that determines the properties of quantum-field or gravitational theory, and this holds even if we consider brane cosmology as an intermediate limit of an even higher-dimensional fundamental theory of nature. To be more specific we recall here that the underlying idea of holographic dark energy is that one cannot have more energy in a volume than the mass of a black hole of the same size. In braneworld models, where the spacetime dimension is more than 4, black holes will in general be D -dimensional [15,16], no

matter what their 4D effective (mirage) effects could be. Therefore, although holography itself can be applied in arbitrary subspaces, its specific cosmological application which eventually gives rise to holographic dark energy holds only for the main space, i.e., the bulk. Subsequently, this bulk holographic dark energy will bring forth an effective 4D dark energy with “inherited” holographic nature, and this one will be present in the Friedmann equation of the brane. Completing these self-consistent cogitations, the brane Friedmann equation should arise from the full dynamics, too.

The generalized holographic dark energy can lead to either radically different or exactly identical 4D behavior, comparing to that obtained in conventional 4D literature [1–8]. In Section 2 we applied this bulk holographic dark energy in a general braneworld model, with an induced gravity term and a perfect fluid on the brane. In this case, the low energy evolution of the single brane leads to effective 4D holographic dark energy behaving either as a phantom, quintessence or quintom, identically to conventional 4D calculations. However, as we have mentioned, it is obvious that the interpretation and explanation of this behavior is fundamentally different. The reason for the coinciding results in this specific example is that the extra dimension is arbitrary large, imposing no restrictions on the application of holographic dark energy. The only necessary assumption of our calculation is the use of the 4D future event horizon as the cosmological length of holographic behavior. This can be justified since in this anisotropic model the 5D event horizon cannot be found analytically, and furthermore, under the requirement of the recovery of conventional evolution on the brane, in agreement with observations, the 5D future event horizon cannot be significantly different than its 4D counterpart. Note however, that in other braneworld models, where the whole space is FRW, dS or AdS [3,22], the D -dimensional future event horizon can be easily calculated, and this would lead to an exact application of the bulk holographic dark energy.

From this discussion it becomes evident that a finite bulk would radically affect the 4D behavior of dark energy. For example, a two brane model with constant interbrane distance would lead to a constant dark energy, and the physical interpretation is that it is impossible to have an arbitrary large bulk black hole in this case. However, one could built a two-brane model where the branes moving apart. The subject is under investigation [24].

In this work we present a restored version of holographic dark energy in brane cosmology, and we argue that it has to be considered in the bulk and not in the brane. As an example we use a single-brane model and we show that, in the low energy limit, the 4D effective holographic dark energy behavior coincides with that predicted by conventional 4D calculation. However, its behavior in more complicated braneworld models can be significantly different.

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